Phenomenological Evidence for Gluon Depletion 
in \(pA\) Collisions

Rudolph C. Hwa\(^1\), Ján Pišút\(^2\) and Neva Pišútová\(^2\)

\(^1\)Institute of Theoretical Science and Department of Physics 
University of Oregon, Eugene, OR 97403-5203, USA

\(^2\)Department of Physics, Comenius University, SK-84215, Bratislava, Slovakia

Abstract

The data of \(J/\psi\) suppression at large \(x_F\) in \(pA\) collisions are used to infer the existence of gluon depletion as the projectile proton traverses the nucleus. The modification of the gluon distribution is studied by use of a convolution equation whose non-perturbative splitting function is determined phenomenologically. The depletion factor at \(x_1 = 0.8\) is found to be about 10% at \(A = 100\).

In two previous papers [1, 2] we have examined the possibility that the phenomenon of \(J/\psi\) suppression in heavy-ion collisions could, to a large extent, be caused by the depletion of gluons before the hard subprocess of \(c\bar{c}\) production. Apart from showing that such an effect cannot be excluded, no definitive conclusion can be made on the magnitude of the effect. In this paper we focus on \(p-A\) collisions and show that the data [1] on \(\alpha(x_F)\) can be used to infer that gluon depletion in the projectile proton is not negligible.

The \(J/\psi\) suppression problem has been well reviewed in recent years for nucleus-nucleus collisions [4]-[6]. For \(p-A\) collisions the small \(x_F\) region has been studied in [7] by considering the time- and energy-dependences of the effective absorption cross section. In [8] the \(x_F\) range is extended to higher values in the framework of a classical model of charmonium absorption. In neither of these papers are the partons' degrees of freedom considered explicitly. Since \(c\bar{c}\) pairs are produced by gluon annihilation, we pay particular attention to the evolution of the gluon distribution of the projectile as it traverses the nucleus. The approximate absence of dilepton suppression and the consequent implication that the quark distribution is nearly unaltered by the nuclear medium lead some to expect that the gluon distribution would be unaltered also. However, such a view is based on the validity of DGLAP evolution of the parton distribution functions [9]. We adopt the reasonable alternative view that the evolution in a nucleus is different from that of pQCD at high \(Q^2\); indeed, we shall let the data guide us in determining the proper dynamics of the low-\(Q^2\) non-perturbative process.

The Fermilab E866 experiment measured the \(J/\psi\) suppression in \(p-A\) collisions at 800 GeV/c with a wide coverage of \(x_F\) [3]. The result is given in terms of \(\alpha(x_F)\), which is defined by the formula

\[
R(x_F, A) = \frac{\sigma_A(x_F)}{A \sigma_N(x_F)} = A^{\alpha(x_F)-1} , \tag{1}
\]
where $\sigma_{N,A}$ is the cross section for $J/\psi$ production by a proton on a nucleon ($N$) or on a nucleus ($A$). In [3] a simple parametrization of $\alpha(x_F)$ for $J/\psi$ production is given:

$$\alpha(x_F) = 0.952(1 + 0.023x_F - 0.397x_F^2)$$

(2)

for $-0.1 < x_F < 0.9$. It is our aim here to explore the implication of Eq. (2) on the evolution of the gluon distribution.

Since the semihard subprocess of $g + g \rightarrow c + \bar{c}$ is common for $p-N$ and $p-A$ collisions, they cancel in the ratio $R(x_F, A)$ so the $x_F$ dependence can come from three sources: (a) the ratio of the gluon distribution in the projectile passing through a nucleus to that in a free proton, $G(x_F, A)$, (b) nuclear shadowing of gluons in the target, $N(x_F, A)$, and (c) hadronic absorption of the $c\bar{c}$ states after the semihard subprocess, $H(x_F, A)$. Putting them together, we have

$$R(x_F, A) = G(x_F, A)N(x_F, A)H(x_F, A).$$

(3)

$G(x_F, A)$ and $N(x_F, A)$ are ignored in [7, 8]. Since $x_F < 0.25$ in [7], there is not much dependence on $x_F$ to be ascribed to $H(x_F, A)$, but in [8], where the full range of $x_F$ is considered, $H(x_F, A)$ is forced to carry the entire $x_F$-dependence by a fitting procedure, resulting in an unreasonably short octet lifetime. Our approach by including $G(x_F, A)$ and $N(x_F, A)$ in Eq.(3) is therefore complementary to the work of [7, 8].

