High energy photon emission rate from matter created in Pb + Pb collisions at CERN SPS energies is evaluated. The evolution of matter from the initial state up to freeze-out has been treated within the framework of (3+1) dimensional hydrodynamic expansion. We observe that the photon spectra measured by the WA98 experiment are well reproduced with hard QCD photons and photons from a thermal source with initial temperature \( \sim 200 \) MeV. The effects of the spectral changes of hadrons with temperature on the photon emission rate and on the equation of state are studied. Photon yield for Au + Au collisions at RHIC energies is also estimated.

PACS: 25.75.+r;12.40.Yx;21.65.+f;13.85.Qk

I. INTRODUCTION

Ultra-Relativistic collisions of heavy nuclei have brought us within reach of creating and studying various aspects of bulk deconfined matter, known as quark-gluon plasma (QGP), which so far was believed to exist in the microsecond old universe or possibly in the cores of neutron or quark stars. We are at a very interesting situation in this area of research where the Super Proton Synchrotron (SPS) era has drawn to a close and the first results from the Relativistic Heavy Ion Collider (RHIC) have started to appear. Already from the results of the Pb run at the SPS quite a few of the signatures of QGP, e.g., \( J/\Psi \) suppression, strangeness enhancement etc., are reported to have “seen” unmistakable hints of the existence of QGP [1]. Electromagnetic probes, viz., photons and dileptons have long been recognized as the most direct probes of the collision [2–8]. It has also been emphasized that owing to the nature of their interaction they undergo minimal scatterings and are by far the best markers of the entire space-time evolution of the collision.

The single photon data, obtained from Pb-Pb collisions at CERN SPS reported by the WA98 Collaboration [9] have been the focus of considerable interest in recent times [10–12]. In this work we will analyze this data in a detailed and realistic framework using a reasonable set of parameters. Let us first identify the possible sources of “excess” photons above those coming from the decays of pseudoscalar \( \pi^0 \) and \( \eta \) mesons, as provided by the data. Firstly, one has the prompt photons coming from the hard collisions of initial state partons in the colliding nuclei. These populate the high transverse momentum region and can be estimated by perturbative QCD. The thermal contribution depends on the space-time evolution scenario that one considers. In the event of a deconfinement phase transition, one first has a thermalized QGP which expands and cools, reverts back to hadronic matter, again expands and cools and eventually freezes out into hadrons which are mainly pions. Photon emission in the QGP occurs mainly due to QCD annihilation and Compton processes between quarks and gluons. In order to estimate the emission from the hadronic matter we will consider a gas of light mesons viz. \( \pi, \rho, \omega \) and \( \eta \). Reactions among these lead to the emission of photons.

It has been emphasized that the properties of vector mesons may change appreciably because of interactions among the hadrons at high temperatures and/or densities (see the reviews [5,13–16]). This modifies the rate of photon emission as well as the equation of state of the evolving matter. Among various models for vector mesons available in the literature [5], we examine two possibilities for the hadronic phase in this paper: (i) no medium modifications of hadrons, and (ii) the scenario of the universal scaling hypothesis of the vector meson masses [14]. In principle, we can think of another scenario (iii) the large collisional broadening of the vector mesons [15]. Both (ii) and (iii) can reproduce the enhancement of the low-mass dileptons measured by CERES Collaboration at CERN SPS, but the scenario (iii) has been found to have a negligible effect on the emission rate for photons [5]. The effect of temperature dependent mass as described in case (ii) has also been incorporated in the equation of state (EOS) of the hadronic matter undergoing a (3+1) dimensional expansion. We will see that the resulting photon spectra reproduce the experimental data quite well.

There is still substantial debate on the order of the phase transition as well as the value of the critical temperature. To address this aspect we will also consider a scenario where the system begins to evolve from a high temperature phase where all hadron masses approach zero (pion mass is fixed at its vacuum value). As the system expands and cools, the hadrons acquire masses as in case (ii) till freeze out. Incorporation of medium modified masses and the equation of state in this case also provides a reasonable explanation of the data.

The paper is organized as follows. In the following section we describe the static photon emission rate and the equation state. In section 3 we discuss the results and finally in section 4 we present summary and discussions.
II. FORMULATION

A. Photon Emission

The lowest order processes contributing to hard thermal photon emission from quark gluon plasma are the QCD Compton and annihilations processes [17,18]. It has been shown recently [19] that the two-loop contribution leading to bremsstrahlung and $q\bar{q}$ annihilation with scattering contribute to the same order as the lowest order processes. The total rate of emission at temperature $T$ per unit four-volume is given by

$$E \frac{dR}{d^3p} = \frac{5}{g} \frac{\alpha_s}{2\pi^2} \exp(-E/T) \left[ \ln \left( \frac{2.912 E}{g^2 T} \right) + 10 \frac{(J_T - J_L)}{\pi^3} \{ \ln 2 + \frac{E}{3T} \} \right] \quad (1)$$

where $J_T \simeq 4.45$ and $J_L \simeq -4.26$. The QCD coupling $'g'$ is given by

$$\alpha_s \equiv \frac{g^2}{4\pi} = \frac{6\pi}{29 \ln(8T/0.16)} \quad (2)$$

for two quark flavours [20].

