SIMULATIONS OF ACCRETION FLOWS CROSSING THE LAST STABLE ORBIT

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ABSTRACT

We use three dimensional magnetohydrodynamic simulations, in a pseudo-Newtonian potential, to study geometrically thin accretion disc flows crossing $r_{ms}$, the marginally stable circular orbit around black holes. We concentrate on vertically unstratified and isothermal disk models, but also consider a model that includes stratification. In all cases, we find that the sonic point lies just inside $r_{ms}$, with modest magnetic field amplification observed inside the last stable orbit. The gradient of the specific angular momentum of the flow, $(dl/dr)$, is close to zero within $r_{ms}$, despite the presence of continuing magnetic stress in the plunging region. These results are in general agreement with expectations based on traditional disk models, but differ from recent results obtained from simulations of geometrically thick disks. For thin disks, we find that the use of a zero-torque boundary condition, at the last stable orbit, provides a reasonable approximation to the numerical results.

Subject headings: accretion, accretion disks – black hole physics – MHD – stars: neutron – hydrodynamics – instabilities

1. INTRODUCTION

The existence of a marginally stable orbit is a distinctive feature of accretion flows onto black holes. Within $r_{ms}$, stable circular orbits do not exist, and gas inevitably plunges on a short timescale into the hole. The location of the marginally stable orbit, which lies at $r_{ms} = 6GM/c^2$ for a non-rotating hole, varies strongly with the spin parameter $a$ for Kerr black holes. This variation, together with the assumption of a large change in the emission properties of the gas interior and exterior to $r_{ms}$, is central to all attempts to measure $a$ from observable quantities for both galactic (Zhang, Cui & Chen 1997) and supermassive black holes (Iwasawa et al. 1996; Dabrowski et al. 1997; Bromley, Chen & Miller 1997). Neutron stars, if they are sufficiently compact, could also possess a last stable orbit, though we will not consider this possibility further here.

The presence of gas on plunging orbits within $r_{ms}$ can have observable consequences. It potentially modifies the profile of the relativistic iron Kα line that has been observed in the X-ray spectra of some Seyfert galaxies (Tanaka et al. 1995; Reynolds & Begelman 1997; Young, Ross & Fabian 1998), and may create detectable absorption signatures (Nandra et al. 1999). However, it has generally been believed that gas within $r_{ms}$ does not have any dynamical importance, since the infall rapidly becomes supersonic a short distance inside the marginally stable orbit. Whatever complexities may develop in the flow subsequently are then causally disconnected from the disk at larger radius. This line of reasoning suggests that for modelling the disk (by which we mean the region of the flow outside the last stable orbit), it suffices to impose a zero-torque boundary condition at $r_{ms}$ and ignore the interior region altogether. For the details of disk models constructed in this manner, we refer the reader to Abramowicz & Kato (1989), and references therein.

This simple hydrodynamic picture would be oversimplified if magnetic fields inside $r_{ms}$ were strong enough to maintain a connection between the plunging region and the disk. A recent analysis by Krolik (1999) showed that if magnetic fields, generated in the disk, remain frozen into the gas as it inspirals within $r_{ms}$, they grow rapidly to become dynamically important. Related ideas have also been proposed by Gammie (1999). In addition to altering the zero-torque boundary condition for the disk, the presence of extremely strong fields in the plunging region could have several potentially observable consequences (Agol & Krolik 2000). For example, the radiative efficiency of the disk, $\epsilon = L/Me^2$, could be substantially increased. The analysis of Krolik (1999), however, depends upon a split of the accretion flow into two distinct regions, a disk at $r > r_{ms}$ in which magnetic dynamo processes (Balbus & Hawley 1991; Brandenburg et al. 1995; Stone et al. 1996; for a review see e.g. Hawley & Balbus 1999) maintain a turbulent state, and a plunging region at $r < r_{ms}$ in which it is assumed that magnetic flux is frozen into the fluid. This split is clearly an approximation, because the same turbulent processes that act to generate and destroy magnetic fields in the disk itself will continue to operate until some finite distance inside the last stable orbit (Paczynski 2000). This may alter the conclusion that rapid field growth is inevitable. The analytic analysis further assumes that the flow is axisymmetric and steady-state. This is also only approximately the case in a turbulent accretion flow, and needs to be tested via numerical simulations.