The nuclear shadowing problem has been studied in detail by Eskola et al. [10, 11], using the deep inelastic scattering data of nuclear targets at high $Q^2$. On the basis of DGLAP evolution they can determine the parton distributions at any $Q^2 > 2.25$ GeV$^2$. The results are given in terms of numerical parametrizations (called EKS98 [11]) of the ratio $N^i_A(x, Q^2) = f_{i/A}(x, Q^2)/f_i(x, Q^2)$, where $f_i$ is the parton distribution of flavor $i$ in the free proton and $f_{i/A}$ is that in a proton of a nucleus $A$. We shall be interested in the ratio for the gluon distributions only at $Q^2 = 10$ GeV$^2$, corresponding to $c\bar{c}$ production, and denote it by $N(x, A)$. From the numerical output of EKS98 we find that a simple formula can provide a good fit to within 2% error in the range $40 < A < 240$ and $0.01 < x < 0.12$; it is

$$N(x, A) = A^\beta(x),$$

(4)

where

$$\beta(\xi(x)) = \xi (0.0284 + 0.0008\xi - 0.0041\xi^2),$$

(5)

with $\xi = 3.912 + \ln x$. Thus the $A$ dependence is minimal at $\xi = 0$, corresponding to $x = 0.02$.

The variable $x$ in Eq.(4) is the gluon momentum fraction in a nucleon in the nucleus, usually referred to as $x_2$. Both $x_F$ in Eq.(1) and $x_2$ in Eq.(4) are to be converted to the $x_1$ variable for the projectile nucleon, using

$$x_F = x_1 - x_2, \quad x_1x_2 = \tau \equiv M_{J/\psi}^2/s,$$

(6)

so that a part of Eq. (3) can be rewritten as

$$R(x_F, A)/N(x_2, A) = A^{\alpha(x_F(\tau)) - \beta(x_2(\tau))}. $$

(7)
In our approach we treat \( H(x_F, A) \) as having negligible dependence on \( x_F \). This is a point of view that is opposite to those in [7, 8], which ignore the effect of \( N(x_2, A) \) and \( G(x_1, A) \). Since both of those approaches result in conclusions that suggest the existence of unaccounted mechanisms responsible for the enhanced suppression in \( R(x_F, A) \) at large \( x_F \), our approach is the complementary one that places the emphasis on the gluon depletion mechanism. Of course, if the \( x_F \) dependence of \( H(x_F, A) \) were independently known, its incorporation in our analysis is straightforward. For us here, we identify the \( x_1 \) dependence of \( G(x_1, A) \) in Eq. (3) with that in Eq. (7), which is completely known, and proceed to the study of the phenomenological implication on gluon depletion.

In the spirit of DGLAP evolution, even though the effect of a nuclear target on the projectile gluon distribution is highly non-perturbative, we now propose an evolution equation on the gluon distribution \( g(x, z) \), where \( z \) is the path length in a nucleus. For the change of \( g(x, z) \), as the gluon traverses a distance \( dz \) in the nucleus, we write

\[
\frac{d}{dz} g(x, z) = \int_x^1 \frac{dx'}{x'} g(x', z) Q\left(\frac{x}{x'}\right),
\]

where \( Q(x/x') \) describes the gain and loss of gluons in \( dz \), but unlike the splitting function in pQCD, it cannot be calculated in perturbation theory. Equation (8) is similar to the nucleonic evolution equation proposed in [12], except that this is now at the parton level. Instead of guessing the form of \( Q(x/x') \), which is unknown, we shall use Eq. (7) to determine it phenomenologically.

