Antonio Alfonso-Faus  
E.U.I.T. Aeronáutica  
Plaza Cardenal Cisneros s/n Madrid 28040 SPAIN  
E-mail:aalfonso@euita.upm.es

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The fine structure constant \( \alpha \) includes the speed of light as given by  
\[
\alpha = \frac{e^2}{4 \pi \varepsilon_0 c \hbar}.  
\]

It is shown here that, following a \( \text{TH}_{\varepsilon\mu} \) formalism, interpreting the permittivity \( \varepsilon_0 \) and permeability \( \mu_0 \) of free space under Lorentz local and position invariance, this is not the case. The result is a new expression  
\[
\alpha = \frac{e^2}{4 \pi \hbar}  
\]
in a new system of units for the charge that preserves local and position invariance. Hence, the speed of light does not explicitly enter in the constitution of the fine structure constant. The new expressions for the Maxwell’s equations are derived and some cosmological implications discussed.

La constant de la structure fine insère aussi la vitesse de la lumière en accord avec la formule  
\[
\alpha = \frac{e^2}{4 \pi \varepsilon_0 c \hbar}.  
\]

On démontre avec ce travail que, suivant le formulisme \( \text{TH}_{\varepsilon\mu} \) et interprétant la permittivité \( \varepsilon_0 \) et la perméabilité \( \mu_0 \) du vide selon l’invariant de Lorentz local et de position, cette formule n’est pas l’adéquate. La nouvelle expression es  
\[
\alpha = \frac{e^2}{4 \pi \hbar}.  
\]
dans un système d’unités neuf pour la charge électrique, système qui préserve l’invariant local et de position. Par conséquent, la vitesse de la lumière ne rentre pas dans la constitution du constant de la structure fine. On déduit les nouvelles expressions des équations de Maxwell et on débat certaines implications cosmologiques.

**Key words:** fine structure constant, speed of light, Lorentz local and position invariance, equivalence principle, Maxwell equations, cosmology.

**I. INTRODUCTION**

The correct expression for the permittivity of free space \( \varepsilon_0 \), and consequently for the permeability of free space \( \mu_0 \), is of fundamental importance when dealing with laboratory and cosmological work. By correct we mean the forms that preserve both Local Lorentz Invariance (LLI) and Local Position Invariance (LPI). We define both following Will (see [1]).

By LLI we mean the aspect of the Einstein’s Equivalence Principle that postulates the same predictions for identical local non-gravitational test experiments, performed in two freely falling frames located at the same event in space-time, but moving relative to each other.

By LPI we mean the independence from the space-time location of the frame, when observing the results of local non-gravitational test experiments.

The \( \text{TH}_{\varepsilon\mu} \) formalism, devised by Lightman and Lee ([2]), can be used to implement the Einstein’s Equivalence Principle, as presented by Will ([1]). This author proves that a necessary and sufficient condition for both Local Lorentz and Position Invariance to be valid is given by

\[
\varepsilon_0 = \mu_0 = \left( H_0 / T_0 \right)^{1/2}  
\]

for all events.

In our work here we use this relation to obtain Local Lorentz and Position Invariance. Since the product \( \varepsilon_0 \mu_0 \) is equal to \( c^{-2} \), then the relation (1.1) implies

\[
\varepsilon_0 = \mu_0 = \frac{1}{c}  
\]

It is extremely important that we retain \( c \) different from unity in all equations. By doing so we keep open to a possible variation of \( c \) with cosmological time as a result of the expansion of the Universe, e.g. Alfonso-Faus ([3]), or any other cause. We are not dealing here with the constancy or time variation of \( c \). But to be able to find the correct basic physical relations, in particular as related to the fine structure constant, we must keep \( c \) in the expressions.

We know that in the electrostatic system of units (e.g. Jackson ([4])) one takes \( \varepsilon_0 = 1 \), and \( \mu_0 = c^{-2} \). In the electromagnetic system of units \( \varepsilon_0 = c^{-2} \) and \( \mu_0 = 1 \). Both are wrong from an invariance point of view, and the same happens with the rationalized mks system. In the gaussian and Heaviside-Lorentz systems one takes \( \varepsilon_0 = \mu_0 = 1 \) which corresponds to take \( c = 1 \). Here we are presenting a system of electromagnetic units consistent with local and position invariance even in the case of a time varying \( c \). By doing so we keep the physical insight of the presence of \( c \) in electromagnetic properties.
One of the results is that the speed of light does not enter explicitly in the intrinsic constitution of the fine structure constant.

