On Galaxy-Cluster Sizes and Temperatures

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1 August 2000

ABSTRACT
We show that the distribution of the sizes and temperatures of clusters can be used to constrain cosmological models. The size-temperature (ST) distribution predicted in a flat Gaussian cluster-abundance-normalized $\Omega_0 = 0.3$ model agrees well with the fairly tight ST relation observed. A larger power-spectrum amplitude $\sigma_8$ would give rise to a larger scatter about the ST relation as would a larger value of $\Omega_0$ and/or long non-Gaussian high-density tails in the probability density function. For Gaussian initial conditions, the ST distribution suggests a constraint $\sigma_8 \Omega_0^{0.26} \sim 0.76$. The ST relation is expected to get tighter at high redshifts. In the process, we derive a simple formula for the halo formation-redshift distribution for non-Gaussian models.

We also suggest that the discrepancy between the naive zero-redshift ST relation and that observed may be due, at least in part, to the fact that lower-mass clusters form over a wider range of redshifts.

Key words: cosmology: theory - large scale structures - clusters of galaxies

1 INTRODUCTION

Galaxy clusters are now being widely used as probes of cosmological and structure-formation models. For example, the abundance of galaxy clusters has been used to constrain the amplitude $\sigma_8$ of the power spectrum and the nonrelativistic-matter density $\Omega_0$ in models with an initially Gaussian distribution of density perturbations (Evrard 1989; Henry & Arnaud 1991; Bahcall & Cen 1992; Bahcall & Cen 1993; Lilje 1992; Oukbir & Blanchard 1992; White, Efstathiou & Frenk 1993; Viana & Liddle 1996; Eke, Cole & Frenk 1996; Viana & Liddle 1999), as well as in models with long non-Gaussian tails (Robinson, Gawiser & Silk 2000; hereafter RGS).

In this paper, we show that the scatter in galaxy cluster scaling relations can be used to constrain cosmological and structure formation models. Specifically, we focus on the relatively small scatter of the relation between X-ray isophotal size and emission-weighted intracluster-medium mean temperature $T_X$ demonstrated in Mohr & Evrard (1997; hereafter ME97). We illustrate how this scatter should depend on $\sigma_8$ and $\Omega_0$, and how it is affected by the introduction of a non-Gaussian distribution of perturbations with a long tail of high-density peaks. Our work on the size-temperature (ST) relation follows prior analytic work by Kitayama & Suto (1996) (although they focussed primarily on other cluster properties) and employs the framework for relating the ST relation to the underlying dark matter properties as discussed in Mohr et al. (2000; hereafter M00).

The small scatter is heuristically expected if clusters form at rare high-density peaks in a Gaussian primordial distribution. Clusters that form earlier should be denser when they are first virialized and so they should have smaller radii for a given mass, or similarly, smaller radii for a given temperature. In this way, any dispersion in the formation redshifts for clusters of a given mass should yield a spread in the ST relation. If clusters come from rare Gaussian peaks, then the spread in formation redshifts should be small; given the rapidly dying Gaussian tails, it is unlikely that any cluster of a given mass observed today was formed at a redshift much earlier than the others. However, if the distribution had long non-Gaussian tails (as would be required to significantly boost the cluster abundance; see Fig. 1) or if clusters formed from peaks that were not quite so rare (e.g., $> 2\sigma$ rather than $> 3\sigma$ peaks), then clusters of a given mass observed today should have had a much broader distribution of formation redshifts and thus a much broader distribution of sizes (for a given mass or temperature).

