CLASSIFYING REPORTED AND ‘MISSING’ RESONANCES ACCORDING TO THEIR P AND C PROPERTIES

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Abstract

The Hilbert space $H^{3q}$ of the three quarks with one excited quark is decomposed into Lorentz group representations. It is shown that the quantum numbers of the reported and "missing" resonances fall apart and populate distinct representations that differ by their parity or/and charge conjugation properties. In this way, reported and "missing" resonances become distinguishable. For example, resonances from the full listing reported by the Particle Data Group are accommodated by Rarita–Schwinger (RS) type representations $\{\frac{3}{2}, \frac{3}{2}\} \otimes \{\frac{1}{2}, 0\} \oplus \{0, \frac{1}{2}\}$ with $k = 1, 3, \text{and } 5$, the highest spin states being $J = 3/2^−, 7/2^+, \text{and } 11/2^+$, respectively. In contrast to this, most of the “missing” resonances fall into the opposite parity RS fields of highest-spins $5/2^−, 5/2^+, \text{and } 9/2^+$, respectively. Rarita–Schwinger fields with physical resonances as lower-spin components can be treated as a whole without imposing auxiliary conditions on them. Such fields do not suffer the Velo–Zwanziger problem but propagate causally in the presence of electromagnetic fields. The pathologies associated with RS fields arise basically because of the attempt to use them to describe isolated spin-$J = k + \frac{1}{2}$ states, rather than multispin-parity clusters. The positions of the observed RS clusters and their spacing are well explained through the interplay between the rotational-like $\frac{1}{2} (\frac{k}{2} + 1)$-rule and a Balmer-like $\frac{1}{(k+1)^2}$ behavior.

1. Baryons as Multispin-Parity Clusters

1.1. Introductory remarks

One of the basic quality tests for any model of composite systems is the level of accuracy reached in describing the excitation spectrum of its simplest object. Recall, that quantum mechanics was established only after the successful description of the...
spectrum of the hydrogen atom. Also one of the main successes of quantum electrodynamics as a gauge theory, the explanation of the Lamb shift of the electron levels in the hydrogen atom, was related to a spectroscopic observable. The history of spectroscopy is basically the history of finding the degeneracy symmetry of the spectra (see Ref. 1 for a related historical review). It is therefore, natural, to expect from the models of hadron structure that they should supply us with a satisfactory description of baryon excitations. In that respect, the knowledge on the degeneracy group of baryon spectra appears as a key tool in constructing the underlying strong interaction dynamics. To uncover it, one has first to analyze isospin by isospin how the masses of the resonances from the full listing in Ref. 2 spread with spin and parity. Such an analysis has been performed in our previous work, 3 where it was found that Breit-Wigner masses reveal on the $M/J$ plane a well pronounced spin-and parity clustering. There, it was further shown that the quantum numbers of the resonances belonging to a particular cluster fit into Lorentz group representations of the type $\{\frac{1}{2}, \frac{1}{2}\} \otimes \{(\frac{1}{2}, 0) \oplus \{0, \frac{1}{2}\}\}$, known as Rarita–Schwinger (RS) fields. To be specific, one finds the three RS clusters with $k = 1, 3, \text{and } 5$ in both the nucleon ($N$) and $\Delta$ spectra. These representations accommodate states with different spins and parities and will be referred to as multispin-parity clusters. To illustrate this statement it is useful to recall that the irreducible representations (irreps) $\kappa$ of the Lorentz group yield in its Wick rotated compact form $O(4)$, four-dimensional space inversion operation $P$ as

$$P\kappa_{\eta;l} = \eta e^{i\pi l} \kappa_{\eta;-l-m}, \quad l = 0^\eta, 1^-\eta, \ldots, (\sigma - 1)^-\eta, \quad m = -l, \ldots, l. \quad (1)$$

In coupling a Dirac spinor, $\{\frac{1}{2}, 0\} \otimes \{0, \frac{1}{2}\}$, to the Coulomb multiplets $\{\frac{k}{2}, \frac{k}{2}\}$ from above, the spin ($J$) and parity ($P$) quantum numbers of the baryon resonances are created as

$$J^P = \frac{1}{2}^\eta, \frac{1}{2}^-\eta, \frac{3}{2}^-\eta, \ldots, \left(\frac{k + 1}{2}\right)^-\eta, \quad k = \sigma - 1. \quad (2)$$

The RS fields are finite-dimensional nonunitary representations of the Lorentz group which, in being described by totally symmetric traceless rank-$k$ Lorentz tensors with Dirac spinor components, $\psi_{\mu_1\mu_2\cdots\mu_k}$, have the appealing property that spinorial and four-vector indices are separated. They satisfy the Dirac equation according to:

$$(i\partial \cdot \gamma - M)\psi_{\mu_1\mu_2\cdots\mu_k} = 0. \quad (3)$$

In terms of the notations introduced above, all reported baryons with masses below 2500 MeV, are completely accommodated by the RS fields $\psi_\mu$, $\psi_{\mu_1\mu_2\mu_3}$, and $\psi_{\mu_1\mu_2\cdots\mu_5}$, having states of highest spin-$3/2^-$, $7/2^+$, and $11/2^+$, respectively (see Figs. 1 and 2). In each one of the nucleon, $\Delta$, and $\Lambda$ hyperon spectra, the RS cluster of lowest mass is always $\psi_\mu$. For the nonstrange baryons, the $\psi_\mu$ cluster is followed
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The clustering of the reported nucleon excitations in terms of the Rarita–Schwinger multispinors. Here, $\psi_1$ describes the first $P_{11}$, $S_{11}$, and $D_{13}$ resonances. The second $P_{11}$, $S_{11}$, and $D_{13}$ states together with the first $P_{33}$, $D_{15}$, $F_{15}$, and $F_{17}$ states belong to $\psi_{\mu\nu\rho\sigma}$. Finally, the third $S_{11}$, $P_{11}$, and $D_{13}$ resonances, the second $P_{33}$, $D_{15}$, $F_{15}$, and $F_{17}$ states, together with the first $G_{17}$, $G_{19}$, $H_{19}$ and $H_{1,11}$ states belong to $\psi_{\mu\nu\rho\sigma\tau\nu}$. The two “missing” $F_{17}$ and $H_{1,11}$ resonances have been denoted by the open square $\Box$.

by $\psi_{\mu_1\mu_2\mu_3}$, and $\psi_{\mu_1\mu_2\cdots\mu_5}$, while for the $\Lambda$ hyperons a parity doubling of the resonances starts above 1800 MeV. In the following we will extend the notation of the RS clusters to include isospin ($I$) according to $\sigma^2 I, \eta$. For example, the first nucleon cluster is denoted by $2^{1+}_I$, while $2^{3+}_I$ and $2^{0+}$ stand in turn for the corresponding $\Delta$- and $\Lambda$-hyperon ones. From Eqs. (1) and (2) follows that the $2^{1+}_I$ clusters, where $I = 1/2, 3/2$, and 0, always unite the first spin-$1/2^+$, $1/2^-$, and $3/2^-$ resonances.

