An all optical implementation of quantum information processing with semiconductor macroatoms is proposed. Our quantum hardware consists of an array of semiconductor quantum dots and the computational degrees of freedom are energy-selected interband optical transitions. The proposed quantum-computing strategy exploits exciton-exciton interactions driven by ultrafast sequences of multi-color laser pulses. Contrary to existing proposals based on charge excitations, the present all-optical implementation does not require the application of time-dependent electric fields, thus allowing for a sub-picosecond, i.e. decoherence-free, operation time-scale in realistic state-of-the-art semiconductor nanostructures.

The introduction of quantum information/computation (QIC)\(^1\) as an abstract concept, has given birth to a great deal of new thinking about how to design and realize quantum-information processing devices. This goal is extremely challenging: one should be able to perform, on a system with a well-defined quantum state-space (the computational space), precise quantum-state preparation, coherent quantum manipulations (gating) of arbitrary length, and state detection as well. It is well known that the major obstacle to implement this ideal scheme is decoherence: the spoiling of the unitary character of quantum evolution due to the uncontrollable coupling with environmental, i.e., non-computational, degrees of freedom. Mostly due to the need of low decoherence rates, the first proposals for experimental realizations of quantum-information processing devices originated from specialties in atomic physics\(^2\)-\(^5\), in quantum optics\(^6\),\(^7\), and in nuclear and electron magnetic-resonance spectroscopy\(^8\),\(^9\). On the other hand practically relevant quantum computations require a large number of quantum-hardware units (qubits) that are known to be hardly achievable in terms of such systems. In contrast, in spite of the serious difficulties related to the “fast” decoherence times, a solid-state implementation of QIC seems to be the only way to benefit synergistically from the recent progress in ultrafast optoelectronics\(^10\) as well as in meso/nanostructure fabrication and characterization\(^11\). Among the proposed solid-state implementations one should mention those in superconducting device physics\(^12\),\(^13\), in electron physics\(^14\), and in meso- and nanoscopic physics\(^15\). In particular the first semiconductor-based proposal relies on spin dynamics in quantum dots (QDs), it exploits the low decoherence of spin degrees of freedom in comparison to the one of charge excitations. On the other hand, as originally envisioned in 16, gating of charge excitations could be performed by exploiting present ultrafast laser technology, that allows to generate and manipulate electron-hole quantum states on a sub-picosecond time-scale: coherent-carrier-control\(^17\),\(^18\). In this respect decoherence times on nano/microsecond scales can be regarded as “long” ones.

Based on this idea a few implementations have been recently put forward\(^19\),\(^20\); However, while in these proposals single-qubit operations are implemented by means of ultrafast optical spectroscopy, the control of two-qubit operations still involves the application of external fields and/or microcavity-mode couplings, whose switching times are much longer than decoherence times in semiconductor QDs. It clearly follows that such proposals are currently out of reach in terms of state-of-the-art optoelectronics technology. As already pointed out in\(^16\), in order to take full advantage from modern ultrafast laser spectroscopy one should be able to design fully optical gating schemes able to perform single- and two-qubit operations on a sub-picosecond time-scale.

Following this spirit, in this paper we propose the first all-optical implementation with semiconductor macroatoms. To overcome the above mentioned limitations, a novel scheme is presented, where two-qubit coupling is controlled by the many-body quantum state of the few-electron system only, without resorting to slowly-varying external fields and cavity-mode couplings. To this end, we shall exploit few-electron effects in semiconductor macroatoms\(^11\), i.e., exciton-exciton interactions in coupled QDs driven by multicolor sequences of ultrafast laser pulses. The microscopic origin of such exciton-exciton dipole coupling is the same of the Forster process exploited by Quiroga and Johnson\(^21\) for the generation of entangled states in coupled QDs. More specifically, our analysis is based on a realistic, fully three-dimensional, description of multi-QD structures, whose many-body electron-hole Hamiltonian can be schematically written as:

\[
\mathbf{H} = \mathbf{H}_0 + \mathbf{H} = (\mathbf{H}_c + \mathbf{H}_{ce}) + (\mathbf{H}_{cl} + \mathbf{H}_{env})
\]

