Field theory model giving rise to "quintessential inflation" without the cosmological constant and other fine tuning problems

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A field theory is developed based on the idea that the effective action of yet unknown fundamental theory, at energy scale below the Planck mass $M_p$, has the form of expansion in two measures:

$$ S = \int \! d^4x [\Phi L_1 + \sqrt{-g}L_2] $$

where the new measure $\Phi$ is defined using the antisymmetric tensor field $\Phi d^4x = \partial_{[\alpha}A_{\beta\gamma\delta]} dx^\alpha \wedge dx^\beta \wedge dx^\gamma \wedge dx^\delta$. A shift $L_1 \rightarrow L_1 + \text{const}$ does not affect equations of motion whereas a similar shift when implementing with $L_2$ causes a change which in the standard GR would be equivalent to that of the cosmological constant (CC) term. The next basic conjecture is that the Lagrangian densities $L_1$ and $L_2$ do not depend on $A_{\mu\nu\lambda}$. The new measure degrees of freedom result in the scalar field $\chi = \Phi/\sqrt{-g}$ alone. The constraint appears that determines $\chi$ in terms of matter fields. After conformal transformation to the new variables (Einstein frame), all equations of motion take canonical GR form of equations for gravity and matter fields and, therefore, the models we study are free of the well-known "defects" that distinguish the Brans-Dicke type theories from the Einstein’s GR. All novelty is revealed only in an unusual structure of the effective potentials and interactions which turns over our intuitive ideas based on our experience in field theory. For example, the greater $\Lambda$ we admit in $L_2$, the smaller effective inflaton potential $U(\phi)$ will be in the Einstein picture. Field theory models are suggested with explicitly broken global continuous symmetry, which in the Einstein frame has the form $\phi \rightarrow \phi + \text{const}$. The symmetry restoration occurs as $\phi \rightarrow \infty$. A few models are presented where the effective potential $U(\phi)$ is produced with the following shape: for $\phi \lesssim -M_p$, $U(\phi)$ has the form typical for inflation model, e.g. $U = \lambda \phi^4$ with $\lambda \sim 10^{-14}$; for $\phi \gtrsim -M_p$, $U(\phi)$ has mainly exponential form $U \sim e^{-a\phi/M_p}$ with variable $a$: $a = 14$ for $-M_p \lesssim \phi \lesssim M_p$, that gives possibility for nucleosynthesis and large-scale structure formation; $a = 2$ for $\phi \gtrsim M_p$, that implies the quintessence era. There is no need in any fine tuning to prevent appearance of the CC term or any other terms that could violate the flatness of $U(\phi)$ at $\phi \gg M_p$. $\lambda \sim 10^{-14}$ is obtained without fine tuning as well. Quantized matter fields models, including spontaneously broken gauge unified theories, can be incorporated without altering the mentioned above results. Direct coupling of fermions to the inflaton resembles the Wetterich’s (1995) model but it does not lead to any observable effect at the late universe. SSB does not raise any problem with CC at the late universe.

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Recent high-redshift and CMB data [1] suggest that a small effective cosmological constant gives a dominant contribution to the energy density of the present universe. Among the attempts to describe this picture, the idea to profit by the properties of a slow-rolling scalar field (quintessence model) [2]- [8] seems to be the most attractive and successful. In such approach, the present vacuum energy density $\rho_{\text{vac}} \sim 10^{-47}(GeV)^4$ has to be imitated by the energy density of a slowly rolling scalar field down its potential $U(\phi)$ which presumably approaches zero as $\phi \to \infty$. However all known quintessence models contain two fundamental problems:

1. The cosmological constant problem [9], [10] remains in the quintessence models as well: particle physics and cosmology must give a distinct mechanism that enforces the effective cosmological constant to decay from an extremely large value in the very early universe to the extremely small present value without fine tuning of parameters and initial conditions.

2. All known quintessence models are based on the choice of some specific form for the potential $U(\phi)$. For example, in the most popular models either the inverse power low or the exponential form for $U(\phi)$ is supposed. The general feature of the potentials needed to realize quintessence consists in the demand for $U(\phi)$ to be flat enough as $\phi$ is large enough in order to provide conditions for the slow-roll approximation. However it is not clear what happens with other possible terms in the potential, including quantum corrections [11]. In fact, the potential may for instance contain terms that constitute a structure of polynomials in $\phi$ (and $\phi^n \ln \phi$) and they are not negligible as $\phi$ is large enough, unless an extreme fine tuning is assumed for the mass and self-couplings [11]. For example, the restriction of the flatness conditions on the quartic self-interaction $\lambda \phi^4$ is $\lambda \ll 10^{-120}(\frac{M_p}{\phi})^2$, that implies an extreme fine tuning as well.

In this paper I am going to present a field theory model that resolves the above fine tuning problems and besides that, this model is able to give a broad range of tools for constructive answering few more important questions:

3. In the framework of a model where potential $U(\phi)$ of the exponential or inverse power low (or there combinations [8]) form plays the role of a quintessence potential as $\phi$ is large enough, the question arises what is the cosmological role of such $U(\phi)$ as $\phi$ is close to zero or negative. If some other scalar field (inflaton) is responsible for an inflation of the early universe, then a field theory has to explain why the potential $U(\phi)$ of the scalar field $\phi$ is negligible as $\phi$ is close to zero or negative. However, if the same quintessence field $\phi$ plays also the role of the inflaton [12], [13] (in the early universe) then again a field theory has to explain [14] an origin of the relevant effective potential. Of course this is a nontrivial problem. For example, Peebles and Vilenkin [13] have presented an interesting model of a single scalar field that drives the inflation of the early universe and ends up as quintessence. They adopt the monotonic potential $U(\phi) = \lambda m^4 [1 + (\phi/m)^4]$ for $\phi < 0$, $\frac{\lambda m^4}{1 + (\phi/m)^4}$ for $\phi \geq 0$ (1)

where $\alpha = \text{const} > 0$ (for example 4 or 6) and the parameters $\lambda = 10^{-14}$ and $m = 8 \times 10^5 GeV$ were adjusted in [13] to achieve a satisfactory agreement with the main observational constraints. It is well known [15] that such extremely small value of $\lambda$ is dictated in the $\lambda \phi^4$ theory of the chaotic inflation scenario by the necessity to obtain a density perturbation $\frac{\delta \rho}{\rho} \sim 10^{-5}$ in the observable part of the universe. In other words, the potential of this "quintessential inflation" model includes both the fine tuning required by the inflation of the early universe and the fine tuning dictated by the quintessential model of the late universe. As it is pointed out in Ref. [13], it seems also to be an unnatural feature of this model that a small mass $m = 8 \times 10^5 GeV \ll M_p$ must appear in the potential of the inflaton field $\phi$, interacting only with gravity. And finally, one should apparently believe that such sort of "quintessential inflation" potential must be generated by some field theory without fine tuning. These problems are typical for the quintessential inflation type models [12], [13].

4. It is known that the intriguing problem for quintessence going by the name the "coincidence problem" [16] can be avoided in the framework of the quintessence models that make use "tracker potentials" [8]. The exponential potential with $a = \text{const}$

$$U(\phi) = U_0 e^{-a\phi/M_p}$$ (2)

is a special example of a tracker solution [8]. It is well known [2], [17], [7] (see also Ref. [18] where a similar result was achieved in the context of Kaluza-Klein-Casimir cosmology), that in the spatially flat models with such potential,

\[1\]$^1$We use the notation $M_p$ for the reduced Planck mass: $M_p = (8\pi G)^{-1/2}$. 

the ratio of the scalar field $\phi$ energy density to the total matter energy density rapidly approaches a constant value determined by $a$ and the matter equation of state. However the strong constraint on $\Omega_\phi$ dictated by cosmological nucleosynthesis ($\Omega_\phi \lesssim 0.2$) [6], [7], [19] predetermines for the $\phi$-fraction to remain subdominant one in the future that apparently contradicts the observable accelerated expansion. A possible resolution of this problem proposed by Wetterich [6] consists in the idea that $a$ in (2) might be $\phi$-dependent. In that case one can again conclude that it would be very attractive to develop a field theory model where the exponential potential (2) with an appropriate $\phi$-dependent $a$ is generated in a natural way.

5. Since the mass of excitations of the $\phi$-field has to be extremely small in the present-day universe ($m_\phi \lesssim H_0 \sim 10^{-33} eV$), possible direct couplings of $\phi$ to the standard matter fields should give rise to very long-range forces which do not obey the equivalence principle [20]. To prevent such undesirable effects, the very strong upper limits on the coupling constants of the quintessence field to the standard matter fields have to be accepted without any known reason: an attempt to construct a model where an unbroken symmetry could support zero mass of $\phi$-excitations [21] inevitably runs against the necessity to start from a trivial potential [20]; without knowledge of a mechanism for the breaking of this symmetry, such small coupling constants may be introduced into a theory only by hand.

It will be shown in this paper that one can answer all the above questions 1-5 in the framework of the field theory model based on the hypothesis that the effective action of the fundamental theory at the energy scales below the Planck mass can be represented in a general form including two measures and respectively, two Lagrangian densities

$$S = S_1 + S_2$$

$$S_1 = \int \Phi L_1 d^4x$$

$$S_2 = \int \sqrt{-g} L_2 d^4x$$

Here $\sqrt{-g}$ is the standard measure of integration in the action principle of Einstein’s General Relativity (GR) and other gravitational theories making use the general coordinate invariance. The measure $\Phi$ is defined using the antisymmetric tensor field $A_{\mu \nu \lambda}$

$$\Phi d^4x = \partial_{[\alpha} A_{\beta \gamma \delta]} dx^\alpha \wedge dx^\beta \wedge dx^\gamma \wedge dx^\delta$$

and (4) is also invariant under general coordinate transformations. Notice that the measure $\Phi$ is a total derivative and therefore a shift $L_1 \rightarrow L_1 + \text{const}$ does not affect equations of motion whereas a similar shift when implementing with $L_2$ causes a change which in the standard GR would be equivalent to that of the cosmological constant term. The next basic conjecture is that the Lagrangian densities $L_1$ and $L_2$ do not depend of $A_{\mu \nu \lambda}$. In this paper I refer to this theory as the "two measures theory" (TMT). The main features of TMT have been studied in series of papers [22]-[25].

In Ref. [25] we have explicitly presented a broad class of TMT models free of the cosmological constant problem. By this statement we meant there that the models possess all three following features:

a) A conformal Einstein frame exists where all equations of motion have canonical form of the Einstein’s GR equations, that is the models are free of the well-known defects of the scalar-tensor theories. All difference from GR consists in an unusual structure of the effective potential and interactions.

b) The energy density of the true vacuum state appears to be zero without any sort of fine tuning for infinite number of initial conditions. This takes place also in models (including gauge models) where the true vacuum state is realized due to a spontaneous symmetry breaking (SSB).

c) The models allow for the very early universe undergoes an inflationary stage driven by the scalar field (inflaton).

2Notice that Eq. (4) has a form of the coupling of the "space-filling" brane [26] with the 4-form $\partial_{[\alpha} A_{\beta \gamma \delta]}$ and a further coupling with the Lagrangian density $L_1$ (for details see Ref. [23]). In Refs. [22]- [25] we have also studied the models where the measure $\Phi$ in the $d$-dimensional space-time is defined via $d$ independent "measure scalar fields" which is a particular realization of the model (6). The theory based on the independent measure scalar fields has some specific features that can appear to be important. We will return to this question in Discussion.
Unfortunately, in order to reach the standard equations of motion for gravity and all matter fields in the Einstein frame, we were forced in [25] to start from the very nonlinear (in matter fields) Lagrangian densities $L_1$ and $L_2$. In the most advanced models of Ref. [25] (see Secs. III-V therein) such sort of nonlinearity was needed for example in order to generate the standard fermionic mass term in the Einstein frame.\(^3\) Because of that and some other reasons which will be discussed in detail in the present paper, the quantization of the matter fields in the framework of studied in Ref. [25] TMT models seems to be an inaccessible problem.

In the present paper we will study a broad class of new TMT models that are able to resolve all the above mentioned problems 1-5. Constructing these models we will use also an idea that the theory possesses an explicitly broken continuous global symmetry \(^4\) which, in the variables of the Einstein frame, takes a form $\phi \rightarrow \phi + \text{const}$. It will be shown that just in the framework of these models it is possible to define an appropriate background in a natural way. Then starting from the original TMT action (3)-(5) with the standard degree of nonlinearity in matter fields we obtain in the Einstein frame the standard matter field theory in curved background. After this, the matter fields quantization in such a background is an absolutely standard well known procedure [29]. Studying interrelation between resolution of the problems 1-5 and the possibility for matter fields quantization in TMT is actually one of the main aims of this paper.

II. HOW THE TWO MEASURES THEORY (TMT) WORKS

To demonstrate general features of TMT let us consider a simple model with the scalar field $\phi$

$$L_1 = -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu \nu} \phi,_{\mu} \phi,_{\nu} - V_1(\phi)$$  \hspace{1cm} (7)

$$L_2 = V_2(\phi)$$  \hspace{1cm} (8)

The case where $V_2(\phi) \equiv \text{const}$ was studied in Ref. [25] and the general case was studied by Guendelman in Ref. [27]. The TMT gives desirable results if we proceed in the first order formalism (metric $g_{\mu \nu}$ and connection $\Gamma^\lambda_{\mu \sigma}$ are independent variables as well as the antisymmetric tensor field $A_{\mu \nu \lambda}$) and $R(\Gamma, g) = g^{\mu \nu} R_{\mu \nu}(\Gamma)$, $R_{\mu \nu}(\Gamma) = R_{\lambda \mu \nu \lambda}(\Gamma)$ and

$$R_{\mu \nu \sigma}^\lambda(\Gamma) \equiv \Gamma^\lambda_{\mu \nu, \sigma} + \Gamma^\lambda_{\alpha \sigma} \Gamma^\alpha_{\mu \nu} - (\nu \leftrightarrow \sigma).$$  \hspace{1cm} (9)

At this stage no specific form for $V_1(\phi)$ and $V_2(\phi)$ is assumed.

Varying the action (3)-(5) with respect to $A_{\mu \nu \lambda}$ we obtain

$$\epsilon^{\mu \nu \alpha \beta} \partial_\beta L_1 = 0$$  \hspace{1cm} (10)

which means that

$$L_1 = -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu \nu} \phi,_{\mu} \phi,_{\nu} - V_1(\phi) = sM^4 = \text{const},$$  \hspace{1cm} (11)

where $sM^4$ is an integration constant, $s = \pm 1$ and $M$ is a constant of the dimension of mass.

Performing the variation with respect to $g^{\mu \nu}$ we get

$$-\frac{1}{\kappa} R_{\mu \nu}(\Gamma) + \frac{1}{2} g^{\mu \nu} \phi,_{\mu} \phi,_{\nu} - \frac{1}{2\chi} V_2(\phi) g_{\mu \nu} = 0$$  \hspace{1cm} (12)

where the scalar field $\chi$ is defined by

$$\chi \equiv \frac{\Phi}{\sqrt{-g}}$$  \hspace{1cm} (13)

Another approach to resolution of this problem, based on the use of a spontaneously broken continuous global symmetry, was studied in Ref. [28].

TMT models where this symmetry is only spontaneously broken have been studied by Guendelman in Refs. [27] and [28].
\[
V_1(\phi) + sM^4 - \frac{2V_2(\phi)}{\chi} = 0
\]  

(14)

We have defined \( \chi \) and \( \Phi \) to be of the same sign. To avoid problems which could appear if the measure \( \Phi \) becomes singular (\( \Phi = 0 \)), in what follows we must care about such choice of \( V_1, V_2 \) and \( sM^4 \) that the constraint (14) provides for \( \chi \) to be positive definite. Then \( \Phi \) will be positive definite as well.\(^5\) If for example, \( V_2(\phi) \) is positive definite then \( V_1(\phi) + sM^4 \) must be non-negative. We will revert to this question later.