Indeed, the relative $\pi N$ momentum $L$ takes for $l = 0^+$ the value $L = 1^+$ and corresponds to the $P_{21,1}$ state, while for $l = 1^-$ it takes the two values $L = 0^-$, and $L = 2^-$ describing in turn the $S_{21,1}$ and $D_{21,3}$ resonances. The natural parity of the first $O(4)$ harmonics reflects the arbitrary selection of a scalar vacuum through the spontaneous breaking of chiral symmetry. Therefore, up to the three lowest $N$, $\Delta$, and $\Lambda$ excitations, chiral symmetry is still in the Nambu–Goldstone mode. The Fock space of the $2^{1+}_I$ clusters will be denoted in the following by $F^+$. Note, that in this context the first $P_{11}$ and $S_{11}$ states do not pair, because their internal orbital angular momenta differ by one unit, instead of being equal but of opposite parities. All the remaining nonstrange baryon resonances have been shown in Ref. 3 to belong to either $4^{2+}_{2I,-}$, or $6^{2+}_{2I,-}$. They have been viewed to reside in a different Fock space, $F^-$, which is built on top of a pseudoscalar vacuum. For example, one finds all the seven $\Delta$-baryon resonances $S_{31}$, $P_{31}$, $F_{33}$, $D_{33}$, $D_{35}$, $F_{35}$ and $F_{37}$ from the $4^{2-}_{3,-}$ cluster to be squeezed within the narrow mass region from 1900 MeV to 1950 MeV, while the $I = 1/2$ resonances paralleling them, of which only the $F_{17}$ state is still “missing” from the data, are located around $1700_{-50}^{+20}$ MeV. Therefore, the $F_{17}$ resonance is the only nonstrange state with a mass below 2000 MeV which
The clustering of the reported $\Delta$ excitations in terms of the Rarita–Schwinger multispinors. Despite of its low mass, the two-star $F_{37}(2000)$ resonance fits well into the third RS cluster. The three “missing” $P_{31}$ and $P_{33}$, and $D_{33}$ resonances from the 2nd cluster have been marked by the open circle $\circ$. Other notations as in Fig. 1.

is “missing” in the present RS classification scheme (compare Figs. 1 and 2).

In continuing by paralleling baryons from the third nucleon and $\Delta$ clusters with $\sigma = 6$, one finds in addition the four states $H_{111}$, $P_{31}$, $P_{33}$, and $D_{33}$ with masses above 2000 MeV to be “missing” for the completeness of the new classification scheme. The $H_{111}$ state is needed to parallel the well established $H_{31}$ baryon, while the $\Delta$-states $P_{31}$, $P_{33}$, and $D_{33}$ are required as partners to the (less established) $P_{211}(2100)$, $P_{13}(1900)$, and $D_{13}(2080)$ nucleon resonances.

The degeneracy group of the nonstrange baryon spectra found in Refs. 3 on the grounds of the successful RS classification is

$$SU(2)_I \otimes SU(3)_C \otimes O(1,3)_L.$$  

Within this scheme, the approximately equidistant cluster spacing of about 200 to 300 MeV between the mass centers of the RS clusters appearing now, is by a factor 3 to 6 larger as compared, for example, to the maximal mass splitting of 50–70 MeV within the $2_{1+}$, $2_{3+}$, $4_{1-}$, and $4_{3-}$ multiplets.

1.2. Multispin-parity clusters versus $SU(6)_{SF} \otimes O(3)_L$ multiplets

Traditionally, hadrons are classified in terms of $SU(6)_{SF} \otimes O(3)_L$ multiplets. This classification is one of the most important paradigms in hadron spectroscopy. According to it, states like, say, the positive parity resonances $P_{13}(1720)$, $F_{15}(1680)$, $F_{35}(1905)$, and $F_{37}(1950)$, are viewed to belong to a $56(2^+)$-plet, the $P_{11}(1710)$ excitation is treated as a member of a $70(0^+)$-plet, while the negative parity baryons $S_{11}(1535)$, $D_{13}(1520)$, $S_{11}(1650)$, $D_{13}(1700)$, and $D_{15}(1675)$ are assigned to a $70(1^-)$-plet. The above examples clearly illustrate that states from our RS clusters separated by only few MeV, such as $D_{15}(1675)$, $F_{15}(1680)$, and $P_{11}(1710)$ from $4_{1-}$, are distributed over three different $SU(6)_{SF} \otimes O(3)_L$ representations, while on the other hand, resonances from different RS “packages” separated by about 200 MeV, such as $D_{13}(1520)$ from $2_{1+}$, and $D_{13}(1700)$ from $4_{1-}$, are assigned to the same
This means that \( SU(6)_{SF} \otimes O(3)_L \) can not be viewed as the degeneracy group of baryon spectra. Further, the \( SU(6)_{SF} \otimes O(3)_L \) multiplets appear approximately only “half-filled” by the reported resonances. Several dozens states are “missing” for the completeness of this classification scheme (see Refs. 2 and 4 for reviews). As long as observed and “missing” states are part of same multiplets, they are indistinguishable from the viewpoint of the underlying \( SU(6)_{SF} \otimes O(3)_L \) symmetry and there is no reason not to believe to the observability of all states. On the contrary, within the RS scheme, observed and “missing” states will fall apart and be attributed to Lorentz multiplets of different space–time properties. Therefore, they will become distinguishable. Within such a scenario, reasons for a possible suppression of states can be sought for.

The considerations from above show that the nonstrange baryons do not exist as isolated higher-spin states but rather as multispin parity clusters described in terms of particular RS representations of the Lorentz group.