As we will see later, the direct QCD photons originating from hard scattering of partons embedded in the colliding nuclei in the very early stages of the collision will have a significant contribution to the total photon yield. They are estimated using perturbative QCD as

$$E \frac{dN}{d^3p} = T_{AA}(b = 0) E \frac{d\sigma_{pp}}{d^3p} \quad (3)$$

where $T_{AA}(b)$ is the nuclear thickness at impact parameter $b$. Its value at $b = 0$ is taken as 220. $\sigma_{pp}$ includes the $pp$ cross-section for Compton and annihilation processes among the partons. At SPS energies one should include the effects of intrinsic $k_T$ distribution of partons [21] leading to substantial enhancement in the photon spectra [12]. This is because of the fact that the colliding partons might have some initial transverse momentum with respect to the incoming hadrons analogous to the Fermi motion of nucleons in the nucleus. In practice such an effect is implemented by multiplying each of the parton distribution functions appearing in the right hand side of the above equation by a Gaussian function of the type $f(k_T) = \exp[-k_T^2/(2\pi)^2]$ and integrating over $d^2k_T$ (see [12] for details). We use CTEQ5M partons [22] for evaluating the hard QCD photons. The photons from hard QCD processes have been used to normalize the p-A data. An enhanced production in A-A collisions will presumably mark the presence of a thermal source.

To estimate the photon yield from thermal hadronic matter (HM), we have considered an exhaustive set of hadronic reactions and vector meson decays involving $\pi$, $\rho$, $\omega$ and $\eta$ mesons. It is well known [17] that the reactions $\pi \rho \rightarrow \pi \pi \gamma$, $\pi \pi \rightarrow \rho \gamma$, $\pi \pi \rightarrow \eta \gamma$, $\pi \eta \rightarrow \pi \gamma$, and the decays $\rho \rightarrow \pi \pi \gamma$ and $\omega \rightarrow \pi \gamma$ are the most important channels for photon production from hadronic matter in the energy regime of our interest. In the present work we have also considered photon production due to the process $\pi \rho \rightarrow a_1 \rightarrow \pi \pi \gamma$. The invariant amplitudes for all the above reactions are given in Refs. [23,24].

B. Medium Effects

Because of the significantly high temperature of the hadronic matter, in-medium modifications of the constituents may be necessary. Such an effect modifies the static emission rates as well as the equation of state of the evolving matter in a non-trivial way [5,25].

In Ref. [5] the effects of spectral changes of hadrons on the electromagnetic probes have been discussed in detail. It has been observed that the gauged linear and non-linear sigma models and the model with hidden local symmetry do not show effect on photon emissions. In the Walecka model, the universal scaling hypothesis for the vector meson masses as well as the large collisional broadening of vector mesons produce a large enhancement in low mass dileptons. However, the photon emission rate does not suffer substantial medium effect even in these cases, since the spectral function is smeared out in the photon emission rate. Nevertheless, the scaling hypothesis with particular exponent $\lambda = 1/2$ (called the Nambu scaling in [14]) has been seen to enhance photon emission among the others.

To consider the effect of the spectral modifications of hadrons in comparison with the current data, we thus adopt two extreme cases: (i) no medium modifications of hadrons, and (ii) the scaling hypothesis with $\lambda = 1/2$. In case (ii), the parametrization of in-medium quantities (denoted by *) at finite $T$ is

$$\frac{m_V^*}{m_V} = \frac{f_{V}^*}{f_{V}} = \frac{\omega_{V}^*}{\omega_{V}} = \left( 1 - \frac{T^2}{T_c^2} \right)^{\lambda} \quad (4)$$

with $\lambda$ taken as 1/2. $V$ stands for vector mesons ($\rho$ and $\omega$), $f_V$ is the coupling between the electromagnetic current and the vector meson field and $\omega_V$ is the continuum threshold. Mass of the nucleon, $m_N$ also varies with temperature as in eq. (4). (Note that there is no definite reason to believe that all the in-medium dynamical quantities are dictated by a single exponent $\lambda$. This is the simplest possible ansatz [26].) The effective mass of $a_1$ is estimated by using Weinberg’s sum rules [27]. We have seen earlier that the baryon chemical potential has a small effect on the photon yield [25]. Moreover, in the central rapidity region the entropy per baryon is quite large $\sim 40 - 50$ [28,29]. Thus the finite baryon density effects are neglected here.
C. Space-Time evolution