Variation of the action with respect to \( \Gamma^\mu_{\lambda\sigma} \) leads to the equation

\[
-\Gamma^\lambda_{\mu\nu} - \Gamma^\alpha_{\beta\mu}g^{\beta\lambda}g_{\alpha\nu} + \delta^\lambda_{\nu}\Gamma^\alpha_{\mu} + \delta^\lambda_{\mu}\Gamma^\alpha_{\beta}g_{\gamma\nu} - \frac{\delta^\lambda_{\mu}}{\Phi} \frac{\Phi_{,\nu}}{\Phi} = 0.
\]  

(15)

The general solution of Eq. (15) may be represented in the form

\[
\Gamma^\lambda_{\mu\nu} = \{_{\mu\nu}^\lambda\} + \Sigma^\lambda_{\mu\nu}
\]  

(16)

where \( \{_{\mu\nu}^\lambda\} \) are the Christoffel’s connection coefficients of the metric \( g_{\mu\nu} \) and

\[
\Sigma^\alpha_{\mu\nu} = \delta^\alpha_\mu \lambda_{,\nu} + \frac{1}{2} (\sigma_{,\mu} \delta^\alpha_\nu - \sigma_{,\nu} \delta^\alpha_\mu - \sigma_{,\sigma} g_{\mu\nu} g^{\alpha\beta})
\]  

(17)

with \( \sigma = \ln \chi \) and \( \lambda = \lambda(x) \) is an arbitrary function that appears due to the \( \lambda \)-symmetry \([30]\) of the affine curvature tensor (9). If we choose the gauge \( \lambda = \frac{2}{\phi} \), then the antisymmetric part of \( \Sigma^\alpha_{\mu\nu} \) disappears and we get

\[
\Sigma^\alpha_{\mu\nu}(\sigma) = \frac{1}{2} (\delta^\alpha_\mu \sigma_{,\nu} + \delta^\alpha_\nu \sigma_{,\mu} - \sigma_{,\sigma} g_{\mu\nu} g^{\alpha\beta})
\]  

(18)

which contributes to the nonmetricity.

The scalar field \( \phi \) equation is

\[
(-g)^{-1/2} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) + \sigma_{,\nu} \phi_{,\mu} + \frac{dV_1}{d\phi} - \frac{1}{\chi} \frac{dV_2}{d\phi} = 0,
\]  

(19)

In the conformal frame defined by the conformal transformation

\[
g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x) = \chi g_{\mu\nu}(x); \quad \phi \rightarrow \phi; \quad A_{\mu\nu\lambda} \rightarrow A'_{\mu\nu\lambda}
\]  

(20)

the \( \Sigma^\mu_{\alpha\beta} \) contribution into the connection disappears: \( \Gamma^\lambda_{\mu\nu} \rightarrow \tilde{\Gamma}^\lambda_{\mu\nu} = \{\lambda_{\mu\nu}\} \) (here \( \{\lambda_{\mu\nu}\} \) are the Christoffel’s connection coefficients of the Riemannian space-time with the metric \( g'_{\mu\nu} \)). Tensors \( R^\lambda_{\mu\nu\sigma}(\Gamma) \) and \( R_{\mu\nu\sigma}(\Gamma) \) transform to the Riemann \( R^\lambda_{\mu\nu\sigma}(g'_{\alpha\beta}) \) and Ricci \( R_{\mu\nu}(g'_{\alpha\beta}) \) tensors respectively in the Riemannian space-time with the metric \( g'_{\mu\nu} \). After making use the solution for \( \chi \) as it follows from the constraint (14), the scalar field equation (19) in the new conformal frame takes the standard form

\[
\frac{1}{\sqrt{-g'}} \partial_\mu (\sqrt{-g'} g'^{\mu\nu} \partial_\nu \phi) + \frac{dU}{d\phi} = 0
\]  

(21)

where

\[
\frac{dU}{d\phi} = \frac{1}{\chi} \left( \frac{dV_1}{d\phi} - \frac{1}{\chi} \frac{dV_2}{d\phi} \right)
\]  

(22)

\[
U(\phi) = \frac{1}{\chi^2} V_2(\phi) = \frac{1}{4V_2(\phi)} [sM^4 + V_1(\phi)]^2
\]  

(23)

\(^5\)Another possibilities appear in the context of the four ”measure scalar fields” approach to TMT, see Refs. [22], [23].

5
In the same conformal frame, Eq.(12) leads to the canonical Einstein equations

$$R_{\mu\nu}(g_{\alpha\beta}) - \frac{1}{2}g'_{\mu\nu}R(g'_{\alpha\beta}) = \frac{\kappa}{2}T_{\mu\nu}(\phi)$$

where

$$T_{\mu\nu}(\phi) = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g'_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}g'^{\alpha\beta} + g'_{\mu\nu}U(\phi)$$

We conclude that in the conformal frame (20), the equations of motion obtain the standard form of the Einstein’s GR equations for the selfconsistent system of gravity (\(g'_{\mu\nu}\)) and scalar field \(\phi\) with the TMT effective potential \(U(\phi)\). Notice that just \(U(\phi)\) plays the role of the true potential that governs the dynamics of the scalar field \(\phi\) whereas \(V_1(\phi)\) and \(V_2(\phi)\) have no sense of the potential energy densities themselves but rather they generate the potential energy density when either the selfconsistent problem is solved (see Eq. (23)) or the problem is solved in the gravitational background (see Eq. (62) in Sec.V). This is why I use the term pre-potentials \(V_1(\phi)\) and \(V_2(\phi)\). Notice that our choice of the sign in front of the pre-potential \(V_2(\phi)\) is opposite to the usual one that would be in the case of the standard GR. This is doing just for convenience in what follows.

In order to provide disappearance of the cosmological constant, one demands usually that the effective potential is equal to zero at the minimum, i.e. it is necessary that the effective potential and its first derivative are equal to zero at the same point. As a matter of fact this is the essence of the fine tuning problem when a traditional approach to a definition of the true vacuum state is used and the cosmological constant problem is treated in the "old" sense (i.e. the situation with this issue that was when there was no need in a small but non-zero cosmological constant at present). If we want to avoid the necessity to fulfill this fine tuning, TMT gives us such an opportunity.

In fact, independently of the shape of the nontrivial pre-potential \(V_1(\phi)\), infinite number of initial conditions exists for which \(V_1 + sM^4 = 0\) for some value \(\phi = \phi_0\). Let us assume that \(V_1(\phi)\) and \(V_2(\phi)\) are regular at \(\phi = \phi_0\) if \(\chi_0(\phi_0) \neq 0\) and \(V_2(\phi)\) is positive definite. One can see from the constraint (14), which represents \(\chi^{-1}\) as the function of \(\phi\), that \(\chi^{-1}(\phi_0) = 0\). Then it follows immediately from Eqs.(22) and 23) that both \(\frac{d\chi}{d\phi}\) and \(U(\phi)\) are equal to zero at \(\phi = \phi_0\). The TMT effective potential \(U(\phi)\), Eq. (23), is non-negative and \(\phi = \phi_0\) is the absolute minimum of \(U(\phi)\) with the value \(U(\phi_0) = 0\). So, one can conclude (see also Refs. [25], [27]) that in a FRW universe, Eqs.(21)-(25) describe a time evolution where the scalar field \(\phi\) rolls down to the true vacuum state \(\phi_0\) with zero energy density \(U(\phi_0) = 0\) and for this to be happen there is no need of any sort of fine tuning. If we will also assume that the shapes of the pre-potentials \(V_1(\phi)\) and \(V_2(\phi)\) are such that \(U(\phi)\) provides a possibility for an inflation in the early universe then we indeed obtain the mechanism for a solution of the cosmological constant problem.

Notice that in the model just now discussed, the scalar field \(\chi\) skips from \(-\infty\) to \(+\infty\) as \(\phi\) passes through \(\phi_0\) (or, what is the same, \(V_1 + sM^4\) passes through zero). In the Einstein picture with the metric \(g'_{\mu\nu}\), Eq.(20), we do not observe any problem related to this. However in the original frame the metric \(g_{\mu\nu}\) has to be singular as \(\phi = \phi_0\) since the conformal transformation (20) becomes singular at \(\phi = \phi_0\). Moreover, \(g_{\mu\nu}\) has to change its signature as \(\phi\) passes through \(\phi_0\). A possible way to avoid this problem was discussed in Ref. [24]. The idea was to prevent the scalar field \(\chi\) from arriving at an infinite value during a finite time. This idea might be realized for example by taking into account terms quadratic in the scalar curvature.

Another, apparently more realistic approach to the problem might be based on taking into account a matter contribution [27], [28]. In this case the constraint is changed in general and the real vacuum may be not located at the point where \(V_1 + sM^4 = 0\). This model could explain the acceleration of the present universe together with the "cosmic coincidence".

The models we are going to develop in the present paper will be free of the discussed problem.

The assumption that \(V_2(\phi)\) is positive definite will be our choice in what follows.

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6We will discuss later quantum effective potentials. To avoid a possible misunderstanding the term "the TMT effective potential" is used for \(U(\phi)\).

7Of course, there may be such a pre-potential \(V_1\) that there will be also infinite number of initial conditions for which \(V_1 + sM^4 \neq 0\) for any value of \(\phi\). This simple remark will be very important in Sec.IV.B.
I am going to demonstrate now that starting from extremely broad classes of pre-potentials $V_1(\phi)$ and $V_2(\phi)$ in the framework of the same model as in Sec.II, i.e.

$$S = \int d^4x \left[ \Phi \left( -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \phi,_{\mu} \phi,_{\nu} - V_1(\phi) \right) + \sqrt{-g} V_2(\phi) \right]$$

one can observe the generation of the TMT effective potential $U(\phi)$, Eq. (23), which for $\phi$ large enough takes the form of the quintessential potential either of the exponential form or of the negative power low. In Sec.IV we will revert to studying the shapes of the TMT effective potential $U(\phi)$ in all range of $\phi$.

### A. Negative power low form of the TMT effective potential for $\phi$ large enough

Although there exists a possibility for generation of a negative power low potential in the models with dynamical supersymmetry breaking (see, for example [31]), such potential still looks to be exotic one. We will see now that starting from the positive power low pre-potentials $V_1$ and $V_2$, TMT enables to generate the TMT effective potential $U(\phi)$ which has the form of the negative power low potential as $\phi$ is large enough.

#### I. Simple models

Let us start from the simplest model with the pre-potentials $V_1$ and $V_2$ of the form

$$V_1 = m_1^{(4-n_1)} \phi^{n_1}; \quad V_2 = V_2^{(0)} + \frac{1}{4} m_2^{(4-2n_2)} \phi^{2n_2}. \tag{27}$$

where constants $n_1$ and $n_2$ are positive and $m_1, m_2$ are parameters of the dimension of mass. A constant $V_2^{(0)}$ is added to prevent singularity of the TMT effective potential at $\phi = 0$. Positive definiteness of $V_2(\phi)$ implies $V_2^{(0)} > 0$. Notice that in the action (26), the term $V_2^{(0)} \int \sqrt{-g} d^4x$ coincides (with opposite sign) with that which in GR would have the sense of the cosmological constant term. Then the TMT effective potential $U(\phi)$, Eq. (23), is the following

$$U(\phi) = \frac{(sM^4 + m_1^{(4-n_1)} \phi^{n_1})^2}{4V_2^{(0)} + m_2^{(4-2n_2)} \phi^{2n_2}}. \tag{28}$$

Let us consider a few interesting particular cases.

- **The case $n_1 = n_2$**
  In this case the TMT effective potential approaches the nonzero constant $\frac{m_1^8}{m_2^2}$ as $\phi \to \infty$.

The case $n_1 = n_2$ includes the model

$$V_1 = \frac{1}{2} \mu^2 \phi^2, \quad V_2 = V_2^0 + \frac{\lambda}{4} \phi^4 \tag{29}$$

with positive constants $V_2^0$ and $\lambda$ that resembles the massive $-\lambda \phi^4$ scalar field model of the standard field theory. In this model, for the asymptotic value of the TMT effective potential to be of the order of the present vacuum energy density $\rho_{\text{vac}} \sim 10^{-47} \text{GeV}^4$, the mass-like parameter $\mu$ must be of the order $\mu \sim \lambda^{1/4} 10^{-3} \text{eV}$.

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8Adding a constant to $V_1$ is equivalent just to a redefinition of the constant of integration $sM^4$. 

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b. The case $n_2 > n_1$
For $\phi$ large enough, the TMT effective potential has a desirable quintessence form
\[
U \approx \frac{m_1^{2(n_2-n_1)}}{m_2^{2(n_1-n_2)}} \frac{1}{\phi^{2(n_2-n_1)}}
\]
and does not depend on the integration constant.

c. The case $V_1 \equiv 0 (\lambda = 0)$ In this case the TMT effective potential
\[
U = \frac{M^8}{4V_2^0 + m_2^{4-2n_2} \phi^{2n_2}}
\]
is proportional to the integration constant. For $n_2 = 2$ this case includes the model that resembles the massless $-\lambda \phi^4$ scalar field model of the standard field theory. In this case $U(\phi) \simeq M^8/\phi^4$ as $\phi$ is big enough.

2. Broad class of models

The trivial generalization of the simplest model (27) consists of choosing of the pre-potentials $V_1(\phi)$ and $V_2(\phi)$ in the form of polynomials
\[
V_1(\phi) = \sum_{i=1}^{n_1} m_1^{4-i} \phi^i, \quad V_2(\phi) = \sum_{i=0}^{2n_2} m_2^{4-i} \phi^i
\]
(32)

To provide the negative power low form for the TMT effective potential as $\phi$ is large enough, the only necessary restriction is $n_2 > n_1$.

B. The exponential form of the TMT effective potential $U(\phi)$.

1. Simple model

A simple way to realize an exponential asymptotic form of the TMT effective potential $U(\phi)$, Eq.(23), is to define the pre-potentials $V_1$ and $V_2$ as follows:
\[
V_1 = s_1 m_1^4 e^{\alpha \phi/M_p}; \quad V_2 = \frac{1}{4} m_2^4 e^{2\beta \phi/M_p}
\]
(33)

Here $s_1 = \pm 1$ and we assume that $\beta = const > 0$; for $\alpha = const$ it is permitted possibility of both signs. The restrictions formulated after Eq. (14) have to be taken into account. The effective TMT potential corresponding to the pre-potentials (33) is
\[
U = \frac{1}{m_2^2} \left( s_1 m_1^4 e^{-(\beta-\alpha) \phi/M_p} + sM_1^4 e^{-\beta \phi/M_p} \right)^2
\]
(34)

Let us consider a few particular cases.

a. The case $\alpha < 0$. For $(\beta + |\alpha|) \phi \gg M_p$, the TMT effective potential (34) behaves as a decaying exponent proportional to the integration constant
\[
U \simeq \frac{M^8}{m_2^4} e^{-2\beta \phi/M_p}
\]
(35)

This function serves also as the exact form of the TMT effective potential in the model with $V_1(\phi) \equiv 0$. 8
\[ b. \text{The case } \alpha = \beta \]

This case corresponds to a sort of the scale invariant theory studied by Guendelman [27]. In fact, in this case the theory (26) is invariant under global transformations

\[ g_{\mu\nu} \rightarrow e^\theta g_{\mu\nu} \] (36)

\[ A_{\mu\nu\lambda} \rightarrow e^\theta A_{\mu\nu\lambda} \] (37)

whereas the scalar field \( \phi \) undergoes the shift

\[ \phi \rightarrow \phi - \frac{M_p}{\beta} \theta \] (38)

In such a model, the TMT effective potential has the form

\[ U(\phi) = \frac{m_1^8}{m_2^2} \left[ 1 + \frac{s_1}{s} \left( \frac{M}{m_1} \right)^4 e^{-\beta\phi/M_p} \right]^2 \] (39)

and it has been used in Ref. [27] for discussion of possible cosmological applications when the choice \( \frac{s_1}{s} = -1 \) was made. The cosmological applications are based on the observation that \( U(\phi) \) has an infinite flat region as \( \phi \rightarrow \infty \) and \( U \) approaches a non-zero constant \( \frac{m_1^8}{m_2^2} \). The first possibility is related to the very early universe: a slow rolling (new inflationary) scenario might be realized assuming that the universe starts at a sufficiently large value of \( \phi \).

Another scenario discussed by Guendelman in Ref. [27] is based on a possibility for \( \frac{m_1^8}{m_2^2} \) to be very small and this approach has the aim to construct a scenario for the very late universe. In fact, the constant \( \frac{m_1^8}{m_2^2} \) could play the role of the present day vacuum energy density if one chooses for example \( m_2 \sim 10^{16} \text{GeV} \) (close to the GUT scale) and \( m_1 \sim 10^2 \text{GeV} \) (the electro-weak scale). In this scenario there could be a long lived stage with almost constant energy density \( \frac{m_1^8}{m_2^2} \) that will eventually disappear when the universe achieves its true vacuum state with zero cosmological constant. This occurs when the expression in parenthesis in Eq. (39) becomes zero and therefore no fine tuning is needed. It turns out (see Refs. [27], [28]) that in the presence of a matter, which is introduced in a way respecting the global symmetry (36)-(38), the change of the constraint (14) leads to a correlation between \( U(\phi) \) (close but not equal to zero) and the matter energy density.

Notice however that it seems to be impossible in the framework of this model, with \( \alpha = \beta \), to realize a quintessence scenario as \( \phi \rightarrow \infty \).