2. Rarita-Schwinger Fields without Auxiliary Conditions

Originally, the Rarita-Schwinger fields acquired importance in connection to one of the longstanding problems in baryon spectroscopy—the relativistic description of higher-spin states. Since Weinberg’s work on particles with any spin it is of common use to describe spin-\( J \) baryons as the highest-spin states of the Rarita-Schwinger representations of the Lorentz group from Eq. (3). In this way, the spin-\( J = k + \frac{1}{2} \) field appears as the highest-spin state within the multispin-parity cluster and in order to describe it, one has to eliminate all the lower-spin components by suitably chosen auxiliary conditions. In order words, one considers the lower-spin components as redundant, unphysical states. The auxiliary conditions of common use read:

\[
\left( g^{\nu \mu_1} + \frac{1}{M^2} \partial^\nu \partial_\mu_1 \right) \Psi_{\mu_1 \mu_2 \cdots \mu_k} = \Psi^{\nu}_{\mu_2 \cdots \mu_k},
\]

\[
\gamma^{\mu_1} \psi_{\mu_1 \mu_2 \cdots \mu_k} = 0.
\]

The first auxiliary condition is nothing but the Proca equation for a four vector. It is equivalent to \( \partial^\mu \psi_{\mu_1 \mu_2 \cdots \mu_k} = 0 \) and eliminates the time component of the Lorentz vector (pseudovector) associated with the index \( \mu_1 \), while the second condition excludes its longitudinal degree of freedom. As a result, the spinor couples to the purely transversal components of the \( \mu_1 \) vector, and gives rise to a spin-\( 3/2 \) field. In repeating same procedure \( k \) times, the highest-spin state with \( J = k + \frac{1}{2} \) is created.

The auxiliary conditions prevent, therefore, that the minimal helicities \( \pm 1/2 \) of the spin-3/2 field get mixed up with such corresponding to the spin-1/2\( ^+ \), and 1/2\( ^- \) members of \( \psi_{\mu_1} \), respectively.\(^a\) It is well known that the procedure of the exclusion of the lower-spin component from the RS multispin-parity clusters suffers several drawbacks. For example, when minimally coupled to an external electromagnetic field, the propagation of the truncated RS field may violate causality. This drawback is known as the Velo-Zwanziger problem.\(^6\) Furthermore, in trying to incorporate the two auxiliary conditions into the Lagrangian of the RS fields, one is forced to introduce an arbitrary parameter to account for the fact that for off-mass shell particles, the auxiliary conditions are not any longer valid and the exclusion of the lower-spin components not any longer guaranteed. Several solutions to this problem have been suggested over the decades, the only reliable being the concept of supersymmetry.

\(^a\)As long as the multispinor-tensor is totally symmetric with respect to the Lorentz indices, it was sufficient to impose the auxiliary conditions to one index only.
Beyond the RS formalism, other methods for describing higher-spin states have been explored in the literature. In Ref. 7 causal higher-spin propagators have been constructed by Ahluwalia et al. for \((J, 0) \oplus (0, J)\) states and shown to be necessarily nonlinear in the momenta due to the \(p^{2J}\)-dependence of the boost in such representation spaces. This is a promising method, though the techniques for calculating three point functions including besides the spin-\(J\) state, a nucleon and an external vector field, need to be still developed, in particular because the \((J, 0) \oplus (0, J)\) fields do not carry spinor-vector indices. Those states may acquire significant importance in future research on the CPT structure of space–time, especially because their \(C\) and \(P\) properties turn out to be quite different from those of the RS fields. 8

More recently, higher-spin fermion states have been constructed by Moshinsky et al. as eigenstates of the relativistic Dirac oscillator (see Ref. 9 for a review). In exploiting the circumstance, that the elements of the Clifford algebra of the Dirac matrices generate the group \(O(6)\), and that the Hamiltonian contains only some of the generators of its subgroup \(O(5)\), the wave function of a spin-\(J\) state has been expanded into the basis of \(O(5)\) irreps by means of a properly constructed Racah algebra that accounts for the reduction of the \(O(5)\) down to \(O(3)\) through the chain \(O(5)/O(4)/O(3)\). There, the RS fields appear as intermediate and their lower-spin components are automatically projected out by the Racah algebra. This scheme, which has been developed basically for the needs of the relativistic description of many body systems, such like nuclei, has the immediate advantage of being straightforward in calculating various transition matrix elements. The properties of the relativistic harmonic oscillator have been extensively studied by Kim and Noz (see Ref. 10 for a textbook presentation). Applying these ideas to baryon clustering is a task worthy to be explored.

On the other side, the RS spinors without auxiliary conditions have the great advantage that they do not suffer the pathologies indicated above. As we already argued in the previous section, exactly this very case is relevant for baryon spectroscopy. In view of this relevance we here consider the properties of the simplest RS field \(\psi_\mu\) without imposing any subsidiary conditions on it. As noticed by Kruglov and Strashev in Ref. 11, the Dirac equation \((p^\mu \gamma_\mu - M)u_\nu(p) = 0\) for the vector-spinor is equivalent to

\[
(p^\mu \Gamma_\mu - M)U(p) = 0,
\]

where \(\Gamma_\mu = 1 \otimes \gamma_\mu\), and \(U(p)\) is a 16-component vector. The conjugate quantity \(\bar{U}(p)\) is defined as \(U(p)^\dagger \eta\) with \(\eta = \Pi \otimes \gamma_0\) and \(\Pi = \text{diag}(1, -1, -1, -1)\). It can be proven that the matrices \(\Gamma_\mu\) replicate in the 16-dimensional space the anticommutation relations of the Dirac matrices. This allows one to gauge the corresponding Lagrangian, \(\mathcal{L} = \bar{U} p^\mu \Gamma_\mu U - M U U\) in the usual way by introducing the covariant derivative as \(D_\mu = \partial_\mu - ieA_\mu\). The propagator \(S\) of the \(U\) field without auxiliary conditions constructed in this way reads:

\[
S = \frac{p^\mu \Gamma_\mu + M}{2M(p^2 - M^2)}.
\]

It is causal and does not create any problems. It is important to note that also the cluster propagator \(S_{\mu\nu}\) for the \(u_\mu(p)\)-field with Proca’s auxiliary condition,

\[
S_{\mu\nu} = \frac{\left( g_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{M^2} \right)(p^2 + M)}{2M(p^2 - M)}
\]

is causal as both the Dirac and Proca equations do not change form after gauging. It is the second auxiliary condition in Eq. (6) that is responsible for the occurrence of the Velo–Zwanziger problem.
To construct a $N\gamma\psi_{\mu}$ vertex one may notice that the nucleon–photon system $N\gamma$ carries same indices like $\psi_{\mu}$ since the photon transforms as a Lorentz vector $V_{\mu}$, while the nucleon ($N$) is a Dirac spinor. Therefore, the $N\gamma$ system can be converted similarly to Eq. (7) to a 16-dimensional spinor. Same is true for the $N\pi\psi_{\mu}$ vertex, provided the pion is gradiently coupled, so that $(\partial^\mu \pi)N$ transforms as a Lorentz pseudovector, $A_{\mu}$, with Dirac spinor components. With that the description of multispin-parity clusters as intermediate states in processes such like $\pi(\eta)$ photo production off proton is straightforward.