To that end, we first define the moments of \( g(x, z) \) by

\[
g_n(z) = \int_0^1 dx x^{n-2} g(x, z).
\]

Taking the moments of Eq. (8) then yields

\[
dg_n(z)/dz = g_n(z) Q_n,
\]

where \( Q_n = \int_0^1 dy y^{n-2} Q(y) \). It then follows that

\[
g_n(z) = g_n(0) e^{zQ_n},
\]

whose exponential form suggests \( Q_n < 0 \) for the physical process of depletion. The gluon depletion function \( D(y, z) \) is defined by

\[
g(x, z) = \int_x^1 \frac{dx'}{x'} g(x', 0) D\left(\frac{x}{x'}, z\right),
\]

where \( g(x', 0) \) is the gluon distribution in a free nucleon. From Eq. (12) we have \( g_n(z) = g_n(0) D_n(z) \), where \( D_n(z) \) is the moment of \( D(y, z) \). Comparison with Eq. (11) then gives

\[
D_n(z) = e^{zQ_n}.
\]

To relate this result to \( R(x_F, A) \), we first note that \( G(x_F, A) \) in Eq. (3) is, by definition, \( G(x_F, A) = g(x_1, A)/g(x_1, 0) \), where \( x_F \) is expressed in terms of \( x_1 \). It then follows from Eq. (3) that

\[
J(x_1, A) \equiv g(x_1, 0) R(x_F(x_1), A)/N(x_2(x_1), A) = g(x_1, A) H(A).
\]
In relating \( A \) to the average path length \( L \) of the projectile \( p \) through the nucleus, we use \( L = 3R_A/2 = 1.8A^{1/3}\text{fm} \). We then set \( z = L/2 \) for the average distance traversed at the point of \( \bar{c}c \) production. Thus when referring to the last expression of Eq.(14), we write \( J(x_1, A) = g(x_1, z(A)) H(z(A)) \), where \( g(x_1, z) \) is to be identified with that in Eq.(12). Note that the \( A \) dependence of the middle expression in Eq.(14) is, on account of Eq.(7), in terms of \( \ln A \), whereas that of the last expression is in terms of \( z \), or \( A^{1/3} \). Since it is known that \( \ln A \approx A^{1/3} \) for \( 60 < A < 240 \), we shall consider the consequences of Eq.(14) only for \( A \) in that range. We suggest that a revised form of presenting the data, different that in Eq.(1), should be tried in the future.

Taking the moments of \( J(x_1, A) \) and using Eq.(11), we get

\[
\ln J_n(A) - \ln g_n(0) = zQ_n + \ln H(z). \tag{15}
\]

To determine \( Q_n \), it is necessary to use as an input the gluon distribution \( g(x_1, 0) \) in a free proton at \( Q^2 = 10\text{GeV}^2 \). We adopt the simple canonical form

\[
g(x_1, 0) = g_0(1 - x_1)^5, \tag{16}
\]

where the constant \( g_0 \) is cancelled in Eq.(15) due to the definition of \( J(x_1, A) \). In our calculation we set \( g_0 = 1 \). Indeed, the accuracy of \( g(x_1, 0) \) is unimportant, since it enters Eqs.(14) and (15) in ways that render the result insensitive to its precise form. On the basis of Eqs.(7) and (16), \( J(x_1, A) \) is therefore known. The LHS of Eq.(15) can then be computed except for a caveat. To calculate the moments of \( J(x_1, A) \), it is necessary to compute \( \int_0^1 dx_1 x_1^{n-2} J(x_1, A) \). However, \( x_1 \) cannot be less than \( \tau \) in order to keep \( x_2 \leq 1 \) [see Eq.(6)]. Furthermore, Eq.(7) does not provide reliable information on \( J(x_1, A) \) at small \( x_1 \), since the parametrizations of \( \alpha(x_F) \) and \( \beta(x_2) \) are for the variables in ranges that exclude the \( x_1 \to \tau \) limit. Fortunately, that part of the integration in \( x_1 \) can be suppressed by considering \( n \geq 3 \). The part of the integration in the interval \( 0 < x_1 < \tau \) amounts to only about 2% contribution even at \( n = 2 \) (if naive extrapolation is used), so its inaccuracy will be neglected hereafter. Physically, it is the data at high \( x_F \) that we emphasize in our analysis, and that corresponds to the high-\( n \) moments of \( J(x_1, A) \).