In the case \( \alpha = \beta \) the TMT effective potential (39) is not a constant due to the appearance of a non-zero integration constant \( M \), that is actually, due to a spontaneous breaking of the global continuous symmetry (36)-(38). Guendelman noticed [27] that in terms of the dynamical variables used in the Einstein frame, that is \( g'_{\mu\nu} \) and \( \phi \), the symmetry transformations (36)-(38) are reduced to shifts (38) alone (\( g'_{\mu\nu} \) is invariant under transformations (36)-(38)). Thus in terms of the dynamical variables of the Einstein frame, the spontaneous symmetry breaking is just that of the global continuous symmetry \( \phi \rightarrow \phi - \frac{M_p}{\beta} \theta \). It is important that, as it was mentioned in Ref. [27], this global continuous symmetry is restored as \( \phi \rightarrow \infty \).

\[ c. \text{The case } \beta > \alpha > 0. \] This is the most interesting case of the exponential form of the TMT effective potential from the viewpoint of the quintessence scenario. For \( \beta\phi \gg M_p \), the TMT effective potential (34) behaves as a decaying exponent:

\[ U \simeq \frac{m_1^8}{m_2^2} e^{-2(\beta - \alpha)\phi/M_p} \quad \text{as} \quad \beta\phi \gg M_p. \] (40)

If we want to achieve the quintessential form of the TMT effective potential (40) for not too large values of \( \phi \) and with not too big difference in orders of \( m_1 \) and \( M \) (this point will be explained in the next section) then one needs the condition

\[ 0 < \beta - \alpha \ll \beta \] (41)
And of course the most evident argument in favour of this condition consists in the demand to provide the flatness of the \( \phi \)-potential in the late, \( \phi \)-dominated universe where it has to imitate the present "cosmological constant". This is possible only if \( \beta - \alpha \) is less or of order 1 while there are no reasons for \( \beta \) not to be large in general.

Comparing this condition for \( \alpha \) and \( \beta \) with that of the model of Ref. [27] discussed just above, one can observe that the model under consideration can be interpreted as that with a small explicit violation of the global symmetry (36)-(38). Notice that the expression for \( U(\phi) \) as \( \beta \phi \gg M_p \) does not include the integration constant \( M \) and the exponent is proportional to \( \beta - \alpha \). This reflects the fact that such asymmetric behavior of \( U(\phi) \) results from the small explicit violation of the global continuous symmetry (36)-(38).

It is very interesting that although the discussed global continuous symmetry (36)-(38) is broken in this model explicitly, it is also restored as \( \phi \to \infty \), just as in the case \( \alpha = \beta \) with only spontaneous symmetry breaking. In fact, in the "pre-potential sector" \(-\Phi V_1(\phi) + \sqrt{-g} V_2(\phi)\) of the integrand of the action (26), the leading term approaching infinity as \( \phi \to \infty \) is \( \sqrt{-g} m^4 \exp(2\beta \phi/M_p) \) which is invariant under transformations (36)-(38). In terms of the dynamical variables used in the Einstein frame, that is \( g_{\mu \nu} \) and \( \phi \), one can formulate the conclusion that in the model where the condition (41) holds, the approximate global symmetry \( \phi \to \phi - M_p \theta \) is restored as \( \phi \to \infty \).

This observation opens unexpected chance to solve the problem discussed by Carroll [20] (problem 5 in the list of problems in Introduction) which consists of the following. There are no reasons to ignore a possibility that the scalar field \( \phi \) interacts directly to "standard-model" fields. Suppose that such interactions have the form of the coupling \( f_i \phi L_i \), where \( L_i \) is any gauge invariant four-moment operator, \( m \) is a mass scale and \( f_i \) is a dimensionless coupling constant. The flatness of the quintessential potential of the field \( \phi \) means that excitation of \( \phi \) are almost massless. Therefore in the presence of direct interactions of the \( \phi \)-field to the usual matter fields, one has to expect the appearance of the very long-range forces which do not obey the equivalence principle. Observational restrictions on such "fifth force" impose small upper limits on the coupling constants \( f_i \).

To explain smallness of \( f_i \)'s, Carroll proposed to postulate that the theory possesses an approximate global continuous symmetry of the form \( \phi \to \phi + \text{const} \) (the idea similar to what is used in pseudo-Goldstone boson models of quintessence [5] where however, an explicit breaking of the continuous chiral symmetry reduces it to a discrete symmetry). In the framework of Einstein's GR, such exact continuous symmetry is incompatible with a nontrivial potential of the scalar field \( \phi \). This means that if we were working in Einstein's GR, then started from the model with the exact symmetry \( \phi \to \phi + \text{const} \) and therefore with a trivial potential, we would want to achieve a nontrivial, quintessential potential (passed also across a fine tuning purgatory) as a result of some mechanism for a symmetry breaking. Such a picture looks even more problematic one than the fine tuning problem itself. In addition, in the framework of such general idea about a breaking of the symmetry \( \phi \to \phi + \text{const} \), it is impossible to point out the parameters of the theory which could produce, after a symmetry breaking, the small coupling constants \( f_i \).

In contrast to GR, in TMT one can suppose that in yet unknown more fundamental theory the global continuous symmetry (36)-(38) is an exact one and \( \alpha = \beta \). At energies below the Plank mass the symmetry is breaking and it is assumed that the effective action that can describe the relevant physics, has the form of TMT, Eq. (26) (inclusion of the usual matter will be studied in Sec. VI), with the nontrivial pre-potentials (33). The only thing we need from a mechanism for a symmetry breaking consists of a small relative shift of the magnitudes of \( \alpha \) and \( \beta \) satisfying the condition (41). If the symmetry breaking generates couplings of the scalar field \( \phi \) to the usual matter fields, then the corresponding dimensionless coupling constants \( f_i \) must be proportional\(^9\) to some positive power of \( (\beta - \alpha)/\beta \). If the symmetry breaking effect generates additional terms in the pre-potentials they have to include the same type of factors as well. This remark will be used in the next section.

Notice that an unbounded increase of the pre-potentials as \( \phi \to \infty \) does not produce problems, at least on the classical level, since as was already mentioned in Sec. II, the pre-potentials have no sense of a potential energy density. The real potential is the TMT effective potential (23) that in the model under consideration approaches zero according to Eq. (40) as \( \phi \to \infty \). The asymptotic, as \( \phi \to \infty \), restoration of the global continuous symmetries (36)-(38) (which in terms of the dynamical variables of the Einstein frame are just shifts \( \phi \to \phi - M_p \theta \)) is accompanied by approach to zero of the mass of the excitations of the field \( \phi \).

\(^9\)Notice that the exponents in the pre-potentials (33) contain actually dimensional factors \( \alpha M_p^{-1} \) and \( \beta M_p^{-1} \). Therefore the dimensionless parameter that could characterize the symmetry breaking has to be of the form \( (\beta M_p^{-1} - \alpha M_p^{-1})/\beta M_p^{-1} = (\beta - \alpha)/\beta \).
An evident generalization of the pre-potentials (33) that maintains the TMT effective potential behavior (40) as \( \beta \phi \gg M_\nu \), consists of adding to them the terms with lower degree of growth. They may be for example, polynomials in \( \phi \) and/or exponential functions of \( \phi \). In such a case the pre-potentials \( V_1(\phi) \) and \( V_2(\phi) \) take the form

\[
V_1(\phi) = P_{n_1}^{(1)}(\phi) + \sum_{i=1}^{r} m_{(1),i}^4 e^{\alpha_i \phi / M_\nu}
\]

\[
V_2(\phi) = \frac{1}{4} \left( P_{n_2}^{(2)}(\phi) + \sum_{i=1}^{q} m_{(2),i}^4 e^{2\beta_i \phi / M_\nu} \right)
\]

Here \( P_{n_1}^{(1)}(\phi) \) and \( P_{n_2}^{(2)}(\phi) \) are polynomials and it is assumed that \( \alpha_1 < \alpha_2 < \ldots < \alpha_r = \alpha, \quad \beta_1 < \beta_2 < \ldots < \beta_q = \beta \), \( m_{(1),i}^4 = s_1 m_4^4 \), \( m_{(2),i}^4 = m_4^2 \). To provide the asymptotic form of the TMT effective potential as it is described by Eq. (40), the only restriction is needed \( \beta > \alpha > 0 \). Then the exponential terms \( s_1 m_4^4 e^{\alpha \phi / M_\nu} \) and \( m_4^2 e^{2\beta \phi / M_\nu} \) are the leading terms in \( V_1(\phi) \) and \( V_2(\phi) \) respectively as \( \phi \to \infty \). Relative contributions of all other terms of \( V_1(\phi) \) and \( V_2(\phi) \) into the TMT effective potential \( U(\phi) \) are exponentially suppressed.

If the additional terms that distinguish (42) and (43) from (33), appear as a result of breaking of the symmetry (36) - (38) (reminded that \( \alpha = \beta \) in the case of the exact symmetry), then all coefficients in (42) and (43), except of \( m_4^2 \), have to be proportional to some positive power of the small parameter \( (\beta - \alpha) / \beta \). For the same reasons as it was in the simple model with \( \beta > \alpha > 0 \), the symmetry (36)-(38) is restored as \( \phi \to \infty \).

Simple reasoning adduced in this paragraph as well as in the paragraph “Broad class of models” of subsection IV.A, does not look like trivial one if we recall that in GR adding any constant and/or increasing (as \( \phi \to \infty \)) term to the potential destined to be a quintessential one, causes a drastic violation of its desirable features: an arbitrary cosmological constant appears and/or the flatness conditions are destroyed if no extreme fine tunings are made. The basis for the resolution of this problem in TMT consists in a possibility to achieve a quintessence form of the effective potential as \( \phi \) is large enough, starting from pre-potentials increasing as \( \phi \to \infty \). As a matter of fact, this is the main advantage of the studied TMT models over the quintessence models formulated in the framework of the standard GR.

In the conclusion it is worthwhile to notice for the following discussion that in all cases considered in this section, \( \chi^{-1} \) as the solution of the constraint (14), asymptotically approaches zero as \( \phi \to \infty \).

### IV. PROBE MODELS: TOWARDS EFFECTIVE TMT POTENTIAL

**OF THE “QUINTESSENTIAL INFLATION” TYPE**

#### A. Some clarifications to the rest part of the paper

The previous sections served a preparatory role in the formulation and solution of the main problems of this paper. In Sec. III our attention was concentrated on the possibilities of TMT to generate without fine tuning the scalar field \( \phi \) potential which for \( \phi \) large enough provides a quintessence. It turns out however that some of such TMT effective potentials can appear to be also well defined to drive the early universe evolution. In this paper I do not aim to look for a precise values of all parameters of the potential that could be able to provide an adequate description of the cosmological evolution from slightly after Planck time up to now and answer all demands of the realistic cosmology. But I do want to exhibit that the field theory models based on TMT provide the existence of a broad spectrum of tools giving us the firm belief that such a potential can be generated without fine tuning. More precisely, in this section I am going to demonstrate that proceeding with the single scalar field and exploring the results of the previous section one can make sure that TMT is able to generate (without any sort of fine tuning) the effective potential of such a form that in the main could answer basic demands of the realistic cosmology.

Such qualitative examination is enough for the purposes of this paper which consist mainly in studying of some basic field-theoretic problems of TMT that turn out to be in the very close interrelation with some fundamental features of the cosmological scenario. The essence of the matter is that generally speaking the price for the success of TMT in the resolution of the cosmological constant problem is serious enough. In fact, in order to incorporate the matter fields into the simplifying picture reviewed in Sec. II in such a way that the TMT effective equations...
of motion of all fields in the Einstein frame would have the form of the equations of motion of the standard field theory based on GR, in Ref [25] we were forced to start from the very nonlinear (in the matter fields) original TMT action. This circumstance together with the non-Riemannian nature of the original action makes the quantization of TMT practically inaccessible problem even on a semiclassical level. Moreover, it was unclear how one can approach a question of matter fields quantization in the background curved space-time. We will see below that with the appropriate choice of the pre-potentials and of the definite (but infinite) class of initial cosmological conditions, it is enough to start from the original TMT action with exactly the same degree of nonlinearity in matter fields as in the standard theory in order to get the standard matter field theory in a background (pseudo-Riemannian) space-time. This appears to be possible to do after the so-called TMT gravitational background will be formulated in Sec.V. Then the matter fields quantization in the TMT gravitational background reduces to the standard procedure of the matter fields quantization in curved (pseudo-Riemannian) space-time [29]. Fortunately, it turns out that the choice of the initial cosmological conditions and the pre-potentials needed to provide such successful construction of the matter field theory in the context of TMT, corresponds to the class of models where the TMT effective potential allows to solve all five problems mentioned in Introduction. Remarkably that these five problems are solved in the context of a cosmological scenario which, in its turn, provides a possibility to define the TMT gravitational background (that in general is not a quite usual problem, see Sec.V).

B. Two big classes of cosmological scenarios

As we already noted in Introduction, the term (4) of the total action (3) is invariant (up to the integral of a total divergence) under a shift \( L_1 \rightarrow L_1 + \text{const} \) or that is equivalent, under a shift \( V_1 \rightarrow V_1 + \text{const} \). After solving Eq. (10) this symmetry is breaking (see Eq.(11)). The integration constant \( s M^4 \) can be either positive or negative (\( s = \pm 1 \)) and taking into account also the form of \( V_1(\phi) \) (which is assumed to be nontrivial) one can divide all possible cosmological scenarios into two big groups.

We will say that a cosmological scenario belongs to the \textit{first class} if one admits to \( V_1(\phi) + s M^4 \) to arrive at zero at some finite value of \( \phi = \phi_0 \). As it was discussed at the end of Sec.II (see also Refs. [25, 27]) in this case the cosmological constant problem is solved without any fine tuning: \( \phi = \phi_0 \) turns out to be the true vacuum state with zero energy density. Scenario of the first class is realized for example if the range of the pre-potential \( V_1(\phi) \) is the entire real axis: \(-\infty < V_1(\phi) < \infty \). Then independently of the value of the integration constant \( s M^4 \) there is a point \( \phi = \phi_0 \) where \( V_1(\phi_0) + s M^4 = 0 \). If range of the pre-potential \( V_1(\phi) \) is not the entire real axis then also infinite number of values of the integration constant \( s M^4 \) exists, providing a first class cosmological scenarios.

However there are three problems hindering construction of a realistic cosmology in this case: 1) It is very hard to adapt such class of scenarios to demands of the present accelerated universe (see however Ref. [28]). 2) It turns out (see the next section) that TMT does not allow to define the gravitational background in the neighbourhood of the point \( \phi = \phi_0 \), that is inadmissible for the true vacuum state. 3) As it was discussed at the end of Sec.II, when \( \phi \) passes through \( \phi_0 \), the conformal transformation (20) becomes singular and the very strange features of the metric in the original frame have to be admitted.

A cosmological scenario belongs to the \textit{second class} if \( V_1(\phi) + s M^4 \) can not arrive at zero at any finite value of \( \phi \). Notice that although the second class scenarios can be realized not for all forms of \( V_1(\phi) \), infinite number of values of the integration constant \( s M^4 \) is allowed here as well. It turns out that in the context of the second class scenarios there are no described above obstacles for constructing realistic cosmological models while all advantages of TMT are kept. This is why we will consider the probe models where only scenarios of the second class are realized.

With positive definite \( V_2(\phi) \), the difference between two classes of cosmological scenarios one can formulate roughly speaking in the following way: in the first class scenarios the true vacuum state with zero effective cosmological constant might be realized at the minimum of the TMT effective potential \( U(\phi) \) at some finite value of \( \phi \); in the second class scenarios the true vacuum state with zero effective cosmological constant might be realized only asymptotically (e.g. as \( \phi \rightarrow \infty \)) if \( U(\phi) \) rolls down to zero as \( \phi \rightarrow \infty \). It is clear that the quintessence belongs to the second class scenarios.

