High energy spectra and zenith angle distributions of the atmospheric muons are calculated for the depths of operation of the underwater neutrino telescopes. The results are compared with the data obtained in the Baikal NT and AMANDA muon experiments. The estimation is performed of the prompt muon contribution underwater with perturbative QCD based models.

I. INTRODUCTION

Considerable literature exists on estimating of the contribution to cosmic ray muon fluxes that originates from decays of charmed hadrons [1–11]. Current data on high energy atmospheric muon flux obtained with many surface and underground detectors are too conflicting to provide the means of discriminating among charm production models (see for the review Ref. [8]).

Both direct and indirect measurements of the atmospheric muon flux at sea level are limited to ∼ 70 TeV for the vertical and to ∼ 50 TeV for about the horizontal. Statistical reliability of these data is still insufficient to evaluate the prompt muon (PM) contribution to the high energy muon flux.

Available energies and accuracy of underground measurements are limited because of the limited detector size and uncertainties in the local rock density. Deep-water installations have the considerable advantages of large detector volume and of the homogeneous matter. So it is pertinent to discuss the potentiality of the large underwater neutrino detectors (AMANDA, Baikal) in the context of the PM flux study in future high energy muon experiments.

In this paper we present calculations on zenith angle dependence of the high energy underwater muon flux considering the PM fraction with the perturbative QCD model (hereafter pQCD) [9]. These pQCD calculations based on MSRD- and CTEQ3 parton distribution functions (PDF) include the next-to-leading order (NLO) corrections to the charm production cross sections.

The pQCD models differ in the renormalization and factorization scales and in the PDF sets being employed in the NLO calculations. A dependence on the PDF of the vertical sea-level prompt muon flux was studied in Ref. [11]. The muon spectra underwater obtained with pQCD models and other type of the charm production models, the quark-gluon string model (QGSM) and the recombination quark-parton one (RQPM), were partly discussed in Ref. [10] (see also [6,8]). Here we would like to accent in particular variations of expected underwater muon fluxes caused by distinctions between the PDF’s used. Besides, we make a comparison between the expected underwater muon flux and zenith angle distributions measured with the Baikal and the AMANDA neutrino telescopes.

II. SEA LEVEL MUON FLUXES

The atmospheric muon energy spectra and zenith angle distributions, both the conventional (π, K) muons and the RQPM contribution, have been computed with the nuclear cascade model [12,13,8]. The differential energy spectra (scaled by $E_{\mu}^3$) of the conventional muons at sea level are shown (solid) in Fig. 1 for the vertical and near horizontal direction together with data of the magnetic spectrometer [14] and indirect measure-

FIG. 1. Sea-level muon fluxes for the vertical and horizontal. The solid lines are for the conventional muons alone. Also are shown the conventional muons plus the prompt muon contribution estimated [9] with the pQCD-1 (dashed), pQCD-2 (dotted), with the model of Volkova et al. [5] (thin) for the vertical, and with RQPM (dot-dashed) for the vertical (lower) and near the horizontal (up).
results obtained with the RQPM (dotted and dashed) both for the vertical direction (lower) and near the horizontal (up). These results enable us to make out the scope of the prompt muon flux predictions.

As is seen from Fig. 1, at $E_\mu \gtrsim 10 \text{–} 20 \text{ TeV}$ none of the above models but the VFGS is consistent with the data of MSU [15] and Frejus [16]. Conversely, none of the charm production models under discussion contradicts the LVD data [17,20]. The VFGS differing in the extent of optimism from the others gives the greatest prompt muon flux, yet compatible with the LVD upper limit [20]. The “crossing energy” $E_\mu^c(\theta)$ (the energy around which the fluxes of conventional and prompt muons become equal) depends on the choice of the PDF set. The vertical crossing energy $E_\mu^c(0^\circ)$ is about $200 \text{ TeV}$ for the pQCD-1 model, that is close to the RQPM prediction ($E_\mu^c \sim 150 \text{ TeV}$). The vertical prompt muon flux predicted with the pQCD-2 model becomes dominant over the conventional one at the energies $E_\mu \gtrsim 500 \text{ TeV}$. Therefore, in order for the differences between the pQCD models to be experimentally found one need to measure up to the muon energies above $\sim 100 \text{–} 200 \text{ TeV}$.

III. MUON FLUXES UNDERWATER

The muon energy spectra and zenith angle distributions deep underwater are calculated with an analytical method [21] (see also Ref. [8]). By this method one can solve the problem of the muon transport through dense matter for an arbitrary ground-level muon spectrum and real energy dependence of differential cross sections for muon-matter interactions. The calculations of the prompt muon fluxes underwater at different zenith angles were performed with the parametrization of the sea level muon differential spectra (pQCD-1,2) taken from [9].