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In this paper, we employ three dimensional magnetohydrodynamic (MHD) simulations to study the properties of the accretion flow as it crosses $r_{\text{ms}}$. A pseudo-Newtonian potential appropriate for a non-rotating black hole (Paczynski & Wiita 1980) is used to mimic the relativistic effect of a last stable orbit within a non-relativistic MHD code. Within this approximation, simulating the evolution of magnetic fields within the plunging region is, if anything, expected to be easier than following the development of turbulence in the disk at larger radii. Within the disk, orbits are stable, and thus in the absence of turbulence a field loop in the plane of the disk will always be sheared out in azimuth until numerical reconnection sets in at the grid scale. Once into the plunging region there is only a finite (but large) amount of shear before the infalling gas reaches the black hole.

The accretion flow within $r_{\text{ms}}$ has already been studied for thick accretion flows by Hawley (2000), who presented global MHD simulations of accretion tori with a geometry similar to that of popular ADAF (Narayan & Yi 1994) and ADIOS (Blandford & Begelman 1999) models for radiatively inefficient flows. This geometry is thought to be appropriate at relatively low accretion rates for both supermassive and stellar mass black hole systems (for observational support see, e.g., Gilfanov, Churazov & Revnivtsev 1999). Significant magnetic stresses within $r_{\text{ms}}$ were obtained in these simulations, and in subsequent higher resolution simulations of the same geometry (Hawley & Krolik 2000). We address here the complementary case of geometrically thin disks, which are the expected mode of accretion at high $\dot{M}$ in both Active Galactic Nuclei, and in stellar mass Galactic black hole sources. Although some of the important properties of thick disks – such as the effective Shakura-Sunyaev (1973) angular momentum transport parameter $\alpha$ – seem to be surprisingly similar to thin disks (Hawley 2000), there is no general reason for any similarity. Indeed, the conditions at $r_{\text{ms}}$ are expected to differ for geometrically thin and thick disks (Popham & Gammie 1998; Paczynski 2000).

2. NUMERICAL METHODS

2.1. The ZEUS hydrodynamics code

We simulate the accretion flow using the ZEUS code (Stone & Norman 1992a, 1992b; Clarke, Norman & Fielder 1994; Norman 2000) to solve the equations of ideal magnetohydrodynamics. ZEUS is an explicit Eulerian finite difference code, formally of second order accuracy, which uses an artificial viscosity to reproduce shocks. Advantages of the code for accretion disk applications include a flexible choice of gridding and co-ordinate systems, and algorithms (Norman, Wilson & Barton 1980) that minimise spurious diffusion of angular momentum relative to mass.

The specific mechanism of magnetic field amplification proposed by Krolik (1999) is independent of the existence of vertical stratification in the disk. For our initial simulations we therefore adopt the computationally easiest option, and consider a vertically unstratified disk (i.e. one where the vertical component of gravity is artificially set to zero), and an isothermal fluid, where the pressure $P$ is given by,

$$ P = \rho c_s^2, $$

where $c_s$ is the sound speed and $\rho$ is the density. For our standard model we choose $c_s$ such that the ratio of the sound speed to the circular orbital velocity at the last stable orbit, $c_s/v_{\text{circ}} = 0.08$. Since the relative disk scale height, $h/r \approx c_s/v_o$, this implies that the disk is ‘thin’ in the sense that radial pressure forces, which scale as $(h/r)^2$, are small compared to the gravitational force. A model with smaller sound speed was also investigated. For better comparison with the thick disk simulations of Hawley (2000), we also ran a variant including the effect of vertical stratification.