In the previous section we showed that exactly this is the case in baryon spectra. There, we showed, that in the excitation spectra of the nonstrange baryons one does not find isolated higher-spin states but rather complete multispin-parity clusters of the Rarita–Schwinger type. The result followed from the analysis of the degeneracy group in the baryon spectra. In the next section we are going to study the RS clustering of baryons on the level of the quark degrees of freedom.

3. Decomposing Three Quark Hilbert Space into Lorentz Group Representations

To get a deeper insight into the structure of baryon spectra, it is quite instructive to consider as an illustrative example how the quantum numbers of the baryon excitations can emerge from a configuration space with one excited quark only. We begin with the simplest space spanned by the $1s$, $1p$, and $2s$-single-particle shells. We use the familiar shell-model notation of the single-particle basis, where $1s$ means the first shell with zero orbital angular momentum (a.m.), $1p$ is the first shell with a unit a.m., etc.

The one-quark excitations give rise to the following orbitally excited two-quark configurations (in standard shell-model notations) of both natural and unnatural parities:

$$\left[ 1s_\frac{1}{2} \otimes 2s_\frac{1}{2} \right]^{l=0^+,1^+}, \left[ 1s_\frac{1}{2} \otimes 1p_\frac{1}{2} \right]^{l=0^-,1^-}, \left[ 1s_\frac{1}{2} \otimes 1p_\frac{3}{2} \right]^{l=1^-,2^-}. \tag{10}$$

Note that we here consider the quark model in the $j-j$ rather than in the LS coupling ordinarily exploited in the traditional $SU(6)_{SF} \otimes O(3)_L$ quark models. Note, further, that the spin-1$^-$ state appears twice in the latter equation as it can result from both the $\left[ 1s_\frac{1}{2} \otimes 1p_\frac{1}{2} \right]$, and $\left[ 1s_\frac{1}{2} \otimes 1p_\frac{3}{2} \right]$ configurations. The quantum numbers $(0^+,1^-)$ fit into the $\{\frac{1}{2},\frac{1}{2}\}^+$ polar Lorentz vector, denoted by $V_{\mu}$

$$V_{\mu} = \left\{ \begin{array}{c} 1 \\ 2 \\ \frac{1}{2} \end{array} \right\}^+ : \left[ 1s_\frac{1}{2} \otimes 2s_\frac{1}{2} \right]^{l=0^+}, \left[ 1s_\frac{1}{2} \otimes 1p_\frac{1}{2} \right]^{l=1^-}. \tag{11}$$

In the following, the index $+/−$ attached to a Lorentz group representation will be used to label natural/unnatural parities. The spin-parity sequence $0^+,1^+,2^−$ fits into $\{1,1\}_-$, the totally symmetric 2nd rank Lorentz pseudotensor $A_{\mu_1\mu_2}$ according to

$$A_{\mu_1\mu_2} = \{1,1\}_- : \left[ 1s_\frac{1}{2} \otimes 1p_\frac{1}{2} \right]^{l=0^-}, \left[ 1s_\frac{1}{2} \otimes 2s_\frac{1}{2} \right]^{l=1^+}, \left[ 1s_\frac{1}{2} \otimes 1p_\frac{3}{2} \right]^{l=2^-}. \tag{12}$$

Finally, the remaining natural parity spin-1$^−$ state from $\left[ 1s_\frac{1}{2} \otimes 1p_\frac{1}{2} \right]$ can be viewed as $\{1,0\}_+$ and it arises out of the multiplicity of the spin-1$^-$ two-quark state in

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Eq. (10). As a result, the Hilbert space under consideration, (it will be denoted by $\mathcal{H}_{1s-2s-1p}^{2q}$) decomposes into Lorentz group representations as follows:

$$\mathcal{H}_{1s-2s-1p}^{2q} = \left\{ \frac{1}{2}, \frac{1}{2} \right\}_+ \oplus \left\{ 1, 1 \right\}_- \oplus \{ 1, 0 \}_+ .$$

(13)

In other words, $\mathcal{H}_{1s-2s-1p}^{2q}$ is decomposed into an odd spin-1$^-$ state and the pair of representations $A_{\mu_1 \mu_2}$, and $V_\mu$, describing in turn states of unnatural and natural parities:

$$\mathcal{H}_{1s-2s-1p}^{2q} = V_\mu \oplus A_{\mu_1 \mu_2} \oplus \{ 1, 0 \}_+ ,$$

(14)

In noting that $\mathcal{H}_{1s-2s-1p}^{2q}$ corresponds to $n = 2$, one finds as quite an interesting result that the principal quantum number $n$ of the Coulomb problem associated with the single particle shells in the 2$q$-Hilbert space gives rise to ultraspherical O(4) harmonics with $\sigma = n$, and $\sigma = n + 1$, respectively. While the $\sigma = n$ ultraspherical harmonics contain O(3) states of natural parities, those with $\sigma = n + 1$ contain unnatural parities states.

In terms of Lorentz-vector index notations one finds

$$\mathcal{H}_{1s-3s-2p-1d}^{2q} = \left\{ 1, 1 \right\}_+ \oplus \left\{ \frac{3}{2}, \frac{3}{2} \right\}_- \oplus \left\{ 1, 0 \right\}_+ \right. \oplus \left\{ 0, 1 \right\}_+ \oplus \{ 2, 0 \}_+ .$$

(15)

In other words, $\mathcal{H}_{1s-3s-2p-1d}^{2q}$ decomposes into

$$\mathcal{H}_{1s-3s-2p-1d}^{2q} = V_{\nu_1 \nu_2} \oplus A_{\mu_1 \mu_2 \mu_3} \oplus F_{\mu \nu} \oplus \{ 2, 0 \}_+ .$$

(16)

Here, the $\{ 1, 0 \}_+ \oplus \{ 0, 1 \}_+$ representation has been described in terms of the totally antisymmetric 2nd rank Lorentz tensor $F_{\mu \nu}$ rather than as a six-component vector. Finally, in including the high-lying 1$f$ and 1$g$ shells, one finds

$$\mathcal{H}_{1s-4s-3p-2d-1f-1g}^{2q} = V_{\mu_1 \ldots \mu_4} \oplus A_{\nu_1 \ldots \nu_5} \oplus \sum_{m=1}^{m=n-2} \left\{ m, 0 \right\}_+ \oplus \{ 0, m \}_+ \oplus \{ 4, 0 \}_+ .$$

(17)

The extension of the 4$s$ – 3$p$ – 2$d$ – 1$f$ single-particle space to include the 1$g$ shell is supported by the empirical observations. In doing so, we effectively produced the quantum numbers of orbital angular momenta $l = 0, \ldots, 4$ which correspond to $n_{\text{eff}} = 5$. The latter notation has been introduced in order to distinguish $n_{\text{eff}}$ from the genuine principal quantum number $n = 5$ of the Coulomb problem associated with the 5$s$ – 4$p$ – 3$d$ – 2$f$ – 1$g$ space.