where \(\mathbf{H}_0\) describes the non-interacting electron-hole system within the nanostructure confinement potential, \(\mathbf{H}_c\) is the sum of the three (electron-electron, hole-hole, and electron-hole) Coulomb-interaction terms, \(\mathbf{H}_{cl}\) describes the
coupling of the carrier system with a classical light field, while $H_{\text{exc}}$ describes the interaction of the carrier system with environmental degrees of freedom, like phonon and plasmon modes of the host material. The latter is responsible for decoherence processes and it will be accounted for within a density-matrix formalism (see below). Their explicit form — involving electron and hole creation/destruction operators for the various single-particle multisubband states — can be found in 22. Here, the carrier-light (cl) coupling term is responsible for the creation/recombination of electron-hole pairs and, as we shall see, it will induce the desired single-qubit rotations. For a given number of electron-hole pairs $N$, a direct diagonalization$^{23}$ of $H_e$ will provide the many-body energy levels and wavefunctions of the interacting electron-hole system, which will exhibit few-carrier effects typical of quasi-0D structures$^{11}$. Given such interacting-carrier states, it is possible to evaluate many-exciton optical spectra, i.e., the absorption probability corresponding to the generic $N \rightarrow N'$ transition.

The above theoretical scheme has been applied to realistic state-of-the-art QD arrays. In particular, as quantum hardware, we consider a GaAs-based structure with in-plane parabolic confinement potential$^{24}$: The square-like carrier confinement along the growth ($z$) direction for electrons and holes is schematically depicted in Fig. 1 for the array unit cell $a + b$. This is tailored in such a way to allow for an energy-selective creation/destruction of bound electron-hole pairs (i.e., excitons) in dots $a$ and $b$. The inter-dot barrier width ($w \sim 50 \text{ Å}$) is such to prevent single-particle tunneling and at the same time to allow for significant inter-dot Coulomb coupling. Moreover, as discussed below (see Fig. 2), in order to induce a significant exciton-exciton dipole coupling, an in-plane static electric field $F$ is applied. We stress that the geometrical and material parameters of the proposed prototypical structure in Fig. 1 are fully compatible with current QD growth and characterization technology$^{25}$. As a starting point, let us discuss the optical response of the semiconductor macromolecule $(a + b)$ in Fig. 1 to a single- as well as a double-pulse sequence. The excitonic $(0 \rightarrow 1)$ optical spectrum in the presence of an in-plane electric field $F = 15 \text{ kV/cm}$ is shown in Fig. 2A, where the two lowest optical transitions correspond to the formation of direct ground-state excitons in dot $a$ and $b$, respectively (see Fig. 1). In contrast, the high-energy peaks correspond to optical transitions involving excited states of the in-plane parabolic potential. Due to the strong in-plane carrier confinement, such low-energy excitonic states are expected to closely resemble the corresponding single-particle ones, thus involving the parabolic-potential ground state only. This is confirmed by the excitonic spectrum (solid curve) in Fig. 2B, which has been obtained limiting our single-particle basis set to the parabolic-potential ground state. As we can see, apart from a small rigid shift, the relative position of the lowest transitions is the same. As anticipated, this suggests to use as a basis of our computational space the set formed by the lowest excitonic transition in each QD.

Let us now come to the biexcitonic spectrum (dashed line in Fig. 2B): it describes the generation of a second electron-hole pair in the presence of a previously created exciton $(1 \rightarrow 2$ optical transitions). Since all the spectra in Fig. 2 have been computed assuming coherent light with a well-defined polarization, the generation of two excitons within the lowest state of the same QD is not allowed. Therefore, the two structures in the biexcitonic spectrum (dashed curves in Fig. 2B) correspond to the formation of an exciton in dot $a$ given an exciton in dot $b$ and vice versa.

The crucial feature in Fig. 2B is the magnitude of the "biexcitonic shift"$^{11}$, i.e., the energy difference between the excitonic and the biexcitonic transition (see solid and dashed curves). For the QD structure under investigation we get energy splittings up to $5 \text{ meV}$, which is by far larger than typical biexcitonic splittings in single QDs and does not depend on our Hilbert-space truncation. This is due to the in-plane static field $F$, which induces a charge separation between electrons and holes. This, in turn, gives rise to significant dipole-dipole coupling between adjacent excitonic states. The physical origin of the biexcitonic shift $\Delta E_c$ is qualitatively described in Fig. 2C, where we show the electron and hole charge distribution corresponding to the biexcitonic ground state. The shift $\Delta E_c$ is the result of the competition between the two repulsive Coulomb interactions ($e_a - e_b$ and $h_a - h_b$) and the attractive ones ($e_a - h_b$ and $e_b - h_a$). As we can see, the charge separation induced by the static field increases significantly the average distance between electrons and holes, thus decreasing their attractive interaction. On the other hand, the repulsive terms are basically field independent. This is the origin of the positive energy difference in Fig. 2B. The biexcitonic shift as a function of the in-plane electric field $F$ is reported as inset in Fig. 2B.