For convenience, let us denote the LHS of Eq.(15) by \( K_n(z) \), i.e., \( K_n(z) \equiv \ln[J_n(z(A))/g_n(0)] \). For sample cases of \( A = 100 \) and 200, they are shown as discrete points in Fig. 1 for \( 3 \leq n \leq 10 \). Instead of performing an inverse Mellin transform on \( K_n(z) \), our procedure is to fit \( K_n(z) \) by a simple formula that can yield \( Q(y) \) by inspection. The fitted curves shown by the solid lines in Fig. 1 are obtained by use of the formula

\[
K_n = -k_0 - \frac{k_1}{n} + \frac{k_2}{n+2}. \tag{17}
\]

Using \( k_i \) and \( k'_i \) to denote the values for the cases \( A = 100 \) and 200, respectively, we have

\[
\begin{align*}
  k_0 &= 1.202, & k_1 &= 4.329, & k_2 &= 11.363 \\
  k'_0 &= 1.375, & k'_1 &= 4.961, & k'_2 &= 13.008
\end{align*}
\]

Because of Eq.(15), the \( n \) dependence of \( K_n \) prescribes the \( n \) dependence of \( Q_n \). Let us therefore write

\[
Q_n = -q_0 - \frac{q_1}{n} + \frac{q_2}{n+2}. \tag{18}
\]
Since Eq.(15) is to be used only for \( A > 60 \), we evaluate it at \( A = 100 \) and 200, and take the difference. Denoting \( z \) by \( z_1 \) and \( z_2 \), respectively, for the two \( A \) values, and with \( \Delta k_i = k_i' - k_i, \Delta z = z_2 - z_1 \), we have

\[
\Delta k_0 = q_0 \Delta z - \ln \frac{H(z_2)}{H(z_1)}, \quad \Delta k_1 = q_1 \Delta z, \quad \Delta k_2 = q_2 \Delta z. \tag{19}
\]

For the hadron absorption factor \( H(z) \) we write it in the canonical exponential form [13], \( H(z) = \exp(-\rho \sigma z) \), where \( \rho^{-1} = (4/3)\pi(1.2)^3 \text{fm}^3, z = 0.9A^{1/3} \text{fm} \), and \( \sigma \) is the absorption cross section. Putting these in Eq.(19), we get (with \( \Delta z = 1.086 \text{fm} \))

\[
q_0 + \rho \sigma = 0.159, \quad q_1 = 0.582, \quad q_2 = 1.515, \tag{20}
\]
in units of \( \text{fm}^{-1} \).

There is a reason why \( q_0 \) and \( \rho \sigma \) enter Eq.(20) as a sum. To appreciate the physics involved, we first note that Eq.(18) implies directly

\[
Q(y) = -q_0 \delta(1 - y) - q_1 y + q_2 y^3. \tag{21}
\]

The first two terms on the RHS above are the loss terms (i.e., gluon depletion), while the last term represents gain (i.e., gluon regeneration). If \( Q(y) \) consists of only the first term, then using it in Eq.(8) gives \( dg(x, z)/dz = -q_0 g(x, z) \), whose solution is of the same exponential form as \( H(z) \). This result is consistent with that in [1], where it is found that simple gluon depletion effect on \( J/\psi \) suppression is of the same nature as the absorption effect. If both are present, the exponents appear as a sum, as in Eq.(20). Our \( Q(y) \) is, however, more complicated. The \(-q_1 y\) term gives rise to depletion that depends on the shape of \( g(x, z) \), while the \( q_2 y^3 \) term generates new gluons at \( x \) from all the gluons at \( x' > x \). The interplay among these terms makes possible a determination of a bound on \( \rho \sigma \).

Recall that the invariant gluon distribution \( g(x, 0) \) is defined in such a way that \( \int_0^1 dx \, g(x, 0) \) is the fraction of a free nucleon’s momentum carried by all gluons. Since gluon depletion in a nucleus implies the decrease of that fraction with \( z \), we require \( \frac{d}{dz} \int_0^1 dx \, g(x, z) < 0 \). From Eq.(8) we then get

\[
\int_0^1 dx \frac{d}{dz} g(x, z) = \int_0^1 dx' g(x', z) \int_0^1 dy Q(y) < 0, \tag{22}
\]