On the other hand, we know that the constancy of the ratio of magnetic to electric forces, as evidenced in the spectra from different distant galaxies, implies that the fine structure constant must be a universal constant. Once we prove that the speed of light does not enter in its constitution, we get as a universal constant the ratio $e^2/\hbar$.

II. THE INVARIANCE OF THE MAXWELL EQUATIONS.

We follow Jackson ([4]) for discussing units and dimensions of electromagnetism. We choose length, mass, and time as independent, basic units. The dimension of the ratio of charge and current will be that of time. Then, the continuity equation for charge and current densities is

\[ \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad (2.1) \]

We consider electromagnetic phenomena in free space, apart from the presence of charges and currents. We have Coulomb’s law

\[ F_1 = k_1 \frac{qq'}{r^2} \quad (2.2) \]

and define the electric field as

\[ E = k_1 \frac{q}{r^2} \quad (2.3) \]

where $k_1$ is a proportionality constant except for possible cosmological time variations. We will call these “constants” by the name of factors. Ampere’s law giving the force per unit length between two infinitely long parallel wires, separated by a distance $d$ and with currents $I$ and $I'$ is

\[ \frac{dF_2}{dt} = 2k_2 \frac{II'}{d} \quad (2.4) \]

where $k_2$ is a factor as in (2.3).

By comparison of the magnitudes of the two mechanical forces (2.2) and (2.4) one has in free space

\[ \frac{k_1}{k_2} = c^2 \quad (2.5) \]

We define the magnetic induction $B$ from Ampere’s law for a long straight wire carrying a current $I$,

\[ B = 2k_2 \frac{\beta I}{d} \quad (2.6) \]

where $\beta$ is a proportionality factor, which has dimensions. Combining (2.1), (2.3), (2.5) and (2.6) one has the dimensions of $E/B$ to be $1/(t\beta)$. As we will see $E$ and $B$ will have the same dimensions, so that $\beta$ has the dimensions of velocity.

Finally, Faraday’s law of induction gives

\[ \nabla \times \vec{E} + k_3 \frac{\partial \vec{B}}{\partial t} = 0 \quad (2.7) \]

On the basis of Galilean invariance one has $k_3 = 1/\beta$. But it can also be proved from the Maxwell’s equations using the fields already defined and taking into account that in the wave equation the velocity of propagation is the speed of light. Maxwell’s equations are then

\[ \nabla \cdot \vec{E} = 4\pi k_1 \rho \quad (2.8) \]

\[ \nabla \cdot \vec{E} + k_3 \frac{\partial \vec{B}}{\partial t} = 0 \]

\[ \nabla \times \vec{B} = 4\pi k_1 \beta \vec{J} + (k_2/k_1) \beta \frac{\partial \vec{E}}{\partial t} \]

\[ \nabla \cdot \vec{B} = 0 \]

where we have the relations between the proportionality factors:

\[ \frac{k_1}{k_2} = c^2 \quad (2.9) \]

\[ k_3 = 1 \]

We only have two factors that must be chosen to completely define the system. We will do it keeping the LLI and the LPI. Also, from the Lorentz force expressed in the gaussian or Heaviside-Lorentz Systems that have $\varepsilon_0$ and $\mu_0$ equal,

\[ \vec{F} = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}) \quad (2.10) \]

hence $E$ and $B$ have the same dimensions. For electromagnetic waves in free space $E = B$ and we keep this result, so that from (2.7) one has

\[ k_3 = c^{-1} \quad (2.11) \]

So far we have kept the same classical discussion as in Jackson ([4]). We are left with only one factor to be defined and this is the characteristic step of this work. From the LLI and the LPI one has the factor in Coulomb’s law:

\[ k_1 = c \quad (2.12) \]
and therefore
\[ k_2 = c^{-1} \] (2.13)