Omitting details we dwell on a factor that may be useful in correcting the underwater muon flux, provided it is crudely estimated with the continuous energy loss approximation (see Ref. [21]). This factor is the ratio $R_{d/c}$ of the integral muon flux $I_{\mu}^{d/c}(E_\mu, h, \theta)$ computed for the discrete (stochastic) muon energy loss, to the flux $I_{\mu}^{\text{cont}}(E_\mu, h, \theta)$ estimated with the continuous loss approximation. In Table I the ratio $R_{d/c}$ is given as a function of the water depth and zenith angle for muon energies above $10 \text{ GeV}$. As is seen the effect of discrete energy loss for the large depth is far from being small: $R_{d/c}$ is $\sim 2$ for the depth value of $\sim 10 \text{ km w. e.}$ The ratio is slightly affected by zenith-angular dependence in itself of the atmospheric muon flux. More precisely, the $R_{d/c}$ depends on the muon “spectral index” that varies weakly with zenith angle, and geometric factor of $\sec \theta$ defining the thickness of water layer $x = h \sec \theta$ that a muon gets over, plays more important role in the $R_{d/c}$ ($h$ indicates the vertical depth in km).

<table>
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FIG. 2. Vertical muon flux as a function of water depth. The lines correspond to the $\pi, K$-muons calculated with the muon residual energy above 1 GeV (solid) and above 20 GeV (dashed).
FIG. 3. Zenith angle distribution of the muon flux underwater measured by Baikal NT-36.

For water the ratio $R_{d/c}$ as a function of the slant depth $x$ can be approximated with accuracy better than $\sim 10\%$ as

\[
\begin{align*}
R_{d/c} &= 0.99 + 0.02x + 6.74 \cdot 10^{-4}x^3, \quad x = 1 \div 12 \text{ km}; \\
R_{d/c} &= 1.43 + 0.054 \exp[(x - 1.19)/3.64], \quad x = 12 \div 35 \text{ km}.
\end{align*}
\]

The effect of the discrete loss is enhanced as the muon energy increases. The energy dependence of the ratio $R_{d/c}$ is adequately illustrated by following: for the depth of 12 km w. e. $R_{d/c} \simeq 2.5$ at $E_\mu = 10$ GeV and $R_{d/c} \simeq 4.0$ at $E_\mu = 1$ TeV.

In Fig. 2 we present a comparison between the expected muon vertical depth–intensity relation in water and the data obtained in underwater experiments (see for review [8], [22]) including recent measurements in the AMANDA-B4 experiment [23]. The calculations are presented for the muon residual energy (threshold of the detection) $E_\mu \geq 1$ GeV (solid) and $E_\mu \geq 20$ GeV (dashed). This difference needs to consider especially for shallow depth.

Figs. 3, 4 show a comparison of the predicted muon zenith angle distribution (without considering the prompt muon contribution) with the measurements in the neutrino telescopes NT-36 [22] and AMANDA [23]. The line in Fig. 3 presents the calculation for the muon threshold energy $E_\mu = 10$ GeV at depth $h = 1.15$ km. Our calculation is in reasonable agreement with the measurements of the NT-36 at all but the angle range $80^\circ$–$84^\circ$. In Fig. 4 upper line relates to the flux at the depth $h = 1.60$ km w. e. calculated for the muon residual energy $E_\mu \geq 20$ GeV, the lower one relates to $h = 1.68$ km w. e. for the same energy threshold. The difference illustrates possible effect of an uncertainty in determining of

FIG. 4. Zenith angle distributions of the muon flux underwater measured with the AMANDA-B4.

FIG. 5. Fluxes of muons above 100 TeV at water depth $h = 1.15$, 2, 3, 4 km (from top to bottom) as a function of cosine of the zenith angle.
The average “trigger depth” [23] relating to the center of gravity in vertical coordinate of all hit optical modules in the AMANDA-B4 experiment. The computed angle distribution agrees fairly well with the AMANDA-B4 data including zenith angles $\theta > 70^\circ$.

The contributions of the $(\pi, K)$ and prompt muons underwater to zenith angle distribution at $E_\mu > 100$ TeV calculated for four values of depths (of 1.15 to 4 km) are shown in Fig. 5. Here we present the results obtained with the pQCD-1 (dashed) and the pQCD-2 (dotted). It is interesting to note that dashed line representing the pQCD-1 prompt muon contribution intersects twice the line of the conventional flux at $\theta = 7 \approx 15$ km: near the vertical and at $\theta \sim 75^\circ$. This can occur because of different zenith angle dependence of the conventional muon flux and the prompt muon one. And this means that at depth of 1.15 km the nearly doubled muon event rate (for $E_\mu > 100$ TeV) would be observed in the $0^\circ - 75^\circ$ range, instead of the rate expected due to conventional muons alone. There is no intersection of the pQCD-2 line at $h = 1.15$ km up to $\theta \sim 85^\circ$. The intersection point shifts to smaller zenith angles with increasing depth. For a depth of 2 km (nearly the AMANDA depth) it is possible to observe prompt muon flux expected with the pQCD-2 at not too large angles, near to $70^\circ$. It should be mentioned that the underwater prompt muon flux will be distorted in a large zenith angle region because the angle isotropy approximation considered for the predictions of the pQCD models, is valid only at $\theta \lesssim 70^\circ$ and $E_\mu \lesssim 10^3$ TeV.