from which follows the negativity condition \( \int_0^1 dy Q(y) < 0 \). Applying this to Eq.(21) gives \( -q_0 - q_1/2 + q_2/4 < 0 \). This corresponds to \( Q_2 < 0 \). We now can return to Eq.(20) and obtain the bound \( \rho \sigma < q_0 + \rho \sigma + q_1/2 - q_2/4 = 0.0718 \text{fm}^{-1} \). With \( \rho = 0.138 \text{fm}^{-3} \) we get \( \sigma < 5.2 \text{mb} \). This result, though satisfactory, is not reliable because, as we have stated earlier, the \( n = 2 \) moment is not accurate, owing to the uncertainty of the integrand near \( x_1 \approx 0 \). Treating \( n \) as a continuous variable in Eq.(18), we find that the maximum of \( Q_n \) occurs at \( n = 3.25 \), from which we can determine the values of \( q_0 \) by requiring \( Q_n = 0 \) for that \( n \), a condition to ensure that \( Q_n(z) \leq 0 \) for all \( n \). The result is \( q_0 = 0.11 \text{fm}^{-1} \). Using this in Eq.(20), we obtain \( \rho \sigma < 0.049 \text{fm}^{-1} \), which implies

\[
\sigma < 3.6 \text{mb}. \tag{23}
\]

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This is a very reasonable bound on the absorption cross section, being roughly half of what is needed for pure absorption [14].

Since the $n$ dependence of $Q_n$ is forced by the $x_1$ dependence of $J(x_1, A)$ that has its origin in the $x_1$ dependence of Eq.(7), the data of Leitch et al. [3] on the $x_F$ dependence of $R(x_F, A)$ can be accommodated in our approach only by the presence of some gluon depletion and regeneration — consistent with the failure of other approaches without such dynamical effect on the gluons [7, 8].

Since it is not easy to see directly from $Q_n$ or $Q(y)$ the magnitude of the effect, we can calculate $g(x_1, z)$, not from Eq.(12), but by fitting the calculated $g_n(z)$ in Eq.(11) with $q_0 = 0.11$ fm$^{-1}$ in Eq.(20), using the formula $g_n(z) = a_1 B(n - 1, 6) + a_2 B(n - 1, 7)$. Then the result yields directly

$$g(x_1, z) = a_1(1 - x_1)^5 + a_2(1 - x_1)^6,$$ (24)

where $a_i$ depends on $z$. For $A = 100$ (200), i.e., $z = z_1$ ($z_2$), we have $a_1 = 0.875$ (0.844) and $a_2 = 0.161$ (0.202), for $g_0 = 1$ in Eq.(16). The result for $G(x_1, z) = g(x_1, z)/g(x_1, 0)$ is then a linear function of $x_1$

$$G(x_1, z) = a_1 + a_2 - a_2 x_1,$$ (25)

which is shown in Fig. 2. It is now evident that gluon depletion suppresses the gluon distribution at medium and high $x_1$, but the unavoidable gluon regeneration enhances the distribution at low $x_1$. The cross-over occurs at $x_1 = 1 - (1 - a_1)/a_2 \approx 0.226$.

An unexpected revelation from this study is that at $x_1 \simeq 0.15$ the gluon distribution is slightly enhanced. Since the $J/\psi$ data measured at CERN-SPS are for $\sqrt{s} \simeq 20$ GeV and thus $x_1 \simeq 0.15$ for $x_F \simeq 0$, the suppression observed in the central region cannot be due to the linear gluon depletion effect discussed in [1]. However, the nonlinear depletion effect for $AA$ collisions discussed in [2] is of a different nature and cannot be ruled out by this result.

Because of the significant enhancement of $g(x_1, z)$ at very small $x_1$, our result suggests that $J/\psi$ suppression at RHIC may not be as severe as one might conclude by a simple extrapolation from the SPS data on the assumption of no QGP formation.

Our analysis has been based on the assumption that the absorption effect is independent of $x_F$. If and when that $x_F$ dependence can be determined independently, the effect can easily be incorporated in our analysis to modify our numerical result — but not our formalism. Until then, our study shows that the $J/\psi$ suppression observed at large $x_F$ in $pA$ collisions [3] strongly suggests the presence of gluon depletion in the beam proton at high $x_1$. The significance of this finding goes beyond the $J/\psi$ suppression problem itself, since it would revise the conventional thinking concerning the role of partons in nuclear collisions.

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References


Figure Captions

Fig. 1 $K_n$ vs $n$ for $A=100$ and 200. The smooth curves are fitted results using Eq.(17).

Fig. 2 $G(x_1,z)$ vs $x_1$ showing the effects of gluon depletion and regeneration for $A=100$ and 200.