COIL-END DESIGN AND END-SPACER MANUFACTURE

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Abstract
One of the main tasks in designing the coil of the superconducting magnet is the coil end optimization and design of end-spacers. In ROXIE the generation of constant perimeter coil ends is automatically done using only a very view user supplied input data. The design principle of the coil end is described in this chapter.

1 Types of coil-ends
For the automatic generation of the coil end region, four options are available:

• Coil end design with or without shims between the turns and conductors placed on the winding mandrel, see Fig. 1.

• Coil end with grouped conductors or with shims between the turns accounting for cable deformations, aligned at the outer radius of the end-spacers. End-spacers featuring shelves to support the turns.

• Coil end for magnets with rectangular cross sections.

• Racetrack coil ends with or without additional straight sections. With this option it is possible to model solenoid and torus magnets in 3 dimensions, see Fig. 2.

Fig. 1: Pair of constant perimeter coils

The geometric positions of bare conductors in xy, yz, sz plane as well as in the 3 dimensions can be printed in the DXF format (DXF, Data eXchange Format originally developed by Autodesc) suitable for
other CAD programs such as EUCLID and AUTOCAD. For numerical field calculation packages such as ANSYS and OPERA3D specialized interfaces are available. 3d peak field calculation in the coil end is possible either for a bare coil or a coil housed in an infinitesimal long iron cylinder. For the calculation of the integrated multipole content the start and the end of the iron yoke can be specified. Note that in this case the start and end of the integration path has to be sufficiently far away from the iron discontinuity as otherwise the applied imaging method gives wrong results. In both cases the iron saturation effects have to be neglected. In the following sections the generation of the geometrical model is described.

2 Conductor positioning in the YZ