C. Models based on the hypothesis that the theory possesses the explicitly broken global symmetry (36)-(38)

The pre-potentials of the general forms (32) or (42), (43) provide the possibility to generate the TMT effective potential \( U(\phi) \) with an asymptotic (as \( \phi \rightarrow \infty \)) quintessence behavior that mimics the current effective cosmological constant. For this to be done there is no need of any sort of fine tuning and the enough condition for this is \( n_2 > n_1 \) in (32) or \( 0 < \beta - \alpha < \beta \) in (42), (43). If however, one wants to extend the range of applicability of the TMT effective
potential of the same single scalar field $\phi$ to satisfy constraints of the realistic cosmology from inflation of the early universe up to the present-day universe, then pre-potentials (32) or (42), (43) are too general and give us excessive amount of means for such an adjusting. I restrict myself by models based on the idea that the action (26) is the effective one of a more fundamental theory at the energy scales below the Planck mass. It seems then to be natural to suppose that transition from the fundamental theory to the effective one is accompanied by breaking of some fundamental symmetries. I will assume that one of such symmetries is the global one (36)-(38) \footnote{Of course, without knowledge of the fundamental theory one can not discuss a mechanism for the symmetry breaking.} \footnote{For comparison one can notice that with such understanding of fine tuning, in the framework of the class of models (32), (41) and footnote 8, I did not succeed in attempts to construct \textit{without fine tuning} a model that could reproduce the main features of the "quintessential inflation" model (see Eq. (1)).}. Such approach to the choice of prepotentials enables to narrow the amount of the suitable versions. In particular, for models leading to the integration constant $M$ of the form (42), (43), will be realized. The Planck mass $M_p$ is chosen as the typical scale for parameters of the dimension of mass corresponding to the limit where the global symmetry (36)-(38) is unbroken. Then the appearance of the mass parameters smaller than $M_p$ is a manifestation of a symmetry breaking by the appropriate terms since those parameters can be represented as $\left(\frac{\beta-\alpha}{\beta}\right)^n M_p, \quad n > 0$. In the framework of such an approach one can conclude that the model is free of a fine tuning if orders of all such mass parameters are not too much differ from $M_p$ (in this connection see also discussions after Eqs. (40), (41) and footnote 8) \footnote{For comparison one can notice that with such understanding of fine tuning, in the framework of the class of models (32), (41) and footnote 8, I did not succeed in attempts to construct \textit{without fine tuning} a model that could reproduce the main features of the "quintessential inflation" model (see Eq. (1)).}

Below we will formulate three models where the simplest cases of the broad class of models with the pre-potentials of the form (42), (43), will be realized. The Planck mass $M_p$ is chosen as the typical scale for parameters of the dimension of mass corresponding to the limit where the global symmetry (36)-(38) is unbroken. Then the appearance of the mass parameters smaller than $M_p$ is a manifestation of a symmetry breaking by the appropriate terms since those parameters can be represented as $\left(\frac{\beta-\alpha}{\beta}\right)^n M_p, \quad n > 0$. In the framework of such an approach one can conclude that the model is free of a fine tuning if orders of all such mass parameters are not too much differ from $M_p$ (in this connection see also discussions after Eqs. (40), (41) and footnote 8) \footnote{For comparison one can notice that with such understanding of fine tuning, in the framework of the class of models (32), (41) and footnote 8, I did not succeed in attempts to construct \textit{without fine tuning} a model that could reproduce the main features of the "quintessential inflation" model (see Eq. (1)).}

1. Model 1

\[
V_1(\phi) = m_1^4 e^{\alpha \phi/M_p}, \quad V_2(\phi) = \frac{1}{4} \left(4V_0^0 + m_2 e^{2\beta \phi/M_p}\right)
\]

With the choice of the parameters $m_2 = M_p, \quad 4V_0^0 = (10^{-3} M_p)^4, \quad m_1 = 10^{-2} M_p, \quad \beta = 7, \quad \alpha = 6$, and with the integration constant $M^4 = (3q \times 10^{-2} M_p)^4$, $0 < q \lesssim 1$ (s = +1), the TMT effective potential $U(\phi)$, Eq. (23), is a monotonically decreasing function with the shape that is convenient to describe in a piecewise form with the following four typical regions:

\[
U(\phi) \approx q^8 M_p^4 \quad \text{for} \quad \phi < -2.2 M_p,
\]
\[
\approx \frac{q^8 M_p^4}{1 + 10^{12} e^{14 \phi/M_p}} \quad \text{for} \quad -2.2 M_p < \phi < -1.8 M_p,
\]
\[
\approx 10^{-12} M_p^4 (1 + 10^{-2} e^{6 \phi/M_p})^2 e^{-14 \phi/M_p} \approx 10^{-12} M_p^4 e^{-14 \phi/M_p} \quad \text{for} \quad -1.8 M_p < \phi < 0.6 M_p,
\]
\[
\approx 10^{-16} M_p^4 e^{-2 \phi/M_p} \quad \text{for} \quad \phi > 1.2 M_p.
\]

(45)

2. Model 2

\[
V_1(\phi) = \frac{1}{2} m_1^2 \phi^2 + m_1^4 e^{\alpha \phi/M_p}, \quad V_2(\phi) = \frac{1}{4} \left(4V_0^0 + m_2 e^{2\beta \phi/M_p}\right)
\]

With the choice of the parameters $m_2 = M_p, \quad 4V_0^0 = (10^{-3} M_p)^4, \quad m_1 = 10^{-4} M_p, \quad m_1 = 10^{-3} M_p, \quad \beta = 7, \quad \alpha = 6$ and with the integration constant $M^4 = (\frac{1}{10^3} 10^{-2} M_p)^4$, (s = +1), the TMT effective potential $U(\phi)$, Eq. (23), is a monotonically decreasing function with the shape that one can describe in a piecewise form with the following three typical regions:

(46)
\[ U \approx \frac{1}{4} \lambda \phi^4, \quad \lambda = 10^{-14} \quad \text{for} \quad \phi < -\frac{1}{3} M_p \]
\[ \approx 10^{-16} M_p^4 \left[ 10^{-1} + \frac{1}{2} \left( \frac{\phi}{M_p} \right)^2 \right]^2 e^{-14\phi/M_p} \quad \text{for} \quad 0 < \phi < 1.1 M_p \]
\[ \approx 6 \times 10^{-28} M_p^4 e^{-2\phi/M_p} \quad \text{for} \quad \phi > 1.4 M_p \]

(47)

where in the interval 0 < \phi < 1.1 M_p the factor in front of the exponential function varies very slowly.

3. Model 3

\[ V_1(\phi) = \frac{1}{2} \mu_1^2 \phi^2 + m_1^4 e^{\alpha \phi/M_p}; \quad V_2(\phi) = \frac{1}{4} \left( 4V_2^0 + \frac{1}{2} \mu_2^2 \phi^2 + m_2^4 e^{2\beta \phi/M_p} \right) \]

(48)

With the choice of the parameters \( m_2 = M_p, \quad 4V_2^0 = (10^{-1} M_p)^4, \quad \mu_1 = 10^{-4} M_p, \quad m_1 = 10^{-3} M_p, \quad \mu_2 = 10^{-2} M_p, \quad \beta = 7, \quad \alpha = 6 \) and with the integration constant \( M^4 = \left( \sqrt[4]{10^{-2} M_p} \right)^4, \quad (s = +1) \), the TMT effective potential, Eq. (23), is a monotonically decreasing function with the shape that one can describe in a piecewise form with the following three typical regions:

\[ U \approx \frac{1}{2} m^2 \phi^2, \quad m = 10^{-6} M_p \quad \text{for} \quad \phi < -0.7 M_p \]
\[ \approx 10^{-16} M_p^4 \left[ 10^{-1} + \frac{1}{2} \left( \frac{\phi}{M_p} \right)^2 \right]^2 e^{-14\phi/M_p} \quad \text{for} \quad -0.6 < \phi < 1.5 M_p \]
\[ \approx 10^{-24} M_p^4 e^{-2\phi/M_p} \quad \text{for} \quad \phi > 1.7 M_p \]

(49)

where in the interval -0.6 < \phi < 1.5 M_p the factor in front of exponential function varies very slowly.

D. Some general features of the models 1 - 3.

As it was already noted, the exact fitting of all parameters to satisfy the requirements of the realistic cosmology is over and above the plan of this paper. Our aim here is rather a demonstration of extremely broad spectrum of tools giving by TMT to solve some fundamental problems of the realistic cosmology.

1) In each of the models 1-3 with the action (26), the global continuous symmetry (36)-(38) is violated by all terms of \( V_1 \) and \( V_2 \) except for the last term of \( V_2 \). The symmetry is restored at the limit \( \phi \to \infty \). All mass parameters (including mass parameters corresponding to "Λ-terms" in each of the models) have orders equal or slightly less than the Planck mass (not but less than the GUT energy scale).

2) One can see that the TMT effective potential \( U(\phi) \) of each of the models 1 - 3 has a region that can be responsible for an inflation of the early universe. Let us refer to this region of \( U(\phi) \) as the "inflationary region" of \( U \).

In model 1 the inflationary region of \( U \) is the infinite interval \( -\infty < \phi < -1.8 M_p \) with practically constant value \( U(\phi) \approx q^8 M_p^4 \) that smoothly passes on a slowly decreasing region. The inflationary region of \( U \) might be responsible for an initial stage of a new inflationary scenario [32].

In models 2 and 3, the inflationary regions of \( U \) have the form of the power low potentials \((\sqrt{\frac{1}{2}} \lambda \phi^4 \text{ and } \sqrt{\frac{1}{2}} m^2 \phi^2 \) respectively) driving the chaotic inflation [15]. Parameters of the pre-potentials are chosen in such a way that the inflationary region of \( U \) satisfies the requirements of the realistic cosmology. It is very important to stress that this can be done without strong tuning of the parameters, in contrast with the GR approach to the chaotic inflation models where the strong enough tuning is needed. The choice of \( \beta \) and \( \alpha \) does not affect practically the inflationary region of \( U(\phi) \).

3) The TMT effective potential \( U(\phi) \) of each of the models 1 - 3 behaves as

\[ U(\phi) \approx \frac{m_8^8}{M_p^4} e^{-2(\beta - \alpha) \phi/M_p} \quad \text{as} \quad \phi > \phi_b = \infty M_p, \]

(50)
where the constant factor $\varpi$ of order 1 is very sensitive to the choice of parameters. Let us refer to this region of $U(\phi)$ as the "quintessential region" since it can serve for the quintessential model of the present universe. One should make here an important remark. The quintessential region of $U$ has the form (50) where the value of $(\beta - \alpha)/\beta \ll \beta$ determines a strength of the symmetry breaking. The choice of $\beta = 1$ and $\beta = 7$ in the models 1-3 has just an illustrative aim and it is not a problem to adjust the value of $\beta - \alpha$ to satisfy the observable value of $\Omega_\phi$ at present.

4) Between the inflationary and quintessential regions, there exists an intermediate region of great interest. The TMT effective potential $U(\phi)$ in the intermediate region can be represented in the general form

$$U(\phi) = f(\phi)M_p^4e^{-2\beta\phi/M_p}$$

(51)

where $f(\phi)$ is a very slowly varying function compared to the exponential factor. There is a remarkable property of the intermediate region of $U$ that provides possibilities for resolution of some fundamental problems of the realistic cosmology: by an appropriate choice of $\beta$ one can achieve a very rapid decreasing of $U(\phi)$ after inflationary epoch that provides conditions for transition to the radiation and matter dominated era. For instance, in model 3 the TMT effective potential $U(\phi)$ at the end of the intermediate region ($\phi \approx 1.5M_p$) is roughly $10^{28}$ times less than at the beginning of the intermediate region ($\phi \approx -0.6M_p$). This property of the intermediate region of $U$ may be very useful for resolution of problems of cosmological nucleosynthesis constraints and large-scale structure formation [6], [7], [19], [8]. The exact shape of the intermediate region of $U$ (steepness and the range of definition) dictated by the realistic cosmology can be adjusted by the choice of the magnitudes of $\alpha$, $\beta$ and dimensional parameters (like $M$, $\mu_1$ etc.).

5) Combining the intermediate and the quintessential regions one can see that the post-inflation region of the TMT effective potential can be represented approximately in the exponential form described by Eq.(2) with $\phi$-dependent parameter $a$

$$a = a(\phi) = 2\beta \quad \text{as} \quad \phi < \phi_b$$

$$a = 2(\beta - \alpha) \quad \text{as} \quad \phi > \phi_b$$

(52)

where $\phi_b$ is a boundary value of $\phi$ between the intermediate and quintessential regions of the TMT effective potential $U(\phi)$. It seems to be very attractive that this result is obtained in a natural way in the framework of the field theory model without any assumptions specially intended for this. The only thing have been assumed is that the model possesses the approximate global continuous symmetry (36)-(38) and the value of $\beta - \alpha \ll \beta$ depending on a strength of the symmetry breaking should not be large in order to provide the flatness of the TMT effective potential in the quintessential region.

6) It turns out that in the models 2 and 3 the shape of $U(\phi)$ in the buffer range between the inflationary and the intermediate region can be very sensitive to variations of the parameters entering into $V_1$ and $V_2$. By means of a suitable change of the parameters one can achieve (without altering the qualitative properties of the discussed above regions), for instance an almost flat shape of $U$ in this buffer range or even successive local minimum and maximum immediately after the inflationary region. One can achieve also the shift of this buffer range from the negative values of $\phi$ (as it is adjusted now) to the range of positive $\phi$. This feature of the models may be very important if for example one wants to realize a scenario where the instant preheating [33] occurs before entry into the intermediate region.

The final remarks concerns the terminology. Since the scalar field $\phi$, in context of models 1-3, dominates both in the very early and in the late universe acting in such a way that the universe expands with acceleration, let us call it the inflaton field following the terminology by Peebles and Vilenkin [13].

V. GRAVITATIONAL BACKGROUND AND INFLATON FIELD $\phi$

The very complicate structure of TMT developed in Refs. [22] - [25] turns a quantization procedure into an extremely complicate problem. This concerns the problem of a quantization of all TMT degrees of freedom. But even the matter fields quantization in TMT seems to be very nontrivial problem. Some reasons of this were discussed briefly in Secs. IVA and IVB.

In the framework of GR, the fundamental role in the procedure of the matter fields quantization belongs to the conception of a gravitational background. Quantum field theory in a gravitational background, i.e. in a given curved (pseudo-Riemannian) space-time, has sense if the energy density produced by quantum fluctuations of matter fields is much less then the Planck energy density. Since the Einstein tensor is proportional to $T_{\mu\nu}$ with the Newton constant as the factor, the reaction of the quantum fields fluctuations through $(T_{\mu\nu})$ on the space-time curvature is small. One can see that this well known question of GR turns into more complicate problem in TMT.

Let us consider here the simple model of Sec. II where the scalar field $\phi$ is the only matter field. The system of equations (21) and (24) has been obtained in Einstein frame after solving the selfconsistent TMT problem (3)-(5), (7),
(8). At first glance, one can follow a standard procedure when ignoring the reaction of the quantum fields fluctuations on the gravity, one regards Eq. (21) as equation of the scalar field with potential (23) in the gravitational background that consists of the metric \(g'_{\mu\nu}\) treated as an external field (For short I will refer to this as "the formal gravitational background"). Then the scalar field quantization in such background would be a well studied problem.

However, in the context of TMT, such standard approach is not always well-grounded. In fact, starting from a (non-degenerate) original metric \(g_{\mu\nu}\) we have to be sure that after conformal transformation (20) to the Einstein frame, the new metric \(g'_{\mu\nu}\) will be also non-degenerate and vice versa. Unfortunately, as we have already discussed at the end of Sec. II, this is not always true: for the first class cosmological scenarios (see Sec. IVB), the conformal transformation (20) becomes singular as \(\phi\)-field arrives at absolute minimum of its TMT effective potential.

In order to elucidate the situation from another point of view it is useful to look at a possible definition of a gravitational background in TMT before conformal transformations (20), i.e. in the original frame. Namely, let us try to define a gravitational background in the same simple model of TMT starting from the original action (3)-(5), (7), (8). We are dealing now with two measures: \(\Phi\), defined by Eq.(6) and \(\sqrt{-g}\). Besides, TMT is formulated in the first order formalism. Therefore, to determine the gravitational background in the original frame we have to fix the metric \(g_{\mu\nu}\), the measure \(\Phi\) by means of the antisymmetric tensor field \(A_{\mu\nu\alpha}\) and the connection. All geometrical objects that constitute the physically admissible gravitational background have to be self-consistent as the geometrical sector of the complete self-consistent gravity + matter system. This means that the connection has to be compatible with \(g_{\mu\nu}\) and measure \(\Phi\), i.e. Eqs. (16) and (18) have to be fulfilled. \(^{12}\) Hence, the gravitational background in the original frame is defined by two fields: the metric \(g_{\mu\nu}\) and the scalar field \(\chi\). We will refer to this as the TMT gravitational background in the original frame. However just the fact that the scalar field \(\chi\) has to be regarded as a fixed external field, is the origin of problems with construction of the TMT gravitational background. Let us discuss the reasons for this.

The constraint (14) does not include the Newton constant or some other very small constant and therefore in contrast to GR, it describes the very strong correlation between scalar field \(\phi\) (inflaton field) and \(\chi\) (geometrical object). So, in the selfconsistent problem, small local space-time fluctuations \(\delta\phi\), where \(\Phi\) (inflaton field) and \(\chi\) (geometrical object) generate fluctuations \(\delta\chi(x)\) of the \(\chi\) field which in general should not be negligible, that is condition

\[
\frac{\delta\chi}{\chi} \ll 1
\]

is not always true. In such a case the \(\chi\) field could not be regarded as the background object and therefore it is impossible to determine the TMT gravitational background approximation.