The central idea in our QIC proposal is to exploit such few-exciton effect to design conditional operations: the presence of an exciton in a quantum dot can prevent the generation of a second exciton by varying the detuning and thus controlling the resonance condition in the photogeneration process. The analysis presented so far suggests the introduction of a simplified model providing an effective description of the electron-hole system within our computational space. To this end let us introduce the excitonic occupation number operators $n_{l}$, where $l$ denotes the generic QD in our array. The two states with eigenvalues $n_l = 0$ and $n_l = 1$ correspond, respectively, to the absence (no conduction-band electrons) and to the presence of a ground-state exciton (a Coulomb-correlated electron-hole pair) in dot $l$; they constitute our single-qubit basis: $|0\rangle_l$ and $|1\rangle_l$. It follows that the initial state $|\psi\rangle = |0\rangle_1$ coincides with the QD ground state in the absence of laser excitations. Moreover, in view of the interband character of our qubit, thermal-activation effects play a very minor role.

The whole computational state-space $\mathcal{H}$ is then spanned by the basis $|n\rangle = \otimes_l |n_l\rangle$, ($n_l = 0, 1$).

The full many-body Hamiltonian $H_e$ in (1) restricted to the above computational space $\mathcal{H}$ reduces to
Here, $\mathcal{E}_l$ denotes the energy of the ground-state exciton in dot $l$ while $\Delta\mathcal{E}_{ll'}$ is the biexcitonic shift due to the Coulomb interaction between dots $l$ and $l'$, previously introduced (see Fig. 2). The effective Hamiltonian in (2) has exactly the same structure of the one proposed by Lloyd in his pioneering paper on quantum cellular automata\textsuperscript{26}, and it is the Model Hamiltonian currently used in most of the NMR quantum-computing schemes\textsuperscript{27}. This fact is extremely important since it tells us that: (i) the present semiconductor-based implementation contains all relevant ingredients for the realization of basic QIC processing; (ii) it allows to establish a one-to-one correspondence between our semiconductor-based scheme and much more mature implementations, like NMR\textsuperscript{27}. According to (2), the two-body, i.e. two-dot, interaction term does not affect the individual single-dot excitonic eigenstates; the exciton-exciton coupling simply results in a spectrum shift $\Delta\mathcal{E}_l$, thus properly describing the strong-confinement regime typical of our QD structures (see Fig. 2). Indeed, as it can be trivially derived from (2), the single-exciton energy $\mathcal{E}_l$ is renormalized by the biexcitonic shift $\Delta\mathcal{E}_{ll'}$, induced by the presence of a second exciton in dot $l'$. The dependence of the single-qubit energy on the occupation of the neighboring sites ($\mathcal{E}_l = \mathcal{E}_l + \sum_{l' \neq l} \Delta\mathcal{E}_{ll'} n_{l'}$) is the crucial ingredient that allows for the implementation of a conditional dynamics required for universal QC. In order to illustrate this idea, let us focus again on the two-QD structure, i.e. two-qubit system, of Fig. 1 and fix our attention on one of the two dots, say dot $b$. The effective energy gap between $|0\rangle_b$ and $|1\rangle_b$ depends now on the occupation of dot $a$. This elementary remark suggests to design properly tailored laser-pulse sequences to implement controlled-not logic gates among the two QDs as well as single-qubit rotations. Indeed, by sending an ultrashort laser $\pi$-pulse\textsuperscript{28} with central energy $\hbar\omega_b[n_a] = \mathcal{E}_b + \Delta\mathcal{E}_{ba} n_a$, the transition $|n_b\rangle_b \rightarrow |1 - n_b\rangle_b$ ($\pi$-rotation) of the target qubit (dot $b$) is obtained if and only if the control qubit (dot $a$) is in the state $|n_a\rangle_a$. Notice that the above scheme corresponds to the so-called selective population transfer in NMR\textsuperscript{27}, alternative procedure used in that field can be adopted to the present proposal as well. In the very same way, by using a laser pulse with central energy $\hbar\omega_a[n_b]$, the role of the target and control qubit is interchanged. By denoting with $U_b^\pi$ the unitary transformation induced by the laser $\pi$-pulse of central frequency $\omega_l[n_l]$, the above conditional gate corresponds to the transformation $U_b^\pi$. Moreover one can easily check that the two-color pulse sequence $U_b^\pi U_b^\pi$, achieves the unconditional $\pi$-rotation of the $l$-th qubit.