Equation (17) shows that any $\mathcal{H}_{1s}^{2q}$ contains fields of natural, $(V_{\mu_1 \mu_2 \ldots \mu_{n-1}})$, and unnatural $(A_{\mu_1 \mu_2 \ldots \mu_n})$ parities, respectively. They are supplemented by several parity symmetric $\{ m, 0 \}_+ \oplus \{ 0, m \}_+$ with $m = 1, \ldots, n - 2$ states, as well as by an odd $\{ n - 1, 0 \}_+$ state.

Representations of the type $\{ m, 0 \}_+ \oplus \{ 0, m \}_+$ deserve special attention. To be specific, the $\{ 1, 0 \}_+ \oplus \{ 0, 1 \}_+$ state, when considered as a totally symmetric (Sym) second order tensor $f^{\alpha}_\beta$ with $SL(2,C)$ spinor ($\xi^\alpha$) components i.e. as $f^{\alpha}_\beta = \text{Sym}^\alpha \xi_\beta$, describes boson and antiboson of same parities (see Ref. 10 for an understandable and yet precisely written textbook on Lorentz group representations). Also the selfconjugate $\left\{ \frac{1}{2}, \frac{1}{2} \right\}_+$ representation predicts this usual type of bosons. In contrast
to this, it was shown in Ref. 8 that the \( \{1,0\} \oplus \{0,1\} \) representation can accommodate an entirely new type of boson whose parity is opposite to its antiparticle. This was achieved by completely giving up the \( SL(2,C) \) tensor–spinor technique for constructing irreducible representations of the Lorentz group. Instead, spin-1 bosons and antibosons were described in terms of a set of fundamental six component \( U-, \) and \( V-\) spinors, respectively. In explicitly constructing then the charge (\( C \)) and parity (\( P \)) conjugation operators within this space as anticommuting 6\( \times \)6 matrices, i.e. \( \{C,P\} = 0 \), the parities of the \( U \) and \( V \) spinors have been proven to be opposite. Such bosons have been termed as Bargman–Wigner–Wightman ones, or, shortly, BWW bosons, a notation which we will adopt in the following. In this way it was demonstrated in Ref. 8 that the \( \{1,0\} \oplus \{0,1\} \) representation, if treated beyond the \( SL(2,C) \) spinor algebra technique, allows for the embedding of entirely new type of bosons. The above example illustrates that specification of the Casimir invariants of the Poincaré group alone, i.e. mass and maximal spin of the representation, do not determine in a unique way the relevant Lorentz multiplet. Only invoking discrete symmetry transformations, such as charge and parity conjugation, fixes the representation space unambiguously (cf. Ref. 12 for a discussion).

With this facts in mind, we now observe that the decompositions of \( H^2q_n \) contain together with representations describing bosons of standard type, also such potentially capable of describing BWW bosons. Finally, the series always end with an odd \( \{n-1,0\}_- \) state of natural parity. The latter one can be considered as the large component of a \( \{n-1,0\} \oplus \{0,n-1\} \) \( U-\)spinor (in the notations of Ref. 8) and reflects the inherent nonrelativistic element of the quarkish shell-model space.

Now one can couple the spectator \( 1s_\frac{1}{2} \) quark to the two-quark configurations and obtain the quantum numbers of the baryons. To be specific, we will carry this procedure for the case of Eq. (10). In doing so, one ends up with a spin-parity sequence containing the ten states

\[
\frac{1}{2}^+; \frac{3}{2}^+; \frac{1}{2}^-; \frac{3}{2}^-; \frac{1}{2}^-; \frac{3}{2}^+; \frac{1}{2}^-; \frac{3}{2}^-; \frac{1}{2}^+; \frac{1}{2}^+; \frac{3}{2}^-.
\]

They can be distributed over three different Lorentz representations. The latter are obtained from coupling the Dirac spinor \( \{\frac{1}{2},0\} \oplus \{0,\frac{1}{2}\} \), to be denoted by \( \psi \) in the following, to the Lorentz tensors in Eq. (13). In doing so, one first finds

\[
V_\mu \otimes \psi =: \left( [1s_{\frac{1}{2}} \otimes 2s_{\frac{1}{2}}]^{l=0^+} \otimes 1s_{\frac{1}{2}} \right)^{l=0^+} + \left( [1s_{\frac{1}{2}} \otimes 1p_{\frac{1}{2}}]^{l=1^-} \otimes 1s_{\frac{1}{2}} \right)^{l=1^-} + \left( [1s_{\frac{1}{2}} \otimes 1s_{\frac{1}{2}}]^{l=1^-} \otimes 1s_{\frac{1}{2}} \right)^{l=1^-}.
\]

In terms of vector–spinor indices the last equation is converted to

\[
\psi_\mu := V_\mu \otimes \psi.
\]

(20)

Here, \( \psi_\mu \) stands for a Lorentz vector with Dirac spinor components. It describes a RS field with a spin-3/2\(^-\) as the state of highest spin. Therefore, at the baryon level, one finds \( \psi_\mu \) to include besides the highest spin-3/2\(^-\) resonance, two more spin-1/2 states carrying opposite parities.

\[
\psi_\mu \rightarrow \left( \frac{1}{2}^+; \frac{1}{2}^-; \frac{3}{2}^- \right).
\]

(21)

The remaining unnatural parity configurations can be accommodated by the \( \{1,1\}_- \otimes \psi \) RS representation. It has the unnatural parity spin-5/2\(^-\) as a
highest spin:
\[ A_{\mu_1 \mu_2} \otimes \psi =: \left( \left[ s_2^{-1} \otimes p_2^1 \right] \otimes 1 \frac{s_2^1}{2} \right)^{l=0,-2} \otimes 1 \frac{s_2^1}{2} \otimes \psi, \]  
\[ \left( \left[ s_2^{-1} \otimes 2s_2^1 \right] \otimes 1 \frac{s_2^1}{2} \right)^{l=1^+} \otimes 1 \frac{s_2^1}{2} \otimes \psi. \]  