One can single out a broad class of situations when the TMT gravitational background approximation has no sense. In the linear approximation in small fluctuations \(\delta\phi\), the constraint (14) results in

\[
\frac{\delta\chi}{\chi} = f(\phi)\delta\phi, \quad \text{where} \quad f(\phi) = \frac{V'_2}{V_2} - \frac{V'_1}{V_1 + sM^4}
\]

Recall that for the first class scenarios, the true vacuum state with zero effective cosmological constant is realized at \(\phi = \phi_0\) where \(V_1(\phi_0) + sM^4 = 0\). Then it follows from Eq. (54) that small fluctuations of \(\phi\) in the neighbourhood of the true vacuum state produce very strong fluctuations of \(\chi\). This means that the conception of the TMT gravitational background has no sense in the context of the first class cosmological scenarios and therefore the problem of the scalar field \(\phi\) quantization remains without answer.

It is not the case for the second class cosmological scenarios where \(V_1(\phi) + sM^4\) does not equal to zero at any finite value of \(\phi\). The models of Sec. IV.C correspond just to the second class cosmological scenarios. It is remarkable that the condition (53) is satisfied for models of Sec. IV.C with extremely high accuracy for all values of \(\phi\):

a. For model 1

\[
0 < f(\phi) < \frac{2 \beta - \alpha}{M_p} = \frac{8}{M_p}
\]

b. For model 2

\(^{12}\)On this stage I restrict myself by models where the matter fields do not contribute to the connection (see Sec. VI.C and Refs. [34], [25]).
\[
\frac{8}{M_p} = \frac{2(\beta - 3)}{M_p} < f(\phi) < \frac{2(\beta + 1)}{M_p} = \frac{16}{M_p}
\]

(56)

c. For model 3

\[
0 < f(\phi) < \frac{15.1}{M_p}
\]

(57)

These numerical results mean that influence of small \( \phi \)-fluctuations on \( \chi \)-field has a typical scale of gravitational interaction. So, one can conclude that for models of Sec. IV.C, the TMT gravitational background is the well-defined object. Notice that in the variables of the Einstein frame, the TMT gravitational background is described by external fields \( g'_{\mu\nu} \) and \( \chi \).

Starting from the selfconsistent problem (26), one can represent the action for the scalar field \( \phi \) in the TMT gravitational background in the following form

\[
S_\phi = \int \sqrt{-g} L(\phi; \chi, g_{\mu\nu}) d^4x
\]

(58)

where

\[
L(\phi; \chi, g_{\mu\nu}) \equiv \chi \left[ \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V_1(\phi) \right] + V_2(\phi)
\]

(59)

Here \( g_{\mu\nu} \) and \( \chi \) play the role of the fixed external fields describing the TMT gravitational background. The connection coefficients and curvature tensor of this background are defined by Eqs. (16), (18) and (9). Such picture implies that \( g_{\mu\nu} \) and \( \chi \) are not affected by small (quantum) fluctuations of \( \phi \).

Performing the conformal transformation (20) to the conformal Einstein frame, we provide that the background connection becomes that of the Riemannian space-time with the metric \( g'_{\alpha\beta} \) (see discussion after Eq. (20)) and therefore we obtain a new description of the same gravitational background that consists now of the pseudo-Riemannian space-time with the metric \( g'_{\alpha\beta} \) and in addition, the background scalar field \( \chi \) enters into the Lagrangian density of the scalar field \( \phi \).

The conformal transformation (20) transforms the scalar field \( \phi \) kinetic term in the action (58), (59) to the canonical form \( \sqrt{-g} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} = \sqrt{-g'} g'_{\mu\nu} \phi_{,\mu} \phi_{,\nu} \) and therefore the action for the scalar field \( \phi \) in the gravitational background described by external fields \( g'_{\mu\nu} \) and \( \chi \), becomes (in the Einstein frame)

\[
S_\phi = \int \sqrt{-g'} \bar{L}(\phi; \chi, g'^{\mu\nu}) d^4x
\]

(60)

where

\[
\bar{L}(\phi; \chi, g'^{\mu\nu}) \equiv \frac{1}{2} g'^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - \bar{\nabla}(\phi; \chi^{-1})
\]

(61)

and

\[
\bar{\nabla}(\phi; \chi^{-1}) \equiv \frac{1}{\chi} V_1(\phi) - \frac{1}{\chi^2} V_2(\phi).
\]

(62)

The appearance of factors \( \chi^{-1} \) and \( \chi^{-2} \) in the expression for the potential (62) leads just to a redefinition of the coupling and mass parameters of the pre-potentials \( V_1(\phi) \) and \( V_2(\phi) \).

Notice that the equation for \( \phi \) corresponding to the action (60)-(62) would coincide with Eq. (21) (with \( dU/d\phi \) determined by Eq. (22)) if \( \chi \)-field in (22) were regarded as an external \( \phi \)-independent field. This notion makes more clear differences between the ”formal gravitational background” and the TMT gravitational background: in the ”formal gravitational background” the term \( dU/d\phi \) in the \( \phi \)-equation (21) is defined by Eq. (22) where \( 1/\chi \) is determined as a function of \( \phi \) by the constraint (14) or, which is the same, one can compute \( dU/d\phi \) directly from the r.h.s. of (23). Thus the classical equations for \( \phi \) depends on what kind of background is used. Of course, quantum effective potentials of \( \phi \)-field in two discussed gravitational backgrounds will be very different in general. Problems related to the \( \phi \)-field quantization in TMT need more systematic studying. But for the aims of the present paper we can restrict ourselves by the above discussion.
VI. INCLUSION OF USUAL MATTER FIELDS

A. Outline of the approach to the problem

As it was already discussed in Sec. IV.A, the inclusion of the ordinary matter fields (like vector bosons, fermions, etc.) in TMT is a very nontrivial problem. In the framework of the first class cosmological scenarios, there was shown in Ref. [25] that the field theory model exists where in the conformal Einstein frame, the classical equations of motion of the gauge unified theories as well as the GR equations are exactly reproduced. The advantage of this model is that the spontaneous symmetry breaking does not generate the cosmological constant term. However a serious defect of this model consists of the necessity to use the artificial enough form of how the gauge fields kinetic terms and the fermions selfinteractions enter in the original action. This creates a situation where it is absolutely unclear how one can approach to the matter fields quantization.

The origin of the problem is practically reduced to the role of the constraint (14) that is modified in the presence of usual matter fields. In fact, matter fields in general contribute to the constraint and then the $\chi$-field becomes depending of matter fields. Therefore when starting with Lagrangians $L_1$ and $L_2$ including the matter fields in a form similar to the canonical one, the resulting matter fields equations of motion in the Einstein picture (obtained with the use of the conformal transformation (20) and its generalization for the fermions) can appear in general to be very nonlinear.

Inclusion of the usual matter fields in the context of the models of Sec. IV.C permits to avoid this problem. In fact, following the idea that the only mass scale typical for the inflaton physics in the limit where the symmetry (36)-(38) is exact, is the Planck mass, and terms that explicitly breaks this symmetry, contain mass parameters only a few orders of magnitude less than $M_p$, we provide a situation where the usual matter fields contributions to the constraint appear to be strongly suppressed in comparison with the inflaton contributions throughout the history of the universe. At the late universe, the unbounded increase of the pre-potentials (as $\phi \to \infty$) reinforces this effect of the relative suppression of the usual matter fields contributions to the constraint. In such cases, the scalar field $\chi$ with high accuracy is determined by the same constraint (14) as it was in the absence of the usual matter fields. This allows, starting from the Lagrangians similar to usual ones, one to keep after transition to the Einstein frame the desirable basic features of the usual matter fields sector. Together with the basic idea about the broken continuous global symmetry (36)-(38) modified to the case of the presence of fermions, this approach provides possibilities for constructing gauge unified theories in the context of TMT and, at the same time, to solve problems 1-5 of Introduction.

B. Action of a gauge abelian model and continuous global symmetry

In the framework of the formulated above general ideas let us consider a toy model that possesses gauge abelian symmetry and contains the following matter fields: a complex scalar field $\chi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2)$, an abelian gauge vector field $A_\mu$ and a fermion $\Psi$. Generalization to non-abelian gauge theories can be performed straightforward.

In the presence of fermions, the vierbein-spin-connection formalism [34], [35] has to be used instead of the first order formalism of Sec. II. The action of the model has the general form as in Eqs. (3)-(5) with

$$L_1 = -\frac{1}{\kappa}R(\omega, V) + \frac{1}{2}g^{\mu\nu}\phi_{\mu}\phi_{\nu} + g^{\mu\nu}(\partial_\mu - ieA_\mu)\xi(\partial_\nu + ieA_\nu)\xi^* - V_1(\phi, |\xi|) +$$

$$+ \frac{i}{2}\overline{\Psi}\left\{\gamma^aV^\mu_a(\bar{\omega}_\mu + \frac{1}{2}\omega^{cd}\sigma_{cd} - ieA_\mu) - (\bar{\omega}_\mu - \frac{1}{2}\omega^{cd}\sigma_{cd} + ieA_\mu)\gamma^aV^\mu_a\right\}\Psi$$

(63)

$$L_2 = V_2(\phi) - \frac{1}{4}g^{\alpha\beta}g^{\mu\nu}F_{\alpha\mu}F_{\beta\nu} - \hbar\overline{\Psi}\xi|e^{\gamma\phi/M_p}$$

(64)

Here the following definitions are used [34]:

$$R(\omega, V) = V^{\alpha\mu}V^{\beta\nu}R_{\mu\nuab}(\omega)$$

(65)

$$R_{\mu\nuab}(\omega) = \partial_\mu\omega_{\nuab} - \partial_\nu\omega_{\muab} + (\omega^c_{\muab}\omega_{r\nu} - \omega^c_{\nuab}\omega_{r\mu})$$

(66)

where $V^{\alpha\mu} = \eta^{ab}V^\mu_b$, $\eta^{ab}$ is the diagonal $4 \times 4$ matrix with elements $(1, -1, -1, -1)$ on the diagonal, $V^a_\mu$ are the vierbeins and $\omega_{\muab} = -\omega_{\muba}$ ($a, b = 0, 1, 2, 3$) is the spin connection.
Pre-potential $V_2(\phi)$ is the same as in the models of Sec. IV.C. Pre-potential $V_1(\phi, |\xi|)$ is chosen in the form
\[ V_1(\phi, |\xi|) = V_1(\phi) + P(|\xi|)e^{\alpha \phi/M_p} \]  
where $V_1(\phi)$ is the same as in the models of Sec. IV.C, that is $e^{\alpha \phi/M_p}$ is a common factor in front of $m_4^2 + P(|\xi|)$ in (67).

The transformations of the continuous global symmetry (36)-(38) are generalized now to the form 
\[ V_{\mu a} \rightarrow e^{-\theta/2} V_{\mu a}; \quad g_{\mu\nu} \rightarrow e^\theta g_{\mu\nu}; \quad A_{\mu\nu\lambda} \rightarrow e^\theta A_{\mu\nu\lambda}; \quad \phi \rightarrow \phi - \frac{M_p^2 \theta}{\beta} \]
\[ \xi \rightarrow \xi; \quad A_{\mu} \rightarrow A_{\mu}; \quad \Psi \rightarrow e^{-\theta/4} \Psi; \quad \overline{\Psi} \rightarrow e^{-\theta/4} \overline{\Psi} \]

The term $\int P(|\xi|)e^{\alpha \phi/M_p} \Phi d^4x$ breaks the symmetry (36)-(38) by the same manner as the pre-potential $V_1(\phi)$. For the "Yukawa coupling type" term
\[ S_{Yuk} = -h \int \overline{\Psi} \Phi |\xi| e^{\gamma \phi/M_p} \sqrt{-g} d^4x \]  
(69)
to be invariant under transformations (68), the parameter $\gamma$ must be $\gamma = \frac{3}{2} \beta$. The value of $\gamma$ preferable from the dynamical point of view will be discussed later\(^\text{13}\) and we will see that $\gamma < 2/\beta$. All other terms describing the usual matter fields are invariant under transformations (68). If $\gamma \neq \frac{3}{2} \beta$ then the symmetry is explicitly broken only by the Yukawa coupling type term and by pre-potentials $V_1(\phi, |\xi|)$ and $V_2(\phi)$. Thus, similar to the models of Sec. IV.C, in the model with the Lagrangian densities (63) and (64), the global continuous symmetry (68) is restored as $\phi \rightarrow \infty$.

Note finally that for "pedagogical" reason we have started from the simplified model where the Yukawa type term appears only with the measure $\sqrt{-g}$. We will see later (see Sec. VI.K) that an additional Yukawa type term in (63), that is with the measure $\Phi$, is needed in order to provide a possibility to avoid the long-range force problem.

### C. Connection

Variation of the action with respect to $\omega_{\mu}^{ab}$ together with the identity [35]
\[ R(\omega, V) \equiv -\frac{1}{4\sqrt{-g}} \varepsilon^{\mu
u\alpha\beta} \varepsilon_{abcd} V_{\alpha}^{\gamma} V_{\beta}^{d} R_{\mu\nu}^{ab}(\omega) \]  
(70)
yields
\[ \varepsilon^{\mu
u\alpha\beta} \varepsilon_{abcd} \left[ \chi V_{\alpha}^{e} D_{\nu} V_{\beta}^{d} + \frac{1}{2} V_{\alpha}^{e} V_{\beta}^{d} \chi_{\nu} \right] + \frac{k}{4\sqrt{-g}} V_{\mu}^{e} \varepsilon_{abcd} \overline{\Psi} \gamma^{5} \gamma^{d} \Psi = 0, \]
(71)
where
\[ D_{\nu} V_{\alpha\beta} \equiv \partial_{\nu} V_{\alpha\beta} + \omega_{\nu}^{a} V_{d\beta} \]
(72)
The solution of Eq. (71) is represented in the form
\[ \omega_{\mu}^{ab} = \omega_{\mu}^{ab}(V) + K_{\mu}^{ab}(\sigma) + K_{\mu}^{ab}(V, \overline{\Psi}, \Psi) \]
(73)
where

\(^{13}\)I would like to stress here a very interesting fact: the form of the $\phi$-dependence of the "Yukawa type" term dictated by the continuous global symmetry (36)-(38), is very similar to a motivated by string theories "nucleon-scalar coupling" discussed by Wetterich [6] in the context of a quintessence type model with exponential potential. Relation to this model will be discussed later in the text in detail.
\[ \omega_{\mu}^{ab}(V) = V_\alpha^a V_\beta^b (\frac{\alpha}{\mu}) - V_\mu^{ab} \partial_\mu V^a_\mu \]  

is the Riemannian part of the connection,

\[ K^{ab}_\mu(\sigma) = \frac{1}{2} \sigma_{,\alpha} (V_\mu^a V_\alpha^b - V_\mu^b V_\alpha^a), \quad \sigma \equiv \ln \chi, \]  

and

\[ K^{ab}_\mu(V, \overline{\Psi}, \Psi) = \frac{K}{8} \eta_{\alpha \beta} V_{\mu \nu} \varepsilon^{\alpha \beta \gamma \delta} \gamma^i \Psi. \]  

D. Matter fields equations in the original frame

The inflaton field \( \phi \) equation is now

\[ \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu \nu} \partial_\nu \phi) + \sigma_{,\mu} \phi \sigma_{,\nu} g^{\mu \nu} + \frac{dV_1(\phi)}{d\phi} - \frac{1}{2} \frac{dV_2}{d\phi} + \frac{\chi}{M_p} P(\phi) e^{\alpha \phi / M_p} = - \frac{h \chi}{2 \sqrt{2 \chi}} \overline{\Psi} e^{\gamma \phi / M_p} \]  

In what follows the unitary gauge will be used and, after a shift

\[ \xi = \frac{1}{\sqrt{2}} \varphi \equiv \frac{1}{\sqrt{2}} (\nu + \bar{\varphi}(x)); \quad \nu = \text{const} \]  

the equation for the matter scalar field \( \varphi \) takes the form

\[ \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu \nu} \partial_\nu \varphi) + \sigma_{,\mu} \varphi_{,\nu} g^{\mu \nu} + \frac{dP(\varphi)}{d\varphi} e^{\alpha \phi / M_p} - e^{-2 \varphi} g^{\alpha \beta} A_\alpha A_\beta = - \frac{h}{\sqrt{2 \chi}} \overline{\Psi} e^{\gamma \phi / M_p} \]  

The equations of motion for \( A_\mu \) and \( \Psi \) fields are respectively

\[ \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu \nu} g^{\nu \beta} F_{\alpha \beta}) + e^{-2 \varphi} \chi \partial_\nu A_\mu = - e \chi \overline{\Psi} \gamma^a V_\nu^a \Psi; \]  

\[ \left\{ i \left[ V_\alpha^{\mu \gamma} \left( \partial_\mu - i e A_\mu \right) + \gamma^a C_{ab}^{\mu \nu} + \frac{1}{4} \omega_{\mu \nu \lambda} \gamma^a \sigma_{cd} + \sigma_{cd} \gamma^a \right] V_\alpha^\nu \right\} + \frac{i}{2} \gamma^a V_\nu^a \sigma_\mu - \frac{h}{\chi \sqrt{2}} e^{\gamma \phi / M_p} \right\} \Psi = 0, \]  

where

\[ C_{ab}^\mu = \frac{1}{2 \sqrt{-g}} \partial_\mu \left( \sqrt{-g} V_\nu^\mu \right). \]  

is the trace of the Ricci rotation coefficients [34]. The \( \overline{\Psi} \) field obeys the similar equation.