Standard choices for the Courant number, $C_0 = 0.5$, and coefficient of artificial viscosity, $C_2 = 2.0$, were used throughout. Second order advection was used for all quantities. The only code scaling which needs to be mentioned is that for time. We adopt units in which the orbital period at $r_{\text{ms}}$, $P_{\text{ms}} = 7.7$.

2.2. Paczynski-Wiita potential

ZEUS is a Newtonian fluid dynamics code which does not model any of the effects of special or general relativity. Within this framework, we use a pseudo-Newtonian gravitational potential (Paczynski & Wiita 1980),

$$ \psi = -\frac{G M}{r - r_g} $$

where $r_g = 2GM/c^2$, to model what is expected to be the dominant relativistic effect around a non-rotating black hole - the existence of an innermost stable orbit at $r_{\text{ms}} = 6GM/c^2$.

2.3. Initial and boundary conditions

We simulate a wedge of the disk extending $30^\circ$ in azimuth in cylindrical polar geometry, $(r, \phi, z)$. The boundary conditions are periodic in $\phi$, and set to outflow at both $r_{\text{min}}$ and $r_{\text{max}}$. Outflow boundary conditions are implemented in ZEUS by setting zero gradients for all flow variables at the boundary. The use of outflow boundary condition is desirable to ensure that the simulations do indeed model magnetic instabilities, rather than purely hydrodynamic instabilities to which relatively narrow annuli, with reflecting boundaries, are known to be susceptible (e.g. Narayan & Goodman 1989). We use a uniform grid in both $\phi$ and $z$. In the radial direction, a uniform grid interior to $r_{\text{ms}}$ is matched smoothly onto a grid at larger radii for which $\Delta r_i = k\Delta r_i$, with $k > 1$ a constant. This concentrates resolution in the region of greatest interest near, and within, the marginally stable orbit.

The initial conditions for the calculation are intended as thin disk analogs of the toroidal configuration used by Hawley (2000). We take a gaussian surface density profile, centered at $r = 2r_{\text{ms}}$, with an azimuthal velocity appropriate to Keplerian rotation. We relax this profile in a preliminary non-magnetic one dimensional calculation, to ensure that it is an accurate numerical equilibrium state. The resulting profile of $\rho$ and $v_\phi$ is then mapped into three dimensions, and a seed magnetic field added. To evaluate the sensitivity of the results to the boundary conditions, three unstratified simulations were run, with different choices of seed field and associated boundary conditions in $z$. Two simulations were run with an initially vertical field imposed at all radii where the surface density exceeded a small threshold value, with periodic boundary conditions.
imposed in the $z$ direction. For the first simulation (the ‘standard’ run) the radial and vertical boundaries were at $r_{\text{min}} = 1$, $r_{\text{max}} = 5$, and $z = \pm 0.3$, in code units where $r_{\text{ms}} = 1.5$. The initial field strength was such that the ratio of gas pressure to magnetic pressure, $\beta_{\gamma} = 5000$. The numerical resolution was $n_r = 150$, $n_z = 32$ and $n_{\phi} = 40$ grid points, with 30 of the radial grid points interior to $r_{\text{ms}}$. The second run simulated a cooler disk (with a sound speed half the previous value), in a larger computational volume, bounded by $1 < r < 10$ and $z = \pm 0.25$. This run used $n_r = 210$, $n_z = 32$ and $n_{\phi} = 48$ grid points, and an initial seed field $\beta_{\gamma} = 800$. Finally, a simulation was run with parameters similar to the first, except starting with an initially azimuthal field, with $\beta_{\gamma} = 100$. For this simulation the vertical boundary conditions were chosen to be reflecting, with the vertical components of both the velocity and the magnetic field set to zero on the boundaries.

Simulations that include vertical stratification are obviously more realistic, but they are also more demanding of computational resources, because the development of magnetically dominated low density regions at high $|z|$ places severe restrictions on the timestep. Numerical tricks can be used to mitigate this problem (Miller & Stone 2000), but for this paper we adopted the simpler approach of considering a volume containing only a small number of disk scale heights. We therefore reran the standard model, including the vertical component of gravity, in a domain bounded by $z = \pm 0.5$. This admits only a couple of scale heights at $r_{\text{ms}}$. For this run we used $n_r = 150$, $n_z = 48$ and $n_{\phi} = 40$ grid points.