We quantify these arguments using a spherical-top-hat-collapse model to relate the virial radius and temperature of a cluster to its mass and formation redshift. We use the formation-redshift distribution for Gaussian perturbations from Sasaki (1994), and we generalize it for an arbitrary initial density distribution. We use a Monte Carlo approach to simulate the ST relation for a variety of parameters, and illustrate in particular how it depends on $\sigma_8$, $\Omega_0$, and $G$, the non-Gaussian multiplicative excess of $> 3\sigma$ peaks introduced by RGS. Our main results are (a) the predicted scatter in the ST relation for Gaussian initial conditions and favored cosmological parameters is found to be fairly consistent with that observed; (b) $G \gtrsim 5$ greatly overpredicts the scatter; (c) the scatter for the non-Gaussian initial conditions required to make the clus-
The solid curve shows a Gaussian distribution $P(y)$ with unit variance, while the broken curve shows a non-Gaussian distribution with the same variance but 10 times as many peaks with $y > 3$. This illustrates (a) how the cluster abundance can be dramatically enhanced with long non-Gaussian tails (since clusters form from rare peaks); and (b) that the dispersion of $y$ for $y > 3$ is much larger for the non-Gaussian distribution than it is for the Gaussian distribution, and this will lead to a larger scatter in the formation redshifts and sizes of clusters of a given mass.

2 INGREDIENTS

2.1 Spherical-Collapse Model

We use the relations of Kitayama & Suto (1996) to relate the cluster virial radius $R_\text{vir}$ and virial temperature $T$ to the mass $M$ and formation redshift $z_f$ (defined to be the redshift at which the cluster collapses). Fig. 2 shows how this model assigns masses and formation redshifts to clusters of given temperatures and sizes assuming that $R_\text{vir} \propto R_\text{vir}$. It is possible to connect more rigorously these cluster dark-matter properties with the observable intracluster medium (ICM) properties in a manner similar to that outlined in M00. Specifically, we assume that $T_X$ is the virial temperature (e.g. Evrard, Metzler & Navarro 1996; Frenk et al. 2000; Bower et al. 2000). We transform from the virial radius to the X-ray isophotal size $R$ using $R \propto R_\text{vir}^{4/3} f_{\text{ICM}}^{2/3}$, where $f_{\text{ICM}}$ is the ICM mass fraction (M00 equations (8) and (10)). The dependence on $f_{\text{ICM}}$ should be included because variations in $f_{\text{ICM}}$ with mass are observed (e.g. Mohr, Mathiesen & Evrard 1999; David, Jones & Forman 1995) and could alter the slope of the ST relation. In the following analysis, we assume $f_{\text{ICM}} \propto T_X^{34}$; however the results of Figs. 4 and 5 are no more than very weakly dependent on the $f_{\text{ICM}}$ functional form.

We normalize the simulated ST relation to the observations by fixing the constant of proportionality so that the most recently collapsed cluster (within the framework of this model) in the observed sample forms at $z_f = 0$.

2.2 Distribution of Halo Masses

Numerical simulations tell us that the Press-Schechter (PS) approach (Press & Schechter 1974) provides a reasonable approximation for the abundance of cluster size halos of a given mass at any given epoch for Gaussian initial conditions (e.g. Lacey & Cole 1994, Gross et al. 1998, Lee & Shandarin 1999), and for a few non-Gaussian initial conditions that have been explored with simulations (Robinson & Baker 1999). The PS approach predicts the number per comoving volume of halos with masses between $M$ and $M + dM$ at redshift $z$ to be (RGS),

$$\frac{dn}{dM} dM = \frac{f \rho_0}{M} P(y(M, z)) \frac{\partial y(M, z)}{\partial M} dM,$$

where $\rho_0$ is the background density, $y = \delta_c(z)/\sigma_M$, $P(y)$ is the primordial probability distribution function normalized to unit variance, $y = \delta_c/\sigma_M$, and $\delta_c(z)$ is the critical overdensity for collapse (see Kitayama & Suto 1996 for accurate analytic fits). Here, $\sigma_M$ is the root-variance of spheres that enclose an average mass $M$, and $f = \int_0^\infty P(y) dy$.