The depth dependence of the muon flux underwater at zenith angle of $\sim 78^\circ$ (Fig. 6) indicates that in case of the pQCD-1 one can observe the doubling of the muon flux at the Baikal depth of 1.15 km for $E_\mu > 100$ TeV. At a depth $\sim 2$ km the same takes place even with the lesser prompt muon flux predicted with the pQCD-2 model.

Fig. 7 shows muon integral energy spectra at a depth $h = 1.15$ km (Baikal) and 2 km (AMANDA) and for $\cos \theta = 0.2 \ (\theta \approx 78.5^\circ)$. Also presented are the predictions of the prompt muon flux issued from the pQCD-1 (dashed) and pQCD-2 (dotted). The crossing energies $E_\mu^c(\theta)$ at the AMANDA depth are less than ones at the Baikal depth by factor of $\sim 3$. In particular, the pQCD-1 model gives $E_\mu^c(\theta \approx 78.5^\circ) \approx 30$ TeV at $h = 2$ km and $E_\mu^c(78.5^\circ) \approx 100$ TeV at depth of 1.15 km. The same quantity calculated with pQCD-2 is of 100 and 250 TeV respectively.

One can see (Fig. 7) that the AMANDA depth ($\sim 2$ km) gives, in a sense, the definite advantage in comparison with the Baikal one. Indeed, in the former case the assumed threshold energy is lesser, the muon flux difference between the pQCD-1 model and the pQCD-2 one is larger (up to two orders of magnitude), and the expected event rate remains approximately equal to the rate at the Baikal depth.

It should be pointed out that muon residual energies below $\sim 10$ TeV and zenith angle $\theta \lesssim 75^\circ$ would be available (see Ref. 10 for a discussion), in the above context, in future high-energy muon experiments with the NESTOR deep-sea detector [24] which is to be deployed at depth of about 4 km.
Energy spectra and zenith angle distributions of the atmospheric muons at high energies have been calculated for the depth from 1 to 4 km that correspond to depths of operation of large underwater neutrino telescopes. The estimation of the prompt muon contribution performed with the pQCD-1, 2 shows that the crossing energy $E_\mu^c$ above which the prompt muon flux becomes dominant over the conventional one, is within the range of $\sim 200$ to $\sim 500$ TeV at sea-level, depending on the choice of the partron distribution functions. For the flux underwater at zenith angle $\sim 78^\circ$ the pQCD-1 model leads to the value $E_\mu^c \simeq 30$ TeV ($h = 2$ km) and $E_\mu^c \simeq 100$ TeV ($h = 1.15$ km). The corresponding crossing energies for the pQCD-2 model are $E_\mu^c \simeq 100$ and $E_\mu^c \simeq 250$ TeV.

The absolute value of the muon flux underwater around $E_\mu^c$ depends visibly on the charm production model. This promising circumstance enables us, in principle, to set restrictions on the charm production cross section from measurements of zenith angle distributions of the muon flux at high energies. In particular, PDF sets under discussion, the MRSD- and the CTEQ3, differing in the small-$x$ behavior of the gluon and sea quark distribution functions, yield inclusive cross sections of charmed patricles produced in nucleon-air collisions and charm production cross sections which vary rapidly in absolute value with increasing energy. Above 100 TeV and for $\cos \theta = 0.2$ these differences result in the prompt muon flux due to the pQCD-1, exceeded the flux issued from the pQCD-2 by a factor of above 4 at $h = 1.15$ km or about 5 times at $h = 2$ km.

In conclusion we outline three probable ways for solving of the prompt muon problem in the underwater experiments. Firstly, one can measure zenith angle dependence of the muon flux in energy region of 50 – 100 TeV (see Fig. 5): the expected event rate with the Baikal NT-200 is about of 200 per year per steradian, supposing that the effective area of NT-200 is of $10^4$ m$^2$ for $E_\mu \geq 100$ TeV [25].

Second, the flux with muon energies $E_\mu \geq 100$ TeV measured as function of depth (say, in depth region about 0.8 – 1.2 km) at given zenith angle ($\sim 78^\circ$), could enable the charm production models to be discriminated (see Fig. 6) at the event rate level of about 200 per year per steradian.

At last, one can attempt extracting information on the prompt muon flux underwater from muon integral spectra being measured at given depth and at given zenith angle (Fig. 7). In this case the event rate is about 7 times as low (with the NT-200 capabilities) compared to above one. It should be remarked to the point that the AMANDA depth of $\sim 2$ km provides some advantage: the threshold energy is lesser, the muon flux difference between the pQCD-1 model prediction and the pQCD-2 one is larger and the expected event rate remains approximately equal to that at the Baikal depth.

We thank V. A. Naumov for useful discussions and comments, St. Hundertmark and Ch. Spiering for kindly providing the table data on muon zenith angle distribution and depth-intensity relation measured in the AMANDA-B4 experiment.

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