We will assume that the produced matter reaches a state of thermal and chemical equilibrium after a proper time $\sim 1$ fm/c. If a deconfined matter is produced, it evolves in space and time till freeze-out undergoing a phase transition to hadronic matter in the process. The $(3+1)$ dimensional hydrodynamic equations have been solved numerically by the relativistic version of the flux corrected transport algorithm [30], assuming boost invariance in the longitudinal direction and cylindrical symmetry in the transverse plane. For the QGP phase the bag model equation of state is used. The effects of the temperature dependent hadronic masses have been taken into account in the EOS through the effective statistical degeneracy [5]. The initial temperature $T_i$ can be related to the multiplicity of the event $dN/dy$ by virtue of the isentropic expansion as [31],

$$\frac{dN}{dy} = \frac{45\zeta(3)}{2\pi^4} \pi R_A^2 a_k T_i^3 \tau_i$$

where $R_A$ is the radius of the system, $\tau_i$ is the initial thermalization time and $a_k = (\pi^2/90) g_k$; $g_k$ being the effective degeneracy of the system. $g_H$, the statistical degeneracy of the hadronic phase, composed of $\pi$, $\rho$, $\omega$, $\eta$, $a_1$ and nucleons, is a temperature dependent quantity in the present work and plays an important role in the EOS. To solve the hydrodynamic equations we need to specify, the initial energy density, which is obtained from $T_i$ through EOS. The functional form of the initial energy density $\epsilon(\tau_i, r)$, is taken from [30]. We take various values for the initial radial velocity and discuss the sensitivity of the results on this quantity.

III. RESULTS

For central collisions of Pb nuclei at 158 AGeV at the CERN-SPS, we assume that QGP is produced at $\tau_i=1$ fm/c which expands both longitudinally as well as in the transverse direction and undergoes a first order phase transition to hadronic matter at $T_f=160$ MeV. Taking $dN/dy=700$ [32], and $g_k = g_{QGP}=37$ for a two-flavour QGP, the initial temperature $T_i$ comes out as 196 MeV. As is typical of a first order phase transition one has a mixed phase of coexisting QGP and hadronic matter which persists till the phase transition is over. Thereafter the hadronic matter expands, cools and freezes out at a temperature $T_f=120$ MeV. The sum total of the photon yields from the QGP phase, the mixed phase and the hadronic phase constitutes the thermal yield.

In Fig. 1, we show only thermal photon spectra originating from quark matter (QM $\equiv$ QGP + QGP part of mixed phase) and hadronic matter (HM $\equiv$ hadronic part of mixed phase + hadronic phase). The solid line corresponds to the case without the medium effect of hadrons (case (i)) and the dashed line corresponds to the case with in-medium hadrons (case (ii)). The case with large collisional broadening shows no deviation from (i). At low $p_T$ the difference between (i) and (ii) is negligible because in this region of phase space most of the photons are emitted from the late stage of the evolution where the in-medium effects are small. The increased photon yield at large $p_T$ is caused by the enhancement in the Boltzmann factor due to the reduction in meson (particularly, $\rho$) masses. However, when we consider the total photon emission this difference of thermal photons at high $p_T$ is masked by the hard photon contribution.

![Fig. 1. Transverse momentum distribution of thermal photons with and without medium effects.](image)

In Fig. 2, results for the total photon emission is shown for three different values of the initial transverse velocity with medium effects as in case (ii). All the three curves represent the sum of the thermal and the prompt photon contribution which includes possible finite $k_T$ effects of the parton distributions. The later, shown separately by the dot-dashed line also explains the scaled $pp$ data from E704 experiment [33]. We observe that the photon spectra corresponding to $v_T(\tau_i, R_A)=0.2$ explains the WA98 data reasonably well. It is found that a substantial fraction of the photons come from mixed and hadronic phase. The contribution from the QGP phase is rather small because of the small life time of the QGP ($\sim 1$ fm/c).
The last statement together with the current uncertainty of the critical temperature $T_c$ poses the following question: Is the existence of the QGP phase essential to reproduce the WA98 data? To study this problem, we have considered two possibilities. (a) pure hadronic model without medium-modifications, and (b) pure hadronic model with scaling hypothesis according to eq.(4). In the former case, the initial temperature $T_i$ is found to be $\sim 250$ MeV for $\tau_i = 1$ fm/c, which appears to be too high for the hadrons to survive. Therefore this possibility should be excluded. On the other hand, the second case with an assumption of $T_i = T_c$ (which is just for simplicity) leads to $T_i \sim 205$ MeV, at $\tau_i = 1$ fm/c, which is not unrealistic. In this case, the hadronic system expands and cools and ultimately freezes out at $T_f = 120$ MeV. The masses of the vector mesons increase with reduction in temperature (due to expansion) according to eq.(4). The results of this scenario for three values of the initial radial velocity including the prompt photon contribution are shown in Fig. 3. The experimental data are well reproduced. Thus, it is fair to say that with a simple hadronic model it is hard to explain the CERN-SPS photon data. Either substantial medium modifications of hadrons or the formation of QGP in the initial stages is necessary to reproduce the data. It is rather difficult to distinguish between the two at present.