In order to test the viability of the proposed quantum-computation strategy applied to the semiconductor-based quantum hardware of Fig. 1, we have performed a few simulated experiments of basic quantum-information processing\textsuperscript{29}. They are based on a numerical solution of the Liouville-von Neumann equation describing the exact quantum-mechanical evolution of the many-exciton system (2) within our computational subspace $\mathcal{H}$ in the presence of environment-induced decoherence processes [see term $\mathcal{H}_{\text{env}}$ in Eq. (1)]. The latter are accounted for in our density-matrix formalism by means of a standard $T_1T_2$ model: we employ a band-to-band recombination time $T_1 = 1\text{ ns}$ and we describe phonon-induced decoherence processes in terms of an exciton-phonon dephasing time $T_2 = 30\text{ ps}$, which is compatible with the experimental values given in 18. Figure 3 shows a simulated sequence of single- plus two-qubit operations driven by the multi-color laser-pulse train depicted in (A). At $t = 0$ the system is in the state $|0, 0\rangle \equiv |0\rangle_a \otimes |0\rangle_b$. The first two-pulse sequence (at $t = 5\text{ ps}$ and $t = 10\text{ ps}$) is tailored in such a way to induce an unconditional $\frac{\pi}{2}$ rotation of the qubit $a$: $|0, 0\rangle \rightarrow (|0, 0\rangle + |1, 0\rangle)/\sqrt{2}$. Since the qubit $b$ is in state $|0\rangle$, the rotation is induced by the first pulse only. At time $t = 15\text{ ps}$ a third pulse induces a conditional $\pi$-rotation of the qubit $b$: $|0, 0\rangle + |1, 0\rangle \rightarrow |0, 0\rangle + |1, 1\rangle$. This last operation plays a central role in any quantum-information processing, since it transforms a factorized state $((|0\rangle + |1\rangle) \otimes |0\rangle)$ into an entangled state. The scenario described so far is confirmed by the time evolution of the exciton occupation numbers $n_a(t)$ and $n_b(t)$ reported in (B) as well as of the square modulus of the state coefficients (in our four-dimensional computational basis) reported in (C).

Let us finally extend the proposed computational scheme to an arbitrary number of QDs. To begin with, due to the short-range nature of the exciton-exciton dipole interaction, we are allowed to consider nearest-neighbor coupling only: indeed, for the specific quantum hardware considered (see Fig. 1), in view of the \(\frac{1}{r^3}\) behaviour and, more important, of their small oscillator strengths, second-neighbor couplings play no significant role. The frequency of the transition $|0\rangle_l \rightarrow |1\rangle_l$ depends now on the state $|n_{l-1}\rangle_l \otimes |n_{l+1}\rangle_{l+1}$ of the two neighboring QDs: $\hbar\omega_l[n_{l-1}, n_{l+1}] = \mathcal{E}_l + \Delta\mathcal{E}_{l,l-1} n_{l-1} + \Delta\mathcal{E}_{l,l+1} n_{l+1}$. By analogy with the two-qubit case, let us denote with $U_b^{n_{l-1}, n_{l+1}}$ the corresponding unitary transformation. It is easy to check that arbitrary unconditional rotations of the $l$-th qubit can be realized by means of the multi-color pulse sequence $\prod_{n_{l-1}, n_{l+1}} U_b^{n_{l-1}, n_{l+1}} = U_b^{1,0} U_b^{0,1} U_b^{1,1} U_b^{1,1}$. Turning to the two-qubits gate one immediately sees that it is important to design our QD array in such a way to avoid the left-right symmetry, i.e., $\Delta\mathcal{E}_{l,l+1} = \Delta\mathcal{E}_{l,l-1}$, which would imply that both nearest neighbors of a given target qubit play the role of control qubit. It is then clear that, in order to obtain genuine two-qubit conditional operations, this potential local degeneracy has to be lifted, if necessary, by a proper choice of the inter-dot distance in the array\textsuperscript{30}. It then follows that, e.g., the pulse sequence $U_b^{1,0} U_b^{1,1}$ realizes the controlled-not between the qubits $l - 1$ and $l$. This completes our strategy for universal quantum computation with the proposed semiconductor-based quantum hardware\textsuperscript{31}.
Let us finally come to the state measurement. In view of the few-exciton character of the proposed quantum hardware, the conventional measurement of the carrier subsystem by spectrally-resolved luminescence needs to be replaced by more sensitive detection schemes. To this end, a viable strategy could be to apply to our semiconductor-based structure the well-known recycling techniques commonly used in quantum-optics experiments\textsuperscript{32}.