The RS field \( \{1, 1\} \otimes \psi \) from the last equation is associated with the totally symmetric rank-2 Lorentz tensor with Dirac spinor components \( \psi_{\mu_1 \mu_2} \)

\[ \psi_{\mu_1 \mu_2} := A_{\mu_1 \mu_2} \otimes \psi. \]  

The \( \psi_{\mu_1 \mu_2} \) field includes, besides the highest spin-5/2 state, also two more spin-1/2, as well as two more spin-3/2 parity duplicated states according to

\[ \psi_{\mu_1 \mu_2} \rightarrow \left( \frac{1}{2}, \frac{3}{2} \right) \]  

Finally, the two more natural parity states \( (\frac{1}{2}^-, \frac{3}{2}^-) \) are obtained from the coupling of the Dirac spinor to the ordinary three vector and are described by means of a three vector with Dirac spinor components, \( \psi_a \) as

\[ \psi_a := V_a \otimes \psi, \quad a = 1, 2, 3. \]  

As a result, the 1s−2s−1p three-quark (3q) Hilbert space contains the two opposite parity RS clusters \( \psi_\mu(x) \) and \( \psi_{\mu\nu}(x) \), respectively:

\[ \mathcal{H}_{1s-2s-1p}^{3q} \rightarrow [V_\mu \oplus A_{\mu_1 \mu_2} \oplus V_a] \otimes \psi. \]  

In proceeding in this way, all the quantum numbers of the excited baryons resulting from a given Hilbert space can be obtained (see Table 1).

Table 1: Decomposition of the three-quark Hilbert space into Lorentz group representations. See text for notations.

<table>
<thead>
<tr>
<th>resonance</th>
<th>( \mathcal{H}_{1s-2s-1p} )</th>
<th>( \mathcal{H}_{1s-3s-2p-1d} )</th>
<th>( \mathcal{H}_{1s-4s-3p-2d-1f-1g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>status</td>
<td>( n = 2 )</td>
<td>( n = 3 )</td>
<td>( n_{\text{eff}} = 5 )</td>
</tr>
<tr>
<td>37 reported, ( 5 ) “missing”</td>
<td>( V_\mu \otimes \psi )</td>
<td>( A_{\mu_1 \mu_2 \mu_3} \otimes \psi )</td>
<td>( A_{\mu_1 \mu_2 \mu_3, \mu_4 \mu_5} \otimes \psi )</td>
</tr>
<tr>
<td>all “missing”</td>
<td>( (2_{2f},+) )</td>
<td>( (4_{2f},-) )</td>
<td>( (6_{2f},-) )</td>
</tr>
<tr>
<td>all “missing”</td>
<td>( A_{\mu_1 \mu_2} \otimes \psi )</td>
<td>( V_{\mu_1 \mu_2} \otimes \psi )</td>
<td>( V_{\mu_1 \mu_2 \mu_3 \mu_4} \otimes \psi )</td>
</tr>
<tr>
<td>|</td>
<td>( (3_{2f},-) )</td>
<td>( (3_{2f},+) )</td>
<td>( (5_{2f},+) )</td>
</tr>
<tr>
<td>all “missing”</td>
<td>( {1, 0}_+ \otimes \psi )</td>
<td>( {2, 0}_+ \otimes \psi )</td>
<td>( {4, 0}_+ \otimes \psi )</td>
</tr>
<tr>
<td>all “missing”</td>
<td>( F_{\mu\nu} \otimes \psi )</td>
<td>( \sum_{m=1}^{3} {m, 0} \otimes {0, m} \otimes \psi )</td>
<td></td>
</tr>
</tbody>
</table>

4. O(4) Degeneracy in \( \mathcal{H}_{3q} \) and Parity Classification of “Missing” Resonances
In comparing now the quantum numbers of the resonances reported in the full list by the Particle Data Group\(^2\) to those contained in \(\mathcal{H}^{3q}\) in Table 1, one immediately realizes that only a part of the possible configurations has been detected so far. Namely, the observed states fitted into the three RS representations from the first row, from which only five resonances are still “missed.” These were the first \(F_{17}\) and \(H_{1,11}\) nucleon states, the third \(P_{31}\), and \(D_{33}\), and the fourth \(P_{33}\) \(\Delta\) states. All the remaining “missing” resonances belong to the representations from the second row on in Table 1. With the exception of the \(\Lambda\) hyperon \(S_{01}(1670)\) and \(D_{03}(1690)\) states, there are no other resonances which can be interpreted as candidates for \(\{1, 0\}_+ \otimes \psi\) configurations.

In connection with this empirical observation, the question arises, how to interpret the absence of the remaining resonances from Table 1 from the spectra. One possibility is, that they are suppressed by some dynamical reasons, another one could be, that they have not yet been observed. While the latter belief is the widely spread one, we here are rather interested in exploring the former option.

One possible way towards explaining a suppression of the resonances lying outside of the \(2_{2I,+}, 4_{2I,\ldots}\), and \(6_{2I,\ldots}\) clusters is to first introduce the O(4) degeneracy of the diquarks and then consider the O(4) bosons (in fact, the Wick rotated O(1,3) bosons) created in this manner as fundamental point like degree of freedom (d.o.f.). The most important O(4) bosons are the Coulomb multiplets

\[
C_{\mu_1 \mu_2 \cdots \mu_k}(x) \simeq \left\{ \frac{k}{2}, \frac{k}{2} \right\} \rightarrow (0^\eta, 1^{-\eta}, \ldots, k^{-\eta}),
\]

for which we coined the term \(C\)-hyperquark. In general, all bosons from Table 1 will be referred to as hyperquarks. The \(C\) hyperquarks behave with respect to space reflection as either Lorentz tensors, or, pseudotensors. In being fundamental point like subbaryonic degrees of freedom, they are created within a Fock space, \(\mathcal{F}_\eta\), by operators, \(C_{\mu_1 \mu_2 \cdots \mu_k : lm}\), acting upon a vacuum, \(|0^\eta\rangle\), of either positive (\(\eta = +\)), or negative (\(\eta = -\)) parity. Here, \(lm\) denote the quantum numbers of the O(3) states accommodated by the O(4) harmonics. To be specific, for \(2_{1,+}\), or, \(V_\mu \otimes \psi\) in equivalent notation, one has

\[
\mathcal{P} C_{\mu : lm} |0^+\rangle = e^{iml} C_{\mu : -lm} |0^+\rangle, \quad l = 0, 1, \quad m = -l, \ldots, l.
\]