We now show our prediction of the photon yield in central collisions of Au nuclei at 200 A GeV at RHIC in Fig. 4. Enhancement due to intrinsic motion of the partons has been ignored at RHIC, because at higher beam energies such effects are small [12]. The initial temperature and thermalization time are taken as 300 MeV and 0.5 fm/c respectively [29]. It is seen that up to $p_T = 2$ GeV, most of the photons are emitted from the thermal source. We have also observed that photons from quark matter make a substantial contribution to the thermal yield (the dash-dotted line in Fig. 4), although it is smaller than the thermal photons from hadronic matter (dotted line in Fig. 4). This is because of the higher initial temperature and larger lifetime of the QGP phase is realized at RHIC compared to that at SPS. The effect of the transverse expansion on the QGP phase is small. However, photons from the hadronic phase, particularly during the late stage of the evolution when the temperature is low, receive a large kick due to the radial expansion and consequently populate the high $p_T$ region, as seen in Fig. 4.
IV. SUMMARY AND DISCUSSIONS

We have evaluated the high energy photon yield from matter created after Pb + Pb collisions at CERN SPS energies. The effects of the variation of hadronic masses on the photon yield have been considered both in the cross section as well as in the EOS. The photon spectra of WA98 collaboration are well reproduced with the formation of QGP phase, although the contribution from this phase is small compared to that from the hadronic phase because of the small life time (\(\sim 1 \text{ fm/c}\), which is of the order of the time scale for strong interaction). The experimental data are also well reproduced in a scenario where the hadronic masses approach zero at a temperature \(\sim 205 \text{ MeV}\); a scenario where the system is driven towards a chirally symmetric phase.

In both the scenarios it is seen that the thermal photons originate from a source with initial temperature \(\sim 200 \text{ MeV}\). Such a value of \(T_i\) is also obtained from the analysis of dilepton data in the intermediate mass region for Pb + Pb collisions at CERN SPS [34].

In spite of the above encouraging situation, a firm conclusion about the formation of the QGP at SPS necessitates a closer look at some pertinent but unsettled issues. We have assumed complete thermodynamic equilibrium for quarks and gluons which may not be entirely realistic for SPS energies (lack of chemical equilibrium will further reduce the thermal yield). From the lattice QCD calculations the value of the critical temperature has been predicted as, \(T_c \sim 170 \text{ MeV}\), for two flavour QGP and \(T_c \sim 250 \text{ MeV}\) for a pure gluonic system [35]. For a system away from chemical equilibrium, the nature of the phase transition is unknown.

We have assumed that \(\tau_i = 1 \text{ fm/c}\) at SPS energies, which may be considered as the lower limit of this quantity, because the transit time (the time taken by the nuclei to pass through each other in the CM system) is \(\sim 1 \text{ fm/c}\) at SPS energies and the thermal system is assumed to be formed after this time is elapsed. Moreover, the photon emission rate from QGP evaluated in Refs. [17–19] obtained by resumming the hard thermal loops is strictly valid for \(g << 1\) whereas the value of \(g\) obtained from eq. (2) is \(\sim 2\) at \(T \sim 200 \text{ MeV}\).

However, at RHIC in addition to the soft processes the semi-hard processes contribute significantly to the initial energy density. The quark and gluon degrees of freedom would play a dominant role leading to less ambiguity in determining the initial temperature through electromagnetic probes at RHIC. It is observed that the contributions from QM is substantial to the total thermal photon yield around \(p_T \sim 2 \text{ GeV}\), because of the long life time of the QGP and high initial temperature.

Acknowledgement: J.A. is grateful to the Japan Society for Promotion of Science (JSPS) for financial support. J.A. and T.H. are also supported by Grant-in-aid for Scientific Research No. 98360 of JSPS.

Eq. (4) for $m_V^*$ is not entirely consistent with the low temperature theorem [36], which says there should be no $O(T^2)$ correction to the mass. Therefore, one cannot take the ansatz too seriously at low $T$. In practical applications, however, $T < 100$ MeV is not relevant in any way since it is below the freeze-out temperature.