The laws of electromagnetism are then given in LLI and LPT form as follows:

Coulomb’s law:
\[ F_1 = e^{qq'} \] (2.14)

Electric field
\[ E = c \frac{q}{r^2} \] (2.15)

Ampere’s law
\[ \frac{dF_2}{dt} = \frac{2}{c} II' \] (2.16)

Magnetic induction
\[ B = \frac{2I}{d} \] (2.17)

and finally Maxwell’s equations are in this form:
\[ \nabla \cdot \vec{E} = 4\pi \varepsilon \rho \] (2.18)
\[ \nabla \cdot \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} = 0 \]
\[ \nabla \times \vec{B} = 4\pi \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \]
\[ \nabla \cdot \vec{B} = 0 \]

III. COSMOLOGICAL IMPLICATIONS

The fine structure constant \( \alpha \) must be a real constant as inferred from the observations of spectra from different distant sources through the ratio of magnetic spin-orbit interaction to the electric interaction. The possible variation reported by Webb et al. ( [5]) is very small for a cosmological time variation at a distance of about red shift one. They report a change of about one part in \( 10^6 \), while here we are considering variations of the same order as the value of the constant. The same consideration applies to the reported result of the electroweak measurement of the strengthening of the electromagnetic coupling (Levine et al. ( [6])): a value of 128.5 as compared with 137.0 represents a small variation in cosmological terms.

Let us assume that \( \alpha \) is a constant. We have proved that it only depends on \( \hbar \) and \( e \). Let us assume that \( \hbar \) and therefore \( e \) are also constants. In terms of the Rydberg constant \( R_\infty \) and the Bohr radius \( a_0 \) one has the following relation (see [7]):
\[ R_\infty \hbar c = 13.6056981 eV = m_e c^2 \alpha^2 \] (3.1)
\[ 2R_\infty \hbar c = \frac{e^2}{4\pi \varepsilon_0 a_0} = 27.2113961 eV \]

Here \( \hbar \) is \( 2\pi \hbar \). If we take this energy as constant then a time variation in \( c \) implies a time variation in the mass of the electron. Also, taking \( \hbar \) and \( e \) constant one has from (3.1)
\[ R_\infty c = \text{const.} \] (3.2)
\[ \varepsilon_0 a_0 = \text{const.} \] (3.3)

Since \( \varepsilon_0 = c^{-1} \) one has the result
\[ a_0 \propto c \] (3.3)

The conclusion is that a time variation in the speed of light, keeping \( \alpha \), \( \hbar \), and \( c \) constants imply that the masses and sizes of the quantum world vary also with time. In particular, if the speed of light decreases with time due to the expansion of the Universe (Alfonso-Faus ( [3])), the sizes of the quantum particles also decrease. Both effects would go together: the expansion of the Universe and the contraction of the quantum world at the same time.

IV. CONCLUSION

We have reviewed all basic electromagnetic relations with the choice \( \varepsilon_0 = \mu_0 = 1/c \) that preserves local Lorentz invariance and local position invariance. One immediate consequence is that there is a factor of \( c \) in Coulomb’s law multiplying the product of the two charges. In this new system of units the speed of light is explicit in the places required by the invariance condition imposed, which gives an insight in the intrinsic relations of physical properties. In particular, the fine structure constant given as \( \alpha = e^2/4\pi \varepsilon_0 c \hbar \) is now, in the new system, \( \alpha c^2/4\pi \varepsilon_0 \hbar \). Hence, the constancy of the fine structure constant, deduced by means of observations of spectra from different distant sources through the ratio of magnetic spin-orbit interaction to the electric interaction, is in fact the constancy of \( \alpha \). This is very important for theories that contemplate time-varying physical “constants” (e.g. Alfonso-Faus ( [3])). In particular, the exact constancy of the electric charge of the electron would imply the exact constancy of Planck’s constant. If there is no cosmological effect on \( e \), then there is no cosmological effect on \( \hbar \), regardless of any possible cosmological effect on \( c \). If there is a cosmological effect on \( c \) then there is a cosmological effect on the masses and sizes of quantum particles, keeping constant \( \hbar, e \) and the atomic energy levels.

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