It is worth noting at the outset that these simulations are not, and are not intended to be, realizations of the same physical situation, and thus the results will differ between them. An initially vertical field gives the fastest possible growth rate of the Balbus-Hawley (1991) instability. Additionally, a non-zero vertical flux boosts the strength of MHD turbulence and increases the effective Shakura-Sunyaev (1973) parameter obtained (Hawley, Gammie & Balbus 1995). Substantially longer timescales are required to reach saturation with an initially azimuthal field. We have run both simulations that used the smaller computational domain until a significant fraction (around a half) of the initial mass had been accreted, and compare the results at this late stage when the flow in the inner regions of the disk, near $r_{\text{ms}}$, has reached an approximate steady state.

3. RESULTS

Fig. 1 shows the growth in the magnetic energy of the radial magnetic field component in the standard unstratified model with an initially vertical ($z$ direction, $\beta_{\gamma} = 5000$) magnetic field. Around 10 orbits of evolution are required to reach a nonlinear state in the inner regions of the flow, during which time the radial magnetic field energy grows exponentially. The growth rate is consistent with the expected growth rate of the Balbus-Hawley instability in this field geometry (Balbus & Hawley 1991), evaluated at the smallest radius that has been seeded with magnetic field. Subsequent to the instabilities saturating, the field energy fluctuates but remains roughly constant, before finally starting to decline as a significant fraction of the disk is accreted. At the point the run was stopped, at $t = 200$, 65% of the initial mass had been accreted. The magnetic field was dominated by the toroidal component, with the ratio of magnetic field energy in the $z$, $r$ and $\phi$ fields being approximately 1:2:30.

The simulation seeded with an azimuthal magnetic field shows similar behavior, but with much slower growth of the magnetic field energy, despite the initially higher ratio of the magnetic energy to the thermal energy. This configuration was run up to $t = 600$, at a somewhat lower resolution than the $z$ field run ($120 \times 32 \times 40$ grid points). By this time just under 40% of the initial disk mass had been accreted.

Fig. 2 shows the evolution of the disk surface density with time, for the standard run with an initial vertical field. The evolution differs somewhat from the standard diffusive evolution of a viscous annulus (e.g. Pringle 1981), due to both the varying amount of time required before the disk at different radii becomes fully turbulent, and due to the development of supersonic infall interior to $r_{\text{ms}}$. However the general trend towards a broadening surface density profile, that remains relatively smooth, is recovered. Spatial fluctuations in $\Sigma$, shown in Fig. 3, are strongly sheared and thus predominantly azimuthal in extent. When normalised to the mean surface density as a function of radius, as in the figure, there is no obvious visual indication of the location of the marginally stable orbit.

The radial and azimuthal velocities as a function of radius obtained in the standard run are shown in Fig. 4. A small radial velocity within the disk itself transitions smoothly into rapid infall within the marginally stable orbit. The sonic point lies somewhat inside $r_{\text{ms}}$ (within a radial distance $\sim h$, the disk scale height that would correspond to the assumed sound speed). Note that the radial velocity slightly outside the marginally stable orbit is already beginning to increase in magnitude, in advance of reaching the actual infall region. Beyond $r \approx 2r_{\text{ms}}$, the radial velocity is outward. This is a consequence of the initial surface density profile and the use of a free outer boundary condition. We have also experimented with runs in which mass was continually injected across the outer boundary, and the disk allowed to evolve until a quasi-steady state was achieved. These runs yielded similar results to those reported here – in particular there was no strong amplification of magnetic field observed interior to $r_{\text{ms}}$. However, with inflow boundary conditions at $r_{\text{out}}$ it is hard to be sure that purely hydrodynamic instabilities, which are known to occur within relatively narrow annuli when the boundary conditions are reflecting, do not contaminate the results. We therefore concentrate attention on the current simulations which follow a ring of gas which is allowed both to accrete and to spread to larger radii.