2.3 Distribution of Formation Redshifts

The objects of mass $M$ observed at some given redshift $z_o$ underwent collapse at a variety of formation redshifts $z_f > z_o$. Sasaki (1994) has shown how the PS formalism leads to an expression for the formation-redshift distribution under the assumption of Gaussian initial conditions and that the merger rate has no characteristic mass scale. His derivation can be generalized in a straightforward fashion to arbitrary $P(y)$. Doing so, we find the distribution of formation redshifts $z_f$ for halos of mass $M$ observed at redshift $z_o$ to be,

$$\frac{dn}{dz_f} = P'(y(M, z_f)) \left[ \frac{\partial y(M, z_f)}{\partial z_f} \right] \left[ P(y(M, z_o)) \right]^{-1}.$$
where \( P'(y) \equiv dP/dy \). Lacey & Cole (1993,1994) have presented an alternative, but somewhat more complicated, formation-redshift distribution that improves upon Sasaki’s assumption of self-similar merging. We will leave the implementation of this alternative distribution and a discussion of the formalism introduced by Percival et al. (2000), to future work, but note that our preliminary investigations, as well as previous results (Viana & Liddle 1996; Buchalter al. 1993; Arnaud & Evrard 1999), so the flux limit is essentially a variety of models in the physics of dark matter.

2.4 Preliminary Estimates

It is straightforward to roughly estimate the effects of non-Gaussian tails on the ST-relation scatter. For a rapidly dying distribution \( P(y) \), the controlling factor in \( dn/dz \) will be \( P'(y) \). For a Gaussian \( P(y) \), the root-variance of \( y \) is 0.282 for the distribution \( P'(y) \) for values of \( y > 3 \), and the mean value of \( y \) is 3.30. For an EdS model, \( y = 1.69(1 + z_f)/\sigma_M \), and \( (1 + z)^{-1} \propto R_{vir} \). Thus, \( \sigma_R/R = (4/3)(\sigma_{R,vir}/R_{vir}) \approx (4/3)(\sigma_y/y) = 0.113 \) for a Gaussian distribution, in surprisingly good agreement with the estimate of the intrinsic scatter of 10% in the ST relation (ME97). For the RGS distribution with \( G = 10 \), the root-variance is 0.896 and the mean value of \( y \) is 3.87 leading to \( \sigma_{R,vir}/R_{vir} \approx 0.31, \) more than twice the observed scatter. Below we will quantify this far more precisely.

3 RESULTS

For any given \( \Omega_0, \sigma_8 \), and \( G \), we perform a Monte Carlo realization of 400 clusters with the mass and formation-redshift distributions given above. We then assign to each of these clusters a size and temperature as outlined in Section 2.1. The ME97 sample to which we compare our calculations is a flux-limited sample. Within this sample, the probability of finding a cluster of luminosity \( L_X \) goes as \( L_X^{-5} \), and \( L_X \) is observed to go as roughly \( T^{2.5} \) to \( T^{3} \) (David et al. 1993; Arnaud & Evrard 1999), so the flux limit is essentially a virial-temperature weighting of \( T^{3.75} \) to \( T^{4.5} \). We thus subject our simulated population of clusters to a \( T^{4.5} \) weighting; our results are not significantly altered for the steeper weighting \( T^{4.5} \).

Fig. 3(a) shows the results of our Monte Carlo for a flat \( \Omega_0 = 0.3 \) model (LCDM) with the value \( \sigma_8 = 0.99 \) inferred from the cluster abundance (Viana & Liddle 1999) and a Gaussian distribution. The data points from ME97 are overlaid. We used a Hubble parameter \( h = 0.65 \), but the results are essentially unchanged for different plausible values of \( h \). Fig. 3(b) illustrates that the scatter in the ST relation is increased if the power-spectrum normalization is higher. In this case, clusters are not quite as rare, and they form over a larger range of redshifts. Fig. 3(c) shows how the scatter is increased as the abundance of high-density peaks is increased. In this case, clusters observed today are also formed over a broader range of redshifts. At this point, we note the apparent similarity between the predictions of the ST distribution of the cluster-abundance-normalized Gaussian LCDM model and the data; the scatter about the ST relation would be broadened considerably with a higher \( \sigma_8 \) or a highly non-Gaussian model.