At this point a few comments are in order. First we stress a very important feature of the proposed semiconductor-based implementation: as for NMR quantum computing, two-body interactions are always switched on\textsuperscript{33}; conditional as well as unconditional dynamics is realized by means of sequences of ultrafast single-qubit operations whose length does not scale as a function of the total number of QDs in the array.

The nanoscale range of the inter-dot coupling we employed for enabling conditional dynamics does not allow for space-selective optical addressing of individual qubit. For this reason, at least for our basic QD molecule $(a + b)$, we resorted to an energy selective addressing scheme. However, extending such strategy to the whole QD array would imply different values of the excitonic transition in each QD, i.e., $E_l \neq E_l'$. This, besides obvious technological difficulties, would constitute a conceptual limitation of scalability towards massive Quantum Computations. This problem can be avoided following a completely different strategy originally proposed by Lloyd\textsuperscript{26} and recently improved in\textsuperscript{34} by properly designed sequences of multicolor global pulses within a cellular-automaton scheme, local addressing is replaced by information-encoding transfer along our QD array.

Finally, a present limitation of the proposed quantum hardware are the non-uniform structural and geometrical properties of the QDs in the array, which may give rise to energy broadenings larger than the biexcitonic shift. However, recent progress in QD fabrication—including the realization of QD structures in microcavities—will allow, we believe, to overcome this purely technological (non conceptual) limitation.

In summary, the first all optical implementation of QIC with a semiconductor-based quantum hardware has been proposed. Our analysis has shown that energy-selected optical transitions in realistic state-of-the-art QD structures are good candidates for quantum-information encoding and manipulation. The sub-picosecond time-scale of ultrafast laser spectroscopy allows for a relatively large number of elementary operations within the exciton decoherence time.
With \( \pi \)-pulse we mean an ultrafast laser pulse whose amplitude is tailored in such a way to induce a \( \pi \) rotation of the interband Bloch vector.

Our time-dependent simulations are based on the realistic state-of-the-art QD structure of Fig. 1: \( E_a = 1.70 \) eV, \( E_b = 1.71 \) eV, \( \Delta E = 2.4 \) meV, which correspond to \( F = 20 \) kV/cm (see Figs. 2 and 3).

The required symmetry breaking can be obtained, e.g., by a repetition of the unitary cell \( a+b \) of Fig. 1 choosing the inter-cell distance such that \( \Delta E_{l,l+1} \neq \Delta E_{l,l-1} \).

Indeed, it is well known that, to this aim, local single- and two-qubit gates suffice. Conditional dynamics between arbitrarily separated qubits can be implemented by sequences of local gates with only a polynomial increase of computational complexity.

This should be compared to the schemes in which two-qubit gates are realized by turning on and off the coupling between subsystems, e.g., by means of slowly-varying fields and cavity-mode couplings.

FIG. 1. Schematic representation of the square-like potential profile for electrons (e) and holes (h) along the growth (z) direction of our QD array. The different well width of dots \( a \) and \( b \) gives rise to different single-particle confinement energies.

FIG. 2. Optical response of the array unit cell \( (a+b) \) in Fig. 1. (A) Excitonic spectrum obtained including the realistic multilevel structure of the in-plane parabolic potential. (B) Excitonic (solid curve) and biexcitonic spectrum (dashed curve) obtained including the in-plane ground state only. (C) Three-dimensional view of the spatial charge distributions of the two electrons (\( e_a \) and \( e_b \)) and holes (\( h_a \) and \( h_b \)) corresponding to the biexcitonic ground state in (B).

FIG. 3. Time-dependent simulation of a single (unconditional) plus a two-qubit (conditional) operations (see text). (A) Multi-color sequence of ultrafast laser pulses used in the simulation, (B) exciton populations (\( n_a \) and \( n_b \)), and (C) square modulus of the four state coefficients as a function of time.