As expected for a thin disk, in which radial pressure forces are small compared to the gravitational force, the azimuthal velocity in the disk, $v_\phi$, is very close to the Keplerian value for orbits in a Paczyński-Witt boundary potential.

The presence of magnetic fields in the flow potentially allows the flow interior to the sonic point to communicate with the exterior disk. Generically, except in unusual circumstances, MHD turbulence driven by the Balbus-Hawley instability saturates at a level where the magnetic pressure in the disk is substantially less than the thermal pressure, typically by one or two orders of magnitude (Hawley, Gammie & Balbus 1995; Brandenburg et
conditions in zulation with an initial As expected, the fields are significantly weaker in the sim-

\[ \mathbf{A} = \sqrt{\frac{B^2}{4\pi \rho}} \]

\[ \alpha_{\text{mag}} = \frac{2}{3} \left( \frac{-B_z B_t}{4\pi \rho c_s^2} \right) \]

which in the disk is just the magnetic contribution to the Shakura-Sunyaev \( \alpha \) parameter. Typical values obtained are a few \( \times 10^{-2} \) for the standard simulation, and some-

what less than \( 10^{-2} \) for the simulation with an initial mag-

\[ \pi \rho c \]

\[ \langle \pi \rho c \rangle \]

in Fig. 7 the specific angular momentum \( l \) of the flow as a function of radius. To reduce fluctuations in \( l \), we aver-

gage the specific angular momentum over several timeslices taken from near the end of the simulations. At a zero-

torque boundary, we expect that \( dl/d\tau \) vanishes. Although there is some variation in \( l \) within the marginally stable orbit, even in the averaged profiles, it is clear from Fig. 7 that for both the unstratified and stratified simulations the specific angular momentum is close to flat within \( r_{\text{ms}} \).

The highest resolution of the runs discussed here is ob-

The existence of this stress inside \( r_{\text{ms}} \), which is one of the predictions of Krolik (1999), has also been seen in previous numerical simulations (Hawley 2000; Hawley & Krolik 2000).

To test whether this stress precludes the use of a zero-torque boundary condition at the last stable orbit, we plot in Fig. 7 the specific angular momentum \( l \) of the flow as a function of radius. To reduce fluctuations in \( l \), we aver-

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4. DISCUSSION

In this paper, we have used MHD simulations to study the transition between a geometrically thin accretion disk, in which inflow is driven by the rate at which turbulence can transport angular momentum outwards, and the un-

stable plunging region interior to the marginally stable orbit. We find that in many respects the transition re-

sembles that expected on the basis of previous analytic and one-dimensional numerical calculations (Paczynski & Bisnovatyi-Kogan 1981; Muchotrzeb & Paczynski 1982; Muchotrzeb 1983; Matsumoto et. al. 1984; Abramowicz & Kato 1989). In particular, we find no evidence, at least in pseudo-Newtonian simulations at the current resolution, for the extremely strong magnetic field amplification and associated dynamical effects discussed by Krolik (1999). Indeed, in these thin disk simulations, a zero-torque boundary condition at the last stable orbit appears to be a good approximation to the numerical re-

results. This differs from the results of geometrically thick accretion flow simulations (Hawley 2000; Hawley & Krolik 2000), in which a continuing decline in the specific angular momentum inside \( r_{\text{ms}} \) was obtained. We do not consider this difference in behavior as especially surprising. Different forces (i.e. pressure gradients) are important in thick discs, and even in vertically averaged models the condi-

tions at the marginally stable orbit are expected to differ between thin and thick accretion flows.