To make these arguments more quantitative as well as survey a larger range of parameters, we have simulated ST relations for a variety of models in the \( \sigma_8-G \) parameter space for both EdS and LCDM models and then used a 2D Kolmogorov-Smirnov (KS) test (Peacock 1983; Press et al. 1996) to compare these with the data. Fig. 4 shows the resulting contours of constant KS significance levels for both \( \Omega_0 = 0.3 \) and \( \Omega_0 = 1 \). The results suggest that the Gaussian cluster-abundance-normalized \( \sigma_8 = 0.99 \) LCDM model provides a good fit to the data.

We heuristically expect that the dependence of the ST scatter on cosmological parameters/models should be similar to that of the cluster abundance; if the peaks that give rise to clusters are rare, we expect little scatter and \textit{vice versa} if clusters are more common. The contours of fixed cluster abundance in Fig. 4 indicate that this is qualitatively correct. We obtain these curves by using a cluster abundance \( n(\geq 6.2 \text{keV}, z = 0.05) = 1.53 \times 10^{+0.16} \times 10^{-7} \text{Mpc}^{-3} h^3 \) (Viana & Liddle 1999) and integrating equation (1) up from the mass associated with a temperature 6.3 keV and a formation redshift \( z_f = 0 \). However, the detailed results also seem to indicate that if \( \Omega_0 \) is fixed, the ST distribution and cluster abundance can be used in tandem to break the degeneracy between \( G \) and \( \sigma_8 \). In fact, combining the two constraints already seems to rule out large deviations from Gaussianity.

Fig. 5 shows the regions of \( \sigma_8-\Omega_0 \) parameter space preferred by the ST relation, as well as the curve in this parameter space suggested by the cluster abundance. From these results, we can ap-
RGS were able to identify for an EdS model, a region in the \( \sigma \) parameter space near \( \sigma = 0.4 \) and \( G = 10 \) in which the predicted cluster abundance was found to agree with that observed. Fig. 6 shows that these parameter choices predict far too much scatter in the ST relation. Allowing for additional sources of scatter in this simulated ST relation would only increase the discrepancy between the model and the observations.

3.1 An Einstein-de-Sitter Universe?

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3.2 High and Intermediate Redshift Results

Clusters that exist at higher redshifts must form from even higher-density peaks than those today. Thus, in a Gaussian model, the scatter in their formation redshifts and thus in their sizes should be even smaller. This is illustrated in Fig. 7. The canonical-model predictions shown in Fig. 7(a) for \( z \approx 0.3 \) seem to be in relatively good agreement with the cluster sample observed so far. Fig. 7(b) shows that the scatter in the ST relation for the canonical model should be very small. Even though the sample of such high-redshift clusters is expected to be small, the predicted scatter is so small that measurement of the sizes of only a handful of clusters could put strong constraints on different sources of scatter (e.g., non-Gaussianity, measurement uncertainties, mergers, galaxy feedback, etc.).

3.3 A size-temperature anomaly?

The ST relation of the low-redshift X-ray flux limited cluster sample has a slope of \( m \sim 1 \), which is considerably steeper than the \( m = 2/3 \) slope expected in a model where all clusters are assumed to have formed at the redshift of observation (ME97, M00). ME97 suggest that a possible explanation for this steeper than expected scaling relation is provided by galaxy feedback. Fig. 3 illustrates that the discrepancy may be due only, or at least in part, to the fact that lower-mass clusters form over a broader range of redshifts, and thus will in general have smaller sizes than they would if they all formed very recently. Visual inspection of Fig. 3 suggests that this is a plausible explanation, especially when the small-number statistics of the observational sample are taken into account. Moreover, the relatively strong dependence of the ST scatter on \( \sigma \) indicates that better agreement than shown in Fig. 3 could be obtained with a slightly different value of \( \sigma \) and/or \( \Omega_0 \) (cf., Fig. 5). The apparent disagreement with ME97’s feedback-free numerical simulations, which show an ST scaling close to the naive scaling (but still steeper in 3 of the 4 cosmologies tested), may have been due to this \( \sigma \) and \( \Omega_0 \) dependence and/or the relatively small-number statistics of their simulations sample. Thus, the apparent deviation of the ST relation slope from the \( m = 2/3 \) expectation is not quite so anomalous.
Figure 7. The ST relation for the LCDM model with $\sigma_8 = 0.99$ and Gaussian perturbations for (a) clusters at $z = 0.3$ and (b) $z = 1$. This illustrates how the scatter should decrease with redshift. The data from Mohr et al. (2000) (that have median redshift 0.3, but cover the range $0.19 < z < 0.54$) are shown in (a).