E. Constraint

As in Sec.II, variation of the action with respect to \( A_{\mu \lambda} \) leads to the equation similar to Eq. (14):

\[ L_1 = L_1 \left( V^{\alpha \mu}, \omega^{ab}_\mu, \phi, \varphi, A_\mu, \Psi, \overline{\Psi} \right) = s M^4. \]  

On the other hand, variation of the vierbeins yields

\[ - \frac{2}{\kappa} V_\nu^{ab} R_{\mu \nu \alpha \beta} (\omega) + \frac{1}{4} \frac{V_\nu^{ab} \omega_{\mu \nu \alpha \beta}}{\mu} (\phi_{,\mu} \phi_{,\nu} + \varphi_{,\mu} \varphi_{,\nu} + e^2 \varphi^2 A_\mu A_\nu) \]

\[ + \frac{i}{2} \overline{\Psi} \left\{ \gamma^b \eta_{ab} \left( \partial_\mu + \frac{1}{2} \omega_{\mu \nu \lambda} \gamma^a \sigma_{cd} - i e A_\mu \right) - \left( \partial_\mu - \frac{1}{2} \omega_{\mu \nu \lambda} \gamma^a \sigma_{cd} + i e A_\mu \right) \gamma^b \eta_{ab} \right\} \Psi \]

\[ - \frac{1}{\chi} V_{\alpha \mu} \left[ \frac{1}{4} g^{\alpha \beta} g^{\mu \nu} F_{\alpha \mu} F_{\beta \nu} - \frac{h}{\sqrt{2}} \overline{\Psi} \varphi e^{\gamma \phi / M_p} \right] - \frac{1}{\chi} g^{\alpha \beta} V^\nu_{\alpha \mu} F_{\alpha \mu} F_{\beta \nu} = 0 \]
Contracting the last equation with $V^a_{\mu}$, making use Eq.(81) and a similar equation of motion for $\overline{\Psi}$, and combining the result with Eq.(83) one can eliminate $R(\omega, V)$ and the result is the constraint

$$sM^4 + V_1(\phi) + P(\varphi)e^{\alpha\phi/M_p} = \frac{2}{\lambda} \left[ V_2(\phi) - \frac{3}{4\sqrt{2}} \hbar \Psi \varphi e^{\gamma\phi/M_p} \right]$$

which is a direct generalization of the constraint (14) to the model we study here.

One of the aims of the toy model (63), (64) consists in a demonstration of a possibility to construct realistic gauge unified theories (like electro-weak and GUT) in the context of models of Sec. IVC. Introducing the scalar field $\xi$ intended for realization of the Higgs phenomenon. Since $P(\varphi)$ and $m_1^2$ appear in the combination $m_1^2 + P(\varphi)$, the constant part of $P(\varphi)$ can be always absorbed by $m_1^2$. Then it is natural to assume\(^{14}\) that $|P(\varphi)| \ll m_1^2$. Later, turning to quantum effective potential, we will discuss a concrete model where $P(\varphi) = \overline{\Psi}^2 \varphi^4$ and then the idea explained in Sec. VLA becomes more clear: the choice of the mass parameters in the models of Sec. IVC allows to provide a situation where $\varphi$-contribution to the constraint (85) is suppressed with respect to the inflaton field $\phi$-contribution and hence it can give only extremely small corrections to the main picture. If fluctuations of fermionic fields are not anomaly large, it is natural to expect that the same conclusion is true for fermionic contribution to the constraint (85) as well. So, the field $\chi$ determined by the constraint (85), in practically interesting cases coincides with the $\chi$-field determined by the constraint (14) which holds in the model free of the usual matter at all. For short, in what follows, when neglecting the usual matter fields contribution to the constraint, we will use the term ”A-approximation”. This notion will be very useful in the next subsection where we are going to represent equations of motion in the Einstein frame.

\(F.\) Equations of motion for the selfconsistent problem in the Einstein frame

In the presence of fermions, the transition to the "Einstein frame" (more suitable term for this case would be the Einstein-Cartan frame) is carried out by the transformations to the new variables [25]

$$V_{\mu\nu}(x) \to V'_{\mu\nu}(x) = \chi^{1/2}(x)V_{\mu\nu}(x); \quad g_{\mu\nu}(x) \to g'_{\mu\nu}(x) = \chi(x)g_{\mu\nu}(x);$$

$$\Psi(x) \to \Psi'(x) = \chi^{-1/4}(x)\Psi(x); \quad \overline{\Psi}(x) \to \overline{\Psi'}(x) = \chi^{-1/4}(x)\overline{\Psi}(x);$$

$$\phi \to \phi'; \quad A_{\mu\lambda} \to A'_{\mu\lambda}; \quad \varphi \to \varphi'; \quad A_{\mu} \to A_{\mu},$$

where $\chi$ is determined by Eq.(85) (see also discussion after Eq.(85)).

In fact, after transition to the new variables defined by the transformations (86), the $\sigma$-contribution (75) to the spin connection (the second term on the r.h.s. of Eq. (73)) is canceled and the transformed spin connection takes the form [25]

$$\omega^{\mu\nu}_{\rho\delta} = \omega^{\rho\delta}_{\mu\nu}(V') + \frac{K}{8} \eta^{\rho\delta} V'_{\mu\nu} \varepsilon^{abcd} \overline{\Psi} \gamma^5 \gamma^4 \Psi'.$$ \hspace{1cm} (87)

that coincides with the well-known solution for the spin connection in the context of the first order formalism approach to the Einstein-Cartan theory [34] where a Dirac spinor field is the only source of a non-riemannian part of the connection. Hence the curvature tensor (66) expressed in terms of the new connection (87) becomes the curvature tensor of such an Einstein-Cartan theory.\(^{15}\)

At the same time, in the equations for fermionic fields (Eq. (81) and similar equation for $\overline{\Psi}$), all terms containing $\sigma_{\mu\nu}$-contributions also disappear [25] in the Einstein-Cartan frame and the result is

\(^{14}\)Recall that $m_1$ appears in the definition of the pre-potential $V_1(\phi)$ in models 1-3 of Sec. IVC, Eqs. (44), (46) and (48) respectively, and the value of $m_1$ are chosen such that $m_1^2 = (10^{-2}M_p)^4$ in the model 1 and $m_1^2 = (10^{-3}M_p)^4$ in models 2 and 3.

\(^{15}\)Notice that in the original frame, the terms including $\sigma_{\mu\nu}$ (recall that $\sigma \equiv \ln \chi$) originate a non-metricity and therefore TMT in the original variables has no form of an Einstein-Cartan theory.
\[
\left\{ i \left[ V^\mu_a \gamma^a \left( \partial_\mu - ieA_\mu \right) + \frac{\gamma_a \gamma^b}{4} \omega^a_{\mu \nu} \right] - \frac{\hbar}{\sqrt{2} \varphi} \frac{e^{\gamma \phi/M_p}}{\chi^{3/2}} \right\} \Psi' = 0 \quad (88)
\]

where
\[
C^{\mu}_{\nu ab} = \frac{1}{2\sqrt{-g'}} \partial_\mu \left( \sqrt{-g'} V^\mu_a \right) \quad (89)
\]
is the trace of the Ricci rotation coefficients in the new variables. Equation for \( \Psi' \) has similar structure. The only difference of these fermionic equations from the standard Dirac equations in the Einstein-Cartan theory [34] where a Dirac spinor field is the only source of a non-riemannian part of the connection, is related to an unusual Yukawa type term and it will be discussed later. Notice that for purposes of realistic particle physics one can neglect the second term in Eq. (87) that leads to a "spin-spin contact interaction" [34] with coupling constant \( M_p^{-2} \). For short, in what follows, when neglecting this interaction, we will use the term "B-approximation".

Other equations of motion in the Einstein-Cartan frame have the following form:
\[
\frac{1}{\sqrt{-g'}} \partial_\mu (\sqrt{-g'} g^{\mu \nu} \partial_\nu \phi) + \frac{1}{\chi} \left[ \frac{dV}{d \phi} - \frac{1}{\chi} \frac{dV_1}{d \phi} + \frac{\alpha}{M_p} P(\varphi) e^{\alpha \phi/M_p} \right] = - \frac{h \gamma}{\sqrt{2} M_p} \Psi' \varphi e^{\gamma \phi/M_p} / \chi^{3/2} ; \quad (90)
\]
\[
\frac{1}{\sqrt{-g'}} \partial_\mu (\sqrt{-g'} g^{\mu \nu} \partial_\nu \varphi) + \frac{\epsilon^{\alpha \phi/M_p}}{\chi} \frac{dP(\varphi)}{d \varphi} - e^2 \varphi g^{\alpha \beta} A_\alpha A_\beta = - \frac{h}{\sqrt{2}} \Psi' e^{\gamma \phi/M_p} / \chi^{3/2} ; \quad (91)
\]
\[
\frac{1}{\sqrt{-g'}} \partial_\mu \left( \sqrt{-g'} g^{\mu \alpha} g^{\nu \beta} F_{\alpha \beta} \right) + \frac{e^2}{2} \varphi^2 g^{\mu \nu} A_\mu = - e \Psi' \gamma^a V^\mu_a \Psi' \quad (92)
\]

It is very important to stress that in the A and B-approximations all matter fields equations, (88), (90)-(92), have the canonical structure of the corresponding matter fields equations in a Riemannian space-time. The only specific features of these equations are concentrated in unusual forms of the effective potentials and some of the interactions.

After some algebraic manipulations with Eq. (84), transition to the new variables by means of (86) and making use the fermionic equation (88) and similar equation for \( \Psi' \), we obtain canonical gravitational equations of the Einstein-Cartan theory. If finally one to write down these equations in the B-approximation, we come to the canonical GR gravitational equations
\[
G_{\mu \nu} = \frac{\kappa}{2} T_{\mu \nu} \quad (93)
\]
where \( G_{\mu \nu} \) is the Einstein tensor of the Riemannian space-time with metric \( g_{\mu \nu} \) and the energy-momentum tensor has a canonical GR structure [29]:
\[
T_{\mu \nu} = \phi_{,\mu} \phi_{,\nu} - \frac{1}{2} g'_{\mu \nu} \phi_{,\alpha} \phi_{,\beta} g^{\alpha \beta} + \frac{1}{\chi^2} V_2(\phi) g'_{\mu \nu} + \varphi_{,\mu} \varphi_{,\nu} - \frac{1}{2} g'_{\mu \nu} \varphi_{,\alpha} \varphi_{,\beta} g^{\alpha \beta} + \frac{1}{4} g'_{\mu \nu} F_\alpha \sigma_{,\rho} g^{\alpha \beta} + \frac{1}{2} g_{\mu \nu} F_{\alpha \beta} g^{\alpha \beta} - F_{\mu \nu} F_{\alpha \beta} g^{\alpha \beta} + e^2 (v + \varphi)^2 \left( A_\mu A_\nu - \frac{1}{2} g'_{\mu \nu} A_\alpha A_\beta g^{\alpha \beta} \right) + \frac{i}{2} \left[ \Psi' \gamma^a V^\mu_a (\mu) \Psi' - (\nabla^a \nabla^b) \gamma^a V^\mu_a \Psi' \right] \quad (94)
\]
where \( \nabla_\mu \Psi' = (\partial_\mu + \frac{1}{2} \omega^{\rho \sigma}_{\mu \nu} \sigma_{,\sigma} - ieA_\mu) \Psi' \) and \( \nabla_\mu \nabla_\nu = \partial_\mu \nabla_\nu - \frac{1}{2} \omega^{\rho \sigma}_{\mu \nu} \nabla_\sigma + icA_\mu \nabla_\nu \).

Notice again that \( \chi \) field entering into Eqs. (88), (90), (91) and (94), is determined by the constraint (85) which in the A-approximation gives
\[
\frac{1}{\chi} = \frac{M^4 + V_1(\phi)}{2V_2(\phi)} \quad (95)
\]

In what follows, all discussion will be performed in the framework of A- and B-approximation.

It is worthwhile to notice that the transformations of the global continuous symmetry (68) expressed in terms of the variables of the Einstein frame, are reduced just to shifts of \( \phi \): \( \phi \rightarrow \phi - \frac{M_p}{\chi} \theta \).
To study the matter fields sector of the system of equations (88) - (95) one has to define an appropriate background. In Sec.V we discussed two different gravitational backgrounds in the model where the usual matter was absent and the inflaton field $\phi$ was the only field of the non-gravitational sector. One can see that if making use of the "formal gravitational background", then it is impossible to write down an effective classical action in curved background giving rise to the system of equations (88), (90) - (92). For example, to provide the appearance of the last term of Eq. (88) and the r.h. sides of Eqs. (90) and (91), such an effective classical action in curved background has to include the "Yukawa coupling type" term $L_{Yuk} = -\frac{h}{\sqrt{2}} \overline{\Psi} \gamma^\mu \gamma^\nu \frac{e^{\gamma_5 / M_p}}{\sqrt{g}} \Psi^\mu \phi / \sqrt{g}$. Working in the "formal gravitational background", we have to insert expression for $\chi$, Eq.(95), into $L_{Yuk}$. But then variation of the inflaton field $\phi$ leads not only to the appearance of the needed terms, but unwanted terms, coming from the variation of $\chi(\phi)$, will appear as well. It is not the case in the framework of the TMT gravitational background since in that case the scalar field $\chi$ is a background one.

If however, we want to construct quantum theory of the usual matter fields then it seems to be natural to start from the approximation where in the addition to the gravitational background, the inflaton field $\phi$ remains practically constant during a typical time of quantum fluctuations of the matter fields. In such a case the mentioned above difference between two definitions of the gravitational background disappears: the background field $\chi$ is determined by the background field $\phi$ via Eq.(95).

So, let us study some features of the particle physics model in the background that, in terms of variables of the Einstein picture, consists of two external fields: $g_{\mu\nu}$ and $\phi$. For short I will refer to this issue as the "particle physics model in the cosmological background".

The effective classical action for the particle physics model corresponding to the system of equations (88), (91) and (92), in the cosmological background, can be chosen in the following form (in the unitary gauge)

$$S_{\text{class}}^{\text{background}} = \int \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \frac{\partial \varphi}{\partial x^\mu} \frac{\partial \varphi}{\partial x^\nu} - V_d(\varphi; \phi) + \frac{e^2}{2} \varphi^2 A_\mu A_\nu g^{\mu\nu} \right]$$

$$- \frac{1}{4} g^{\alpha\beta} g^{\mu\nu} F_{\alpha\mu} F_{\beta\nu} + L_{\text{kin}}(\overline{\Psi}; \Psi', A_\mu) + L_{Yuk}(\overline{\Psi} \Psi' \phi; \phi)$$

where $V_d(\varphi; \phi)$ is the classical TMT effective potential for the matter (Higgs) scalar field $\varphi$ in the presence of the background inflaton field $\phi$:

$$V_d(\varphi; \phi) = P(\varphi) \frac{M^4 + V_1(\phi)}{2V_2(\phi)} e^{\gamma_5 \varphi / M_p};$$

$L_{\text{kin}}(\overline{\Psi}; \Psi', A_\mu)$ is the standard kinetic term for the fermion field in a Riemannian space-time with metric $g_{\mu\nu}$, including also the gauge coupling to the vector field $A_\mu$. And finally, the TMT effective "Yukawa coupling type" term $L_{Yuk}(\overline{\Psi} \Psi' \phi; \phi)$ is

$$L_{Yuk}(\overline{\Psi} \Psi' \phi; \phi) = -\frac{h}{\sqrt{2}} \overline{\Psi} \gamma^\mu \gamma^\nu \frac{e^{\gamma_5 / M_p}}{\sqrt{g}} \Psi^\mu \phi / \sqrt{g} = -\frac{h}{2V_2(\phi)} \left[ M^4 + V_1(\phi) \right]^{3/2} e^{\gamma_5 \varphi / M_p}.$$

**H. Massless scalar electrodynamics model in the cosmological background and SSB**

Up to now the function $P(\varphi)$ was unspecified. Ignoring here technical questions (in particular, the question of renormalizability that requires a non-minimal coupling $\eta R |\varphi|^2$ ), let us attract attention to a quantum effective potential when choosing $P(\varphi) = \frac{27}{4} |\varphi|^4$, $\varphi_0 = \text{const}$. This means that (ignoring the fermion field), we are dealing with massless scalar electrodynamics in curved space-time where the classical potential (the tree approximation) is given by

$$V_d(\varphi; \phi) = \frac{\lambda_0(\phi)}{4!} |\varphi|^4$$

and $\lambda_0(\phi)$ depends on the background field $\phi$: 23
\[ \lambda_0(\phi) = \widetilde{\lambda}_0 k(\phi); \quad k(\phi) = \frac{M^4 + V_1(\phi)}{2V_2(\phi)} e^{\alpha\phi/M_p}. \] (100)

Numerical estimations of \( k(\phi) \) in the models of Sec. IV.C give the following results: \( 0 < k(\phi) < 3.5 \) for model 1; \( 0 < k(\phi) < 1.2 \cdot 10^{-8} \) for model 2; \( 0 < k(\phi) < 3 \cdot 10^{-7} \) for model 3. In all models \( k(\phi) \) asymptotically approaches zero as \( \phi \to \pm \infty \). Thus one can conclude that in all cases \( \lambda_0(\phi) \) is of the same order or less than \( \widetilde{\lambda}_0 \).