Although we have not found any strong dynamical ef-

fxes associated with magnetic fields interior to the last stable orbit, we do see evidence for the essential ingredi-
ents – magnetic field amplification and ongoing magnetic stresses – of recent models that have questioned the validity of a zero-torque boundary condition for black hole accretion (Gammie 1999; Krolik 1999; Agol & Krolik 2000). The differences between these thin disk simulations, and those presented by Hawley (2000) and Hawley & Krolik (2000), are thus quantitative rather than qualitative in nature. We would therefore emphasize that further improvements in the simulations are required to investigate the behavior of the vertically stratified flows more robustly, to extend the simulations to test the evolution of cooler flows crossing the last stable orbit, and to explore the continuum between geometrically thin and thick disks.

The simulations reported here are non-relativistic, and cannot – even crudely – be extended to model any of the additional complexity that arises if the accretion flow is onto a Kerr black hole. Considerable progress has been made in the development of numerical methods for general relativistic hydrodynamics and magnetohydrodynamics (e.g. Font 2000), but these methods are probably not yet able to follow a turbulent disk flow transiting the last stable orbit, and to explore the continuum between geometrically thin and thick disks.

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Fig. 1.— Energy in the radial component of the magnetic field, integrated over the simulation volume, as a function of time. The units on the vertical axis are arbitrary. The initial magnetic field for this simulation was in the \( z \) direction, and the vertical boundary conditions were periodic. The dashed line shows the expected growth rate of the radial magnetic field energy density, for the most unstable (largest \( \Omega \)) mode.
Fig. 2.— Evolution of the disk surface density profile in the simulation with an initially vertical magnetic field geometry. From top down, the slices are plotted at $t = 0$, $t = 50$, $t = 100$, $t = 150$, and $t = 200$. More than half of the mass has been accreted by the end of the simulation.
Fig. 3.— Image of the disk surface density fluctuations, $\Sigma(r, \phi)/\Sigma(r)$. The inner boundary is at 0.66 $r_{\text{ms}}$, the outer boundary at 3.33 $r_{\text{ms}}$. The magnitude of the fluctuations is at the $\sim 10\%$ level. No qualitative changes are noticeable as the flow crosses the marginally stable orbit.
Fig. 4.— Radial and azimuthal velocity at $t = 200$ from the standard simulation. The short dashed curve in the lower panel shows the Keplerian velocity of circular orbits in the pseudo-Newtonian potential.
Fig. 5. — Wave propagation speed as a function of radius, evaluated at $t = 200$ from the standard simulation with and without vertical stratification. In the upper panel, for the unstratified run, the horizontal dotted line shows the sound speed, the short dashed line the mean Alfven speed as a function of radius. The negative of the radial velocity is plotted as the solid curve. The lower panel shows the same quantities plotted for the stratified simulation, with the long dashed line showing the mean Alfven speed in the flow near the disk midplane ($|z| < 0.1$). We also plot the peak Alfven speed at each radius (dot-dashed line), which exceeds the mean value by a substantial factor.
Fig. 6.—The upper panel shows the ratio of the magnetic energy to the thermal energy, as a function of radius, for the standard simulation with an initial $z$ field (dashed line), and for the run with an initial $\phi$ field (solid line). In both cases, several timeslices from near the end of the runs have been averaged together to reduce the magnitude of the fluctuations. The lower panel shows the effective Shakura-Sunyaev $\alpha$ parameter derived from the magnetic torques.
Fig. 7.— The specific angular momentum $l$ as a function of radius. The solid line shows the result for the unstratified simulation with an initial $\phi$ magnetic field, the short dashed and long dashed curves for unstratified simulations with an initial $z$ field but different sound speeds and seed field strength. The dot-dashed line shows the result for the vertically stratified simulation, evaluated in the disk midplane. Note that the simulation with reduced sound speed (long dashed line) evolves more slowly, so that the averaging of several timeslices has not eliminated substantial fluctuations present at larger radius outside $r_{\text{ms}}$. For all these models, to a good approximation, $dl/dr = 0$ at $r_{\text{ms}}$. 
Fig. 8.— The specific angular momentum $l$ as a function of radius, plotted from five timeslices of the stratified run. As in Fig. 7, $l$ is evaluated in the disk midplane.