4 DISCUSSION

We have calculated the ST distribution of clusters with a simple analytic model and focussed in particular on the dependence on the power-spectrum amplitude ($\sigma_8$) and the degree of non-Gaussianity ($G$). We find a fairly sensitive dependence of the ST relation scatter to these two parameters. Thus, the tightness of the ST relation can be used to place valuable constraints on these parameters, as well as on other cosmological parameters. The canonical cluster-abundance-normalized $\Omega_0 = 0.3$ model predicts an ST relation consistent with that observed, but a $\sigma_8$ much larger or smaller would be inconsistent as would a non-Gaussian model that predicts a significant excess of $> 3 \sigma$ high-density peaks. The constraints to $\sigma_8$, $G$, and $\Omega_0$ that arise from the ST distribution should be qualitatively similar to those from cluster abundances, but our preliminary calculations suggest that they may be sufficiently different to provide complementary constraints. The ST relation should become increasingly tight at larger redshifts. Our results also suggest that the discrepancy between the naive $z = 0$ ST relation and the observed ST relation may be due, at least in part, to the fact that lower-mass clusters observed today have formed over a larger range of redshifts than higher mass clusters.

The fact that lower-mass clusters tend to form over a broader redshift range than higher-mass clusters will also tend to steepen the $M_{\text{vir}} - T$ relation beyond the self-similar expectation of $m = 2/3$. Numerical simulations of structure formation within models with non-Gaussian initial conditions or low-$\Omega_0$ open models ought to exhibit this effect. The OCDM256 portion of Fig 4. in Bryan & Norman (1998) indicates that low-mass clusters fall systematically below the best fit $M_{\text{vir}} - T$ relation, consistent with our expectation. It should be emphasized that, in this particular study, the low-mass systems are composed of far fewer particles than the high-mass systems, providing another plausible explanation for structural differences. Further work to investigate departures from self similarity in the cluster population which naturally arise from the spread in formation epochs is clearly required.

Since the overdensity-peak amplitude at which a cluster can form increases at higher redshift, the redshift evolution of the cluster abundance depends on the shape of the primordial density distribution function at high peaks just as the ST scatter does. Thus, in principle, it should be possible to reconstruct the cluster-abundance evolution from the scatter in the ST relation for local clusters.

Although we have used cluster sizes inferred from X-rays to compare with theoretical calculations, the same could be done for the sizes of clusters measured via the Sunyaev-Zeldovich effect, either with or without redshift information (Kamionkowski et al. 2000). Of course, there will invariably be some cluster-formation physics that our current analysis has left out, and numerical simulations may have an advantage in this regard. Note that the only source of scatter in our simulated ST relations is the range of formation epoch, whereas other sources of stochasticity (e.g., measurement uncertainties, mergers, galaxy feedback, etc.) might increase the scatter. However, with our analytic approach, we are able to rule out models that overpredict the scatter, we can sift far more rapidly through a variety of cosmological models and parameters, study the dependence of the ST distributions on these models and parameters, and gain some intuitive feel for how the results arise.

By doing so, we hope to have established that cluster sizes can provide a valuable new probe of cosmological models.

ACKNOWLEDGMENTS

We thank P. Goldreich for asking questions that stimulated this investigation. This research was supported in part by the NSF under grant no. PHY94-07194 to the Institute for Theoretical Physics (Santa Barbara), where part of this research was completed. LV was supported by a TMR grant and thanks Caltech for hospitality.

MK was supported in part by NSF AST-0096023, NASA NAG5-8506, and DoE DE-FG03-92-ER40701. JJJM is supported by Chernandez Fellowship grant PFR-1003, awarded through the Chandra Science Center. The Chandra Science Center is operated by the Smithsonian Astrophysical Observatory for NASA under contract NAS8-39073. AJB was supported by a PPARC studentship.

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