The idea of the point like character of the diquarks, and, consequently, the hyperquarks, has been exploited in the literature to reduce the three-quark Faddeev equations to a two-body quark–diquark Bethe–Salpeter equation (see, for example Refs.\(^{15}\) and \(^{16}\)). However, the last consequence of the point like nature of a quantum object, the parity selection rule in Eq. (29) due to its belonging to a Fock space with a vacuum of fixed parity, was not considered by none of the diquark model versions. The essential difference between the present quark–hyperquark model (QHM) and the customary quark–diquark models (QDM) (see Ref.\(^{13}\) for a digest) is the assumed O(4) clustering of the diquarks and the parity selection rule. To be specific, the operator \(D^\dagger_{21,+}\) which creates the lightest \(C\)-hyperquark is defined as the following linear superposition:

\[
D^\dagger_{21,+} |0^+\rangle = \sum_{lm} c_{lm} C^\dagger_{\mu : lm} |0^+\rangle, \\
\sum_{lm} |c_{lm}|^2 = 1, \\
\langle 0^+|D^\dagger_{21,+}|0^+\rangle = \sum_{lm} c_{lm} R_{2l}(r) Y_{2lm}(\alpha, \theta, \phi).
\]
In Eq. (30) the radial part of the $C$-hyperquark wave function has been denoted by $R_{\alpha l}(r)$, while its angular part has been determined by the four-dimensional harmonics $Y_{\sigma lm}$ defined in the standard way in Ref. 14 as

$$Y_{\sigma lm}(\alpha, \theta, \phi) = i^{\sigma - l - 2l + 1}l! \frac{\sigma(\sigma - l - 1)}{2\pi(\sigma + 1)} \sin\alpha C_{\sigma l}^{\sigma + 1}(\cos\alpha)Y_{\sigma m}^{\sigma l}(\theta, \phi).$$  

(31)

Here, $C_{\sigma l}^{\sigma + 1}(\cos\alpha)$ denote the Gegenbauer polynomials, while $Y_{\sigma m}^{\sigma l}(\theta, \phi)$ are the standard three-dimensional spherical harmonics.

In general, the Lorentz covariant isodoublet spin- and parity-cluster $\sigma_{1,\eta}$, considered as a $C$-hyperquark coupled to a spectator $1s_{1/2}$ quark, is now described as

$$\sigma_{1,\eta} = \psi_{\mu} \otimes \left[ T \otimes \chi^{\frac{1}{2}} \right]^{\tau}, \quad \psi_{\mu} = D_{\sigma_{1,\eta}}^{1 l} b_{1s_{1/2}}^{l} |0\rangle,$$

(32)

where $\chi^{\frac{1}{2}}$ is the ordinary isospinor of the spectator quark, while $T$ stands for the isospin part of the hyperquark wave function. It should not be confused with the isospins attached to diquarks in the traditional quark-diquark models, where the diquark isospin varies with $l$ and is determined from the requirement on a totally antisymmetric 3q wave function. Obviously, for all $\Delta$ states $T$ has to be an isovector. In having once figured out the quantum numbers of the Lorentz multiplets under the guidance of Sect. 3, we here from now on design a model, where the hyperquarks are considered as fundamental degrees of freedom in their own rights. In such a case, antisymmetrization does not matter any longer, because the baryon constituents are essentially different particles.

The $O(4)$ symmetry ansatz for the quark-hyperquark model assumed in the present work is independently supported by the observed rapid convergence of the covariant diquark models in the basis of the Gegenbauer polynomials considered among others in Ref. 15 and 16. It is worthy of being pursued especially because of the quite uncertain experimental status of the “missing” resonances.

Now, the absence of the $3_{2I-}$ cluster associated with $A_{\mu1}^\mu \otimes \psi$ from Eq. (23), may reflect the circumstance, that the Nambu–Goldstone mode of chiral symmetry, known to favor the natural parity for the ground state, extends to the domain of the first $P_{2I,1}$, $S_{2I,1}$, and $D_{2I,3}$ resonances from $H_{1s_{1/2}}-1p_{3/2}$. In this way the scale of the hidden mode of that symmetry is predicted. The unnatural parity of the $4_{2I-}$ and $6_{2I-}$ clusters can be interpreted as a change of the vacuum from scalar to pseudoscalar. Therefore, nucleon excitations to resonances from the $4_{2I-}$ and $6_{2I-}$ clusters can be interpreted as chiral phase transitions. The natural parity $3_{2I+}$, and $5_{2I+}$ clusters could be expunged from $F_{-}$ by that very mode of chiral symmetry realization there. Finally, the absence of the $F_{\mu\nu} \otimes \psi$ cluster from Table 1 could be explained in two different ways. The first one is, that those states are difficult to be excited in the direct $\pi N$ channel, since no totally antisymmetric tensor can be constructed for a pseudoscalar meson. Only in case the $\pi NN$ vertex would partially proceed via intermediate $a_1$ meson states, exciting $F_{\mu\nu} \otimes \psi$ could become possible. In $\gamma N$ excitations, however, such states, if existing at all, could become accessible to measurements. As a second version, one may entertain the possibility, that the $\{m, 0\} \oplus \{0, m\}$ hyperquark configurations in Eq. (17) are of the usual BWW type. In such a case, the charge and parity conjugation properties of the BWW hyperquarks will not any longer match with those of the $\{\frac{3}{2}, \frac{1}{2}\}$ bosons of usual type, and parity conservation through strong interaction could not any longer be guaranteed. Baryons with BWW hyperquarks, would decouple from the standard nucleon excitation channels and be inaccessible to measurements running presently.

5. Mass Formula for the Rarita-Schwinger Clusters
In this section, we shall argue that the algebra of the degeneracy group from Eq. (4) is also partly the spectrum generating algebra. Indeed, the reported mass averages of the resonances from the RS multiplets with \( l = 1, 3, \) and 5 are well described by means of the following simple empirical recursive relation:

\[
M_{\sigma'} - M_{\sigma} = m_1 \left( \frac{1}{\sigma^2} - \frac{1}{(\sigma')^2} \right) + \frac{1}{2} m_2 \left( \frac{\sigma^2 - 1}{2} - \frac{\sigma^2 - 1}{2} \right),
\]

where, again, \( \sigma = k + 1 \).