The computation technics of the effective potential for the "massless" scalar electrodynamics in the one-loop approximation is well-known issue [36]. However, the problem we study here is not quite usual: the quartic coupling "constant" depends actually on the cosmic time via the inflaton field \( \phi \). If one takes into account that in a course of its evolution, the classical field \( \phi \) remains practically constant during a typical time of quantum matter fields fluctuations, then it is natural to consider the problem in the adiabatic approximation. Therefore computing the effective potential we can regard \( \lambda_0(\phi) \) as a constant. Then the computation becomes quite standard. The only additional issue we have to clear up is a possible physical effect that the adiabatically changing \( \lambda_0(\phi) \) might be on the \( \phi \)-effective potential.

One can check that the first point where we encounter necessity to decide this problem, is the renormalization procedure. In fact, performing calculations with the bare coupling constant \( \lambda_0 \) we have no need to think about its adiabatic \( \phi \) dependence. But when we turn to the use of the renormalized (finite) parameter \( \lambda \) defined by \( \lambda_0 = \lambda + \delta \lambda \) where \( \delta \lambda \) is the counter term (which, as one knows, is divergent in perturbation theory), we have to take into account a possible \( \phi \)-dependence of the effective \( \lambda \).

The vector boson loops contribution to the effective potential in the one-loop approximation has the order of \( e^4 \) and does not depend on \( \phi \) (see Eq. (96)). Therefore, just as in the standard scalar electrodynamics, one can claim that in spite of possibility for \( \lambda_0(\phi) \) to be very small, the effective \( \lambda(\phi) \) can not be too small. On the other hand it is important also that \( \lambda(\phi) \) can not be large: since \( \phi \)-dependence of \( \lambda_0 \) acts in the direction of decrease in comparison with \( \widetilde{\lambda}_0 \), there are no reasons for a possible \( \phi \)-dependence of \( \lambda \) to act in the opposite direction.

The scalar loops contribution has the order of \( \lambda^2 \). Therefore, in the same way as in the standard scalar electrodynamics, in the one-loop approximation, one can neglect the scalar loops contribution with respect to the vector boson loops contribution.

The one-loop effective potential for the scalar field \( \varphi \) evaluated at the fixed value of the background inflaton field \( \phi = \phi_1 \) can be written in the form

\[ V_{\text{eff}}(\varphi; \phi_1) = \frac{\lambda(\phi_1)}{4!} \varphi^4 + \frac{3e^4}{(8\pi)^2} \varphi^4 \left( \ln \frac{\varphi^2}{\mu^2} - \frac{25}{6} \right), \] (101)

where

\[ \lambda(\phi_1) = \frac{d^4V_{\text{eff}}}{d\varphi^4} |_{\varphi=\mu}. \] (102)

Let us assume that \( \phi_1 \) is the value of the background inflaton field where \( \lambda(\phi) \) has a maximal possible magnitude (but it is still small!). Suppose also that the renormalization mass \( \mu \) is chosen such that \( \lambda(\phi_1) \sim e^4 \). This can be always done as is well known from the renormalization group analysis [36]. The final form of the effective potential

\[ V_{\text{eff}}(\varphi; \phi_1) = \frac{3e^4}{(8\pi)^2} \varphi^4 \left( \ln \frac{\varphi^2}{\mu^2} - \frac{1}{2} \right), \] (103)

is determined in terms of two free parameters: renormalized gauge coupling constant \( e \) and VEV \( \varphi = \psi \).

To verify whether the change of the value of the background inflaton field \( \phi \) has some physical consequences, let us suppose that we want to repeat the same computation of the one-loop effective potential at another fixed value of the background inflaton field \( \phi = \phi_2 \) where the order of magnitude of \( \lambda(\phi_2) \) is less than \( e^4 \) if we take the same renormalization mass \( \mu \). According to results of the renormalization group analysis [36], one can move \( \lambda(\phi_2) \) to the magnitude of the order of \( e^4 \) by a change in the renormalization mass that does not change the order of magnitude of \( e \). This can be always done if \( \lambda(\phi_2) \) is small. Then the computation of the one-loop effective potential at \( \phi = \phi_2 \) in the same approximation leads to the same effective potential as it was at \( \phi = \phi_1 \), Eq. (103), with the same order of magnitude of the free parameter \( e \). One can conclude therefore that in the used approximation, the \( \phi \)-dependence of \( \lambda \) has no physical effect.

I will ignore on this stage of investigation the fermion loops contribution into the \( \phi \) effective potential. The nonminimal coupling of the Higgs field \( \varphi \) to curvature, which appears in the quantum effective action in curved space-time [37], might have some interesting but, most likely, weak enough effect, and this question exceeds the limits of the present paper.

So, for the usual enough form of the function \( P(\varphi) \), we obtain, in a cosmological background, the effective quantum potential for the scalar (Higgs) field \( \varphi \) typical for dynamically broken gauge theories. Notice again that the term
\[
\frac{\kappa^2}{2} \Phi^2 A_\mu A_\nu g^{\mu\nu} \text{ in Eq. (96) does not depend on the inflaton field } \phi. \text{ Thus, SSB and Higgs phenomenon occur in a standard way.}
\]

I. Yukawa coupling type term and fermion mass

As a result of SSB, the Yukawa coupling type term (98) (see also Eq. (88)) produces the TMT effective fermion mass \( m_f \) depending on the inflaton field \( \phi \):

\[
m_f = m_f(\phi) = \frac{h}{4} v \left[ \frac{M^4 + V_1(\phi)}{V_2(\phi)} \right]^{3/2} e^{\gamma \phi / M_p}.
\] (104)

For \( \phi > M_p \) (the region corresponding to the late universe), the fermion mass becomes

\[
m_f^{(\text{late})} \simeq m_f^{(0)} e^{-[3(\beta - \frac{3}{2}\alpha) - \gamma]|\phi / M_p|}, \quad m_f^{(0)} = 2h v \left( \frac{m_1}{m_2} \right)^6, \quad \text{as} \quad \phi > M_p
\] (105)

We see that in the late universe the fermion mass approaches the nonzero constant \( m_f^{(0)} \) if

\[
\gamma = 3(\beta - \frac{\alpha}{2}).
\] (106)

Notice that if \( \gamma \) indeed satisfies the relation (106) then with the choice as in Sec. IV (i.e. \( \alpha = 6 \) and \( \beta = 7 \)), we obtain \( \gamma = 12 \) that is close to the value of \( \frac{1}{2}\beta = 10.5 \) dictated by the symmetry (68) (see discussion after Eq. (69)) \(^{16}\). So in the framework of our working hypothesis about approximate symmetry (68) one can ensure a successful mass generation for fermions in the present cosmological epoch in a way typical for the standard model and, at the same time, one to keep the direct coupling, Eq.(98), of fermionic matter to inflaton field. This is the very important point and the TMT mechanism providing this result will be discussed in detail in the next subsection.

One has to notice that a formal generalization of the toy (abelian) model we study here, to a non-abelian one (like \( SU(2) \times U(1) \) or \( SU(5) \)) can be performed straightforward. Then we have to worry about scales of the particles mass generated as a result of SSB. In this connection it would be interesting to estimate the order of magnitude of the mass generated for fermions in the present cosmological epoch in a way typical for the standard model and, at the same time, one to keep the direct coupling, Eq.(98), of fermionic matter to inflaton field. This is the very important point and the TMT mechanism providing this result will be discussed in detail in the next subsection.

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Concerning the very early universe, that is for \( \phi < -M_p \), one can see that the model predicts for the TMT effective fermion mass, Eq. (104), to be extremely small: \( m_f \to 0 \) as \( \phi \to -\infty \). For example, in model 3 of Sec. IV.C,

\(^{16}\)The value \( \gamma = 12 \) is as close to \( \frac{1}{2}\beta = 10.5 \) as \( \beta = 7 \) is close to \( \alpha = 6 \).
J. The inflaton field cosmological equation and relation to the Wetterich’s model

This paper does not pursue an object of a complete studying the cosmology of a homogeneous and isotropic universe in the model which on the microscopic level is described by equations of subsection VI.F. We restrict ourselves here by notion that general structure of these equations is canonical one and all specific features are concentrated in two unusual couplings: Eqs. (97) and (98). The first one was discussed in Sec. VI.H in connection with SSB and Higgs phenomenon. The second one gives an interesting result for fermion mass.

Nevertheless, even without entering into detailed study of the cosmological predictions, one can reveal some specific features of the model. Let us turn to the inflaton field equation, (90), which in a homogeneous and isotropic universe takes the form

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{dU}{d\phi} = \frac{\gamma}{M_p} L_{Yuk}. \]  

(107)

where \( H \) is the Hubble parameter and \( \phi \) in \( L_{Yuk} \), Eq. (98), is regarded now as the dynamical field rather than the background one as it was in Sec. VI.G.

Proceeding in the framework of models of Sec. IV.C one can divide the whole region of \( \phi \) into three intervals corresponding to the quintessence epoch, the radiation and matter dominated universe and the inflationary epoch.

For all models of Sec. IV.C, Eq. (107) in the quintessential region (\( \phi \gtrsim 1.7M_p \), see also Sec. IV.D, item 3) takes the form

\[ \ddot{\phi} + 3H \dot{\phi} - c M_p^3 e^{-2\phi/M_p} = -\gamma \frac{m_f^{(late)}}{M_p} \frac{\nabla \Psi'}{\Psi'}, \]  

(108)

where the values \( \beta = 7 \) and \( \alpha = 6 \) have been used; \( \gamma = 12 \) if one chooses (106) to provide constancy of the fermion mass at the late universe; \( m_f^{(late)} \) is defined by Eq. (105); dimensionless constant \( c \) depends on the model: \( c = 2 \cdot 10^{-16} \) in model 1; \( c = 1.2 \cdot 10^{-27} \) in model 2; \( c = 2 \cdot 10^{-24} \) in model 3.

In the intermediate region (see Sec IV.D, item 4) which, as it is expected, should be responsible for the radiation and matter dominated stage, Eq. (107) takes the model-dependent form. But its main features one can demonstrate taking one of the models. For example, in model 3 the equation is the following:

\[ \ddot{\phi} + 3H \dot{\phi} + 10^{-16} M_p^3 f_1(\phi) e^{-14\phi/M_p} = -2\gamma h \frac{V}{M_p} 10^{-12} f_2(\phi) \frac{\nabla \Psi'}{\Psi'} e^{-9\phi/M_p} \quad \text{as} \quad -0.6M_p < \phi < 1.5M_p \]  

(109)

where the value \( \gamma = 12 \) has been substituted to the exponent in the r.h.s. The slow varying dimensionless functions \( f_1(\phi) = [0.1 + 0.5(\frac{\phi}{M_p})^2] \cdot [-1.4 + 2(\frac{\phi}{M_p}) - 7(\frac{\phi}{M_p})^2] \) and \( f_2(\phi) = [0.1 + 0.5(\frac{\phi}{M_p})^2]^{3/2} \) are the only objects in (109) specific for model 3.

And finally, for completeness let us write down the form of the equation in the inflationary region of model 3:

\[ \ddot{\phi} + 3H \dot{\phi} + m^2 \phi = -6\sqrt{2} h \frac{V}{M_p} f_3(\phi) \frac{\nabla \Psi'}{\Psi'} e^{-12|\phi|/M_p} \quad \text{as} \quad \phi < -0.7M_p, \]  

(110)

where \( m = 10^{-6}M_p \) and \( f_3(\phi) = (|\phi|/M_p)^3/[10^4 + 0.5(|\phi|/M_p)^2]^{3/2} \). The r.h.s. of the last equation is practically zero for \( \phi \ll -M_p \) and we recognize the inflaton field equation for the chaotic inflation model [15] with the inflaton potential \( V = \frac{1}{2} m^2 \phi^2 \).

It is convenient to analyse equations (108) - (110) by a comparison with the inflaton equation of Wetterich’s model [6] where the exponential potential for \( \phi \) of the form (2) has been also assumed. Wetterich suggested there an idea that a definite \( \phi \)-dependence of \( a \) in (2) could provide a solution of the problem with constraint from cosmological

\[ m_f \approx h v 10^{-2} e^{-\gamma|\phi|/M_p}, \quad \text{as} \quad \phi < -M_p. \]  

At the same time, the gauge coupling of \( \Psi' \) to \( A_\mu \) (see Eq. (88)) is the standard one and, in particular, it does not depend on the inflaton field \( \phi \).  

\[ ^{17} \text{This can be an interesting example of the model [38] of massless spinor electrodynamics realized as the limit of a massive theory as } \phi \to -\infty. \]
m
fermion and inflaton. Instead of Eqs. (88) and (90) we obtain now.

discussed modification causes in equations of motion in the Einstein frame let us consider here only equations for
\(\phi\) is negligible (for all values of \(\phi\)). From the other hand, \(L_{\text{Wett}}\) describes the \(\phi\)-dependence (and therefore, a cosmic time dependence) of the effective nucleon mass. The only way to avoid such an undesirable effect is to put \(\gamma = 0\) that means just a removal of a direct nucleon-scalar coupling.

In our model we also start from the direct fermion-inflaton coupling that in the original variables is defined by Eq. (69), including in addition the matter (Higgs) scalar field \(\xi\). After SSB, the factor \(h/\sqrt{2}\) (see notations (78)) plays the role of \(m_f(0)\) in \(L_{\text{Wett}}\). Similar to the model [6], the contribution of the direct fermion-inflaton coupling to the inflaton field \(\phi\) equation in the Einstein frame, (107), is also proportional to \(\gamma\). The \(\phi\) - dependence of the effective fermion-inflaton coupling becomes however more complicated in a course of manipulations dictated by TMT. Already on the first stage, when we vary the original action (3)-(5), (63) and (64) with respect to \(\gamma\), one can check that change which \(\tilde{\gamma}\) makes in the constraint consists in the appearance of an additional term \(\gamma\phi/Mp\Phi\Psi\phi\). In such a model the constant fermion mass is also achieved [28]. Having this idea in mind, let us modify our model (3)-(5), (63), (64) with the explicit breaking of the symmetry (68), by including an additional Yukawa coupling type term which enters into the action with measure \(\Phi\).

\(\tilde{S}_{\text{Yuk}} = -\tilde{\hbar} \int \Psi\Psi|\xi|e^{\gamma\phi/Mp}\Phi \Phi dx\). (112)

For this term to be invariant under transformations (68), the parameter \(\tilde{\gamma}\) must be \(\tilde{\gamma} = \frac{1}{2}\beta < \alpha\). The magnitude of \(\tilde{\gamma}\) preferable from the dynamical point of view will be discussed below.

One can check that change which \(\tilde{S}_{\text{Yuk}}\) makes in the constraint consists in the appearance of an additional term \(2\sqrt{2h\Psi}\Psi e^{\gamma\phi/Mp}\) in the l.h.s. of Eq. (85). Since \(\tilde{\gamma}\) must be less than \(\alpha\), the \(\tilde{S}_{\text{Yuk}}\) contribution to the constraint is negligible (for all values of \(\phi\)) compared to first two terms in the l.h.s. of Eq. (85). Among changes which the discussed modification causes in equations of motion in the Einstein frame let us consider here only equations for fermion and inflaton. Instead of Eqs. (88) and (90) we obtain now

\[
\left\{ i \left[ \gamma^\mu \gamma^a (\partial_\mu - ieA_\mu) + \gamma^a C^{ab}_c \frac{i}{4} \varepsilon^{cde} \gamma^b \gamma^d a\mu \right] - \frac{\tilde{\hbar}}{\sqrt{2}} e^{\gamma\phi/Mp} \right\} \Psi' = 0; \quad (113)
\]
The constancy of the fermion mass in the late universe

\[ m^{(\text{late})}_{f,\text{modified}} \simeq v \left( \frac{m_1}{m_2} \right)^2 2^2 \left[ 2h \left( \frac{m_1}{m_2} \right)^4 e^{-3(\beta - \frac{1}{2} - \gamma)\phi/M_p} + \tilde{h} e^{-(\beta - \frac{1}{2} - \gamma)\phi/M_p} \right] \text{ as } \phi > M_p \]  

(115)

is achieved now if the condition

\[ \tilde{\gamma} = \beta - \frac{1}{2} \alpha \]  

(116)

holds together with (106). For \( \beta = 7 \) and \( \alpha = 6 \), the constancy of the fermion mass in the late universe implies that \( \tilde{\gamma} = 4 \) which is as close to \( \frac{7}{2} \beta = 3.5 \) as \( \alpha \) is close to \( \beta \) (see also discussion after Eq. (106) and footnote 15).

Instead of (108), the cosmological inflaton equation in the late universe is now

\[ \ddot{\phi} + 3H \dot{\phi} - cM_p^2 e^{-2\phi/M_p} = -\frac{v}{M_p} \varphi' \left( \frac{m_1}{m_2} \right)^2 2^2 \left[ 2\gamma h \left( \frac{m_1}{m_2} \right)^4 e^{-3(\beta - \frac{1}{2} - \gamma)\phi/M_p} + \tilde{\gamma} h e^{-(\beta - \frac{1}{2} - \gamma)\phi/M_p} \right]. \]  

(117)

Again as it was in the simpler model studied in the previous subsections, with the conditions for constancy of the fermion mass in the late universe, Eqs. (106) and (116), the r.h.s. of Eq. (117) acts as the effective Yukawa coupling of the inflaton to fermionic matter

\[ R_{\text{eff, present}}^{(\text{Yuk, modified})} = -\frac{v}{M_p} \left( \frac{m_1}{m_2} \right)^2 (\beta - \frac{1}{2} \alpha) \left[ 6h \left( \frac{m_1}{m_2} \right)^4 + \tilde{h} \varphi' \right]. \]  

(118)

We see that in the modified model there exists a possibility to prevent the appearance of such danger interaction. To realize this opportunity we have to require

\[ \frac{\tilde{h}}{\tilde{h}} = -6 \left( \frac{m_1}{m_2} \right)^4. \]  

(119)

This is actually strong enough tuning since for instance, in the context of models 2 and 3 of Sec. IV.C, it implies

\[ |\tilde{h}/h| \sim 10^{-12}. \]  

(120)

If we recall that \( \tilde{h} \) and \( h \) are the Yukawa type coupling constants of the Higgs scalar to fermion, it appears to be surprisingly that their ratio has to be of the order of magnitude that shows the degree of the hierarchy problem in GUT: \( m_W/m_X \sim 10^{-12} \).

With conditions (106),(116) and (119), the fermion mass in the late universe becomes

\[ m^{(\text{late})}_{f,\text{modified}} = \frac{2}{3} h v \left( \frac{m_1}{m_2} \right)^2 \text{ as } \phi > M_p \]  

(121)

A possible relation of the discussed question to the hierarchy problem in GUT, as well as other problems that appear in the attempts to generate a realistic unified gauge theories in the context of TMT, will be studied elsewhere.

VII. DISCUSSION AND CONCLUSION

Before summarizing and discussing main results of this paper I would like to stress again that the first impression that the studied models belong to a sort of a scalar-tensor theories, is wrong. The ratio of two measures, that is the scalar field \( \chi \), Eq. (13), is the only object entering into equations of motion and carrying information about the measure \( \Phi \) degrees of freedom. If we restrict ourselves by models where \( L_1 \) is linear in the scalar curvature (see Eqs. (7) and (63)) and \( L_2 \) does not contain curvature, then in the first order formalism, the constraint appears that
A possibility was studied in Ref. [25] (see Sec. VI therein) where a way to prevent the appearance of such a danger term \( \Phi \) is defined by means of four scalar measure fields. In TMT we have a quintessence model of the late universe. However, in contrast to quintessence models studied in the framework of GR or Brans-Dicke type models, in TMT we have a vacuum state realized asymptotically as \( \chi \rightarrow \infty \) at a finite value of \( \phi = \phi_0 \) where \( V_1(\phi_0) + s M^4 = 0 \). As we have seen in Sec. V, in such vacuum the usual conception of the gravitational background becomes invalid: small fluctuations of \( \phi \) cause infinitely large fluctuations of \( \chi \). For the true vacuum state this feature is unacceptable.

For this reason in this paper we studied cosmological scenarios of the second class (see Sec. IV.B) where the true vacuum state is realized asymptotically as \( \phi \sim \infty \). This naturally leads us to a need to apply to a quintessence model of the late universe. However, in contrast to quintessence models studied in the framework of GR or Brans-Dicke type models, in TMT we have a new option: one can choose the prepotentials \( V_1 \) and \( V_2 \) increasing in the late universe (that is as \( \phi > M_p \)). If \( V_2^2/V_2 \) approaches zero as \( \phi \to \infty \), then the TMT effective potential (23) asymptotically approaches zero at the late universe. One can adjust degrees of growth of \( V_1 \) and \( V_2 \) in such a way that the TMT effective potential \( U(\phi) \) will have a desirable flat shape as \( \phi \to \infty \). Unbounded growth of \( V_2 \) as \( \phi \to \infty \) allows adding to \( V_2 \) any constant \( V_2^0 \) without altering \( U(\phi) \) for \( \phi \) large enough (remind that appearance of an additive constant in \( V_1 \) does not affect equations of motion at all). This is actually what we have seen in Sec. III : model with \( U(\phi) \) of the negative power low form for \( \phi \) large enough (Sec. III.A) and model with \( U(\phi) \) of the exponential form for \( \phi \) large enough (Sec. III.B). If appearance of the appropriate term \( \int V_2^0 \sqrt{-g} d^4 x \) in the action (3)-(5) is a result of quantum vacuum fluctuations then we can conclude that in the framework of the described approach to constructing a quintessence model of the late universe, TMT solves the cosmological constant problem.

However, the impression that the described technical details of the approach to the resolution of the cosmological constant problem in TMT settles a question, is premature. One should remind that the last statement about resolution of the cosmological constant problem implies validity of one more basic conjecture formulated in Introduction (after Eq. (6)) and used in all models of the present paper: Lagrangians \( L_1 \) and \( L_2 \) in the original action (3)-(5) do not depend on the measure \( \Phi \) degrees of freedom. In cases when this conjecture is invalid, the cosmological constant problem in TMT can turn into a very nontrivial issue. In fact, till the fundamental theory remains unknown, one can not be sure that the postulated general structure of TMT survives after quantum corrections are taken into account. If it will turns out that the quantum effective action corresponding to the original theory (3)-(5), contains the term \( - \int \Phi \Lambda_{eff} d^4 x \), then in the Einstein frame the latter will generate the real cosmological constant \( \Lambda_{eff} \). This possibility was studied in Ref. [25] (see Sec. VI therein) where a way to prevent the appearance of such a danger term was also discussed. The idea, briefly, is the following. If instead of the antisymmetric tensor field \( A_{\mu \nu \lambda} \), the measure \( \Phi \) is defined by means of four scalar measure fields \( \varphi_a, (a = 1, 2, 3, 4) \),

\[
\Phi \equiv \varepsilon_{a_1 a_2 a_3 a_4} \varepsilon^{\mu \nu \lambda \sigma} (\partial_\mu \varphi_{a_1})(\partial_\nu \varphi_{a_2})(\partial_\lambda \varphi_{a_3})(\partial_\sigma \varphi_{a_4}),
\]

then the action (3)-(5) with \( \varphi_a \) - independent \( L_1 \) and \( L_2 \), is invariant, up to an integral of a total divergence, under transformations \( \varphi_a \to \varphi_a + f_a(L_1) \) where \( f_a(L_1) \) are arbitrary differentiable functions of \( L_1 \). An appearance of the danger term \( - \int \Phi \Lambda_{eff} d^4 x \) in the action would break this local symmetry. Thus, this additional, local symmetry can prevent a generation of the real cosmological constant by quantum corrections to TMT if no anomaly appears.

\[^{18}\text{Taking into account our definition of } V_2(\phi), \text{ Eq. (26), one should notice that the positive } V_2^0 \text{ corresponds to a negative cosmological constant } \Lambda = - V_2^0 \text{ in GR if the term } \int V_2(\phi) \sqrt{-g} d^4 x \text{ would appear in the GR action. For constructing models 1 - 3 of Sec. IV.C, the positive definiteness of } V_2(\phi) \text{ (and therefore the condition } V_2^0 > 0 \text{) was one of the basic assumptions.}\]
b. Resolution of the flatness problem of the quintessence potential. The mechanism for the resolution of the flatness problem of the quintessence potential in TMT (question number 2 of Introduction) is actually the same as the one used for the resolution of the cosmological constant problem. Since the TMT effective potential \( U(\phi) \) takes a quintessence form as \( \phi \to \infty \) (either of an inverse power low form, Sec. III.A or of an exponential form, Sec. III.B) due to the unbounded growth of the leading terms of the pre-potentials \( V_1 \) and \( V_2 \), appearance of any subleading terms (including terms generated by quantum corrections) in \( V_1 \) and \( V_2 \) can not alter the shape of \( U(\phi) \) as \( \phi \) is large enough. There is no any need for coupling constants and mass parameters of the subleading terms to be very small. This is in fact the TMT answer the question raised by Kolda and Lyth [11].

As it was shown by Guendelman [27], [28] the role of the global continuous symmetries \( \phi \to \phi + const \) in TMT belongs to transformations (36)-(38) in the absence of fermions or (68) in the presence of fermions: in terms of the dynamical variables used in the Einstein frame, these transformations are reduced just to shifts of \( \phi \) parametrized as in Eq. (38). In models of Ref. [27], where the exponential form for the pre-potentials (33) with \( \alpha = \beta \) has been used, the global symmetry (36)-(38) is spontaneously broken. And although this symmetry is restored as \( \phi \to \infty \), it is impossible in the framework of such a model to realize a quintessence scenario at \( \phi > M_p \).

We have seen in the present paper that if a small explicit violation of the global continuous symmetries (36)-(38) is present in the TMT original action (26) with the exponential form of the pre-potentials (33), then the TMT effective potential \( U(\phi) \), Eq. (34), for \( \beta \phi \gg M_p \) can be a suitable candidate for a quintessence model with the effective potential (40). The smallness of the explicit symmetry breaking is formulated as a smallness of the dimensionless parameter \((\beta - \alpha)/\beta\), see Eq. (41).

In the absence of a knowledge about the structure of the fundamental theory and without any information about a mechanism leading to an explicit violation of the global continuous symmetry (36)-(38), the quantity \((\beta - \alpha)/\beta\) is the only small parameter that can be used in attempts to modify the action with simple exponential form of the pre-potentials (33), with the aim to give rise to quintessential inflation type models. This can be done by adding terms that disappear as \((\beta - \alpha)/\beta\) tends to zero. This means that coupling constants in such additional terms have to be proportional to some positive power of this small parameter.

The second basic idea is that in the limit \((\beta - \alpha)/\beta \to 0\) (which leads us to the fundamental theory) the only mass parameter of the theory is the Planck mass \( M_p \). This means that the dimensional coupling constants of the symmetry breaking terms have to be powers of the mass parameters \( m \) of the form \( m = [(\beta - \alpha)/\beta]^n M_p, \quad n > 0 \).

In the probe models studied in Sec. IV, we have chosen just for illustration \( \beta = 7, \alpha = 6 \) and hence \((\beta - \alpha)/\beta = 1/7\). Proceeding in the described above way, we reveal a remarkable feature of TMT: it is possible to achieve quite satisfactory quintessential inflation type models (see models 1-3 of Sec. IV) where for parameters adjusting it is enough to use only mass parameters of few orders less than \( M_p \). We interpret this fact as the absence of a need of a fine tuning.

Besides of the generation of the well-defined inflationary and quintessential regions of the TMT effective potential \( U(\phi) \), one more remarkable result consists in the fact that the post-inflationary region of \( U(\phi) \) has the exponential form \( \propto e^{\alpha \phi/M_p} \) with variable \( \alpha \), Eq. (52). This allows to single out a region of \( U(\phi) \), where a familiar approach [6] to a resolution of the problem known as constraint from cosmological nucleosynthesis is realized without any additional assumption.

d. Resolution of the problem of a possible direct coupling of the inflaton field to usual matter. As to the question number 5 of Introduction, the answer is quite clear: if the terms of the form \( f_i \propto \mathcal{E}_i \), described direct couplings of the inflaton field to the usual matter (see Ref. [20]), break the global continuous symmetry (68) they could appear in the original TMT action only with small coefficients \( f_i \propto [(\beta - \alpha)/\beta]^n, \quad n > 0 \).

A direct coupling of the inflaton field to a fermionic matter is of the special interest. In the modified models we have studied in Sec. VI.K, such a coupling enters to the original action in the form of two Yukawa coupling type terms, Eq. (69) and (112). The unbounded increase of \( V_1 \) and \( V_2 \) in the late universe works again in the desirable direction: the contributions of the Yukawa coupling type terms to the constraint (85) are negligible compared to \( V_1 \) and \( V_2 \). As we have seen in detail in Secs. VI.I, VI.J and finally in the most successful, modified model of Sec. VI.K, the Yukawa coupling type interactions can provide the presence of the direct coupling of fermionic matter to inflaton without any observable effect at the late universe (including constancy of the fermion mass at the late universe).

It is worthwhile to notice here that the form of the Yukawa coupling type interactions (69) and (124) might be generalized without altering the results obtained for the late universe. In fact, if for example one modifies these
model of Sec. VI contributes to the constraint, Eq. (85), and hence 1 field $\phi$ turns out that only the second class of the cosmological scenarios (quintessential inflation scenarios belong just to this question of the choice between two large classes of the cosmological scenarios, formulated in Sec. IV.B. It turns out that only the second class of the cosmological scenarios (quintessential inflation scenarios belong just to this class) admits a satisfactory definition of the TMT gravitational background where the quantization of the inflaton field $\phi$ is a standard procedure.

The second TMT problem consists in the quantization of usual matter fields. In particular, fermionic field $\Psi$ in the model of Sec. VI contributes to the constraint, Eq. (85), and hence $1/\chi$ obtained by solving (85), will depend on $\nabla\Psi$. In such a case, equations of motion in the Einstein frame, (88), (90) and (91) would become very nonlinear. In Ref. [25], we have tried to avoid this sort of problems by starting from the original action that was very non-linear in $\nabla\Psi$.

In the present paper, where the inclusion of the usual matter is studied in the context of the models of Sec. IV.C, intended to describe quintessential-inflation scenario without fine tuning, the problem of a non-linearity in matter fields does not appear. The reason is just a way we use to solve the cosmological constant and other fine tuning problems: the parameters of prepotentials $V_1(\phi), V_2(\phi)$ and the integration constant $M^4$ are chosen such that the matter fields contributions to the constraint (85) are negligible compared to $V_1(\phi), V_2(\phi)$ and $M^4$. Then for $1/\chi$ we obtain the expression described by Eq. (95), the same as in the absence of the usual matter. As a result of this, in the Einstein frame the usual matter fields equations in the background have canonical form and their quantization becomes a standard procedure.

f. SSB without generation of the cosmological constant. Reverting to the cosmological constant problem, it is worthwhile to notice in the conclusion that if the scalar (Higgs) field $\varphi$ obtains a non-zero VEV, Eq. (78), the appearance of a constant part in $P(\varphi)$ leads just to a redefinition of $m_1^4$ (see Eq. (85)). It is very important that in models 1 - 3 of Sec. IV.C, $m_1^4$ has the order of $(10^{-2}M_p)^4$ or $(10^{-3}M_p)^4$ which is larger than the GUT scale. The correction we neglect in the l.h.s. of (85) when replace it by (95), becomes of the order of $Q(\tilde{\varphi})/m_1^4$ where $Q$ is a polynomial in $\varphi (|\varphi| \ll \nu < m_1)$ that satisfies the condition $Q(0) = 0$. One can claim therefore that the above redefinition of $m_1^4$ has no any essential influence on the physics. Thus if $|P(\nu)| < m_1^4$, SSB of a gauge symmetry does not affect the magnitude of the effective cosmological constant (in the late universe) imitated by the quintessence potential (50).

Another possibility consists of a supposition that the whole term $m_1^4 e^{\alpha \phi/M_p}$ in the pre-potential $V_1(\phi)$ is generated by SSB. In such a case the quintessence potential becomes

$$U(\phi) \approx \frac{|P(\nu)|^8}{M_p^4} e^{-2(\beta - \alpha) \phi/M_p}$$

(125)

This is the TMT mechanism which together with the shape of $U(\phi)$ in the inflationary region predicted by each of the models 1 - 3 of Sec. IV.C, provides a resolution of one of the most serious aspect of the cosmological constant problem [10]: the need of an enormous fine tuning of initial conditions in models with SSB in order to satisfy the dual requirement of ‘large $\Lambda$ in the past + small $\Lambda$ at present’.
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[14] F. Rosati, "Can the inflaton and the quintessence scalar be the same field?", hep-ph/0002090.