The two mass parameters take the values \( m_1 = 600 \text{ MeV} \), and \( m_2 = 70 \text{ MeV} \), respectively. The first term on the r.h.s. in Eq. (33) is the typical difference between the energies of two single particle states of principal quantum numbers \( \sigma \) and \( \sigma' \), respectively, occupied by a particle with mass \( m \) moving in a Coulomb-like potential of strength \( \alpha_C \) with \( m_1 = \alpha_C^2 m/2\hbar^2 \).

The term

\[
\frac{\sigma^2 - 1}{2} = 2 \frac{k}{2} \left( \frac{k}{2} + 1 \right), \quad \text{with} \quad k = \sigma - 1,
\]

in Eq. (33) is the generalization of the three-dimensional \( j(j+1) \) rule (with \( j = k/2 \)) to four Euclidean dimensions \(^{17}\) and describes a generalized O(4) rotational band. The parameter \( 1/m_2 = 2, 82 \text{ fm} \) corresponds to the moment of inertia \( \mathcal{J} = 2/5MR^2 \) of some “effective” rigid-body resonance with mass \( M = 1085 \text{ MeV} \) and a radius \( R = 1, 13 \text{ fm} \). Therefore, the energy spectrum in Eq. (33) can be considered to emerge from a quark-\( \mathcal{C} \)-hyperquark model with a Coulomb-like potential \((\mathcal{H}_{\text{Coul}})\) and a four-dimensional rigid rotator \((T_{\text{rot}}^{(4)})\). The corresponding Hamiltonian \( \mathcal{H}^{\text{QHM}} \) that is diagonal in the basis of the O(4) harmonics is given by

\[
\mathcal{H}^{\text{QHM}} = \mathcal{H}_{\text{Coul}} + T_{\text{rot}}^{(4)} = -\frac{\alpha_C}{|\mathbf{r}|} + \frac{1}{2\mathcal{J} F^2}.
\]
Here, $\mathcal{F}^2$ denotes a rigid rotator in four Euclidean dimensions as associated with a collective effect there. Note that while the splitting between the Coulomb-like states decreases with increasing $\sigma$, the difference between the energies of the rotational states increases linearly with $\sigma$ so that the net effect is an approximate equidistantly of the baryon cluster levels (see Fig. 3). In extending the Hamiltonian in Eq. (35) to include O(4) violating terms such like $\sim l \cdot s \sim l^2$, or, $\sim r$ (to account for the confinement), and introducing different moments of inertia $J_i$ (to account for possible deformation effects), the O(3) splitting of the O(4) clusters can be studied along the line of the collective models of nuclear structure.\textsuperscript{18}

6. Summary and Discussion

We showed that the nucleon and $\Delta$ resonances, instead of being uniformly distributed in mass, as naively expected on the basis of a $3q$-Hilbert space without degeneracy, form well pronounced spin- and parity-clusters. To be specific, we showed that all the reported nonstrange nucleon resonances with masses below 2500 MeV up to the one “missing” state $F_{17}$ expected to occur around 1700 MeV, and the four more “missing” states $H_{1,11}, P_{31}, P_{33}$, and $D_{33}$ states with masses above 2000 MeV, are accommodated by the RS multispin-parity fields $\psi_{12}, \psi_{1234}, \psi_{1235}$, and $\psi_{123456}$, or, in equivalent notations, the $2_{2L}, 4_{2L}, 6_{2L}$, Lorentz clusters. This structure of the observed baryon excitations suggests using a two-body Hilbert space containing a fundamental point like O(1,3) (or, Wick-rotated O(4)) bosonic degree of freedom and a fundamental fermionic one (quark) rather than three independent quark d.o.f. In this way Lorentz multiplets can appear as bound states. The “missing” states fell into the $3_{2L}, 5_{2L}$, and $5_{2L}$, associated in turn with the RS fields $A_{1234} \otimes \psi, V_{1234} \otimes \psi$, and $V_{12345} \otimes \psi$ from Table 1. Also states with $\{m, m\} \otimes \{0, m\}$ hyperquarks, in our language, are “missing.” While the absence of the “missing” RS clusters from the spectra can be interpreted in terms of exclusion of certain type of parity through the mode of chiral symmetry realization in the different excitation domains, $\{\{m, 0\} \otimes \{0, m\}\} \otimes \psi$ can either be excited by photons, or, suppressed, if they turn out to be of the unusual BWW type. In the idealized case, the C-hyperquark–quark system decouples completely from the remaining $3q$-configurations and the resonances “missing” for the completeness of the nonstrange baryons are only five (see Sec. 2). In reality, some of those couplings may not vanish and few more “missing” resonances can show up. The masses of the RS-clusters and their spacing were shown to follow O(4) rotational bands slightly modified by a Balmer-like term. The collective character of the baryon excitations illustrated in Fig. (3) may indicate that one should not expect to find the complete $3q$-Hilbert space realized in baryon spectra.

Finally, some remarks on how the model suggested here is related to QCD — the gauge theory of strong interaction, are in place. First of all, the SU(2)\textsubscript{11} $\otimes$ O(1,3)\textsubscript{10} symmetry considered by us is simultaneously symmetry of the QCD Lagrangian too, which, in being manifestly Lorentz invariant, is based upon isovector and isosinglet quark degrees of freedoms. In addition, the convenience of the quark–diquark picture is independently supported by an observation recently under debates in the literature. It concerns the fact, that the relativistic generalization of the total orbital angular momentum, $L$, of the three constituent quarks, defined as $\int dx^3 \psi^4 (x \times (-iD)) \psi$, where $\psi = uud$, and $D_i = \partial_i - igA_i^aT^a$ does not satisfy the orbital angular momentum algebra. The introduction of diquark correlations, however, accounts for a solution of this problem (see Ref.\textsuperscript{19} for details). Moreover, the $1/r$ potential used here in combination with a four-dimensional rigid rotator reflects the gluon gauge dynamics of QCD. The Coulomb-like potential was frequently used in quark models as it emerges out of the one-gluon exchange between quarks and is relevant for low energy spectroscopy. However, its contribution to the $3q$-dynamics is of minor use mainly because of the collapse of the spectra with
increasing principal quantum number. We here balanced out the decrease of the cluster-level spacing at higher \( \sigma \)'s by the energy increase of the O(4) rotational states. The significance of the \( 1/r \) potential below 1.5 GeV reflects the importance of the one-gluon exchange in that domain, while the four-dimensional rigid rotator basically determines the baryon spectrum above 1.5 GeV and may point onto an increasing role of the nonperturbative multigluon exchange at this scale.

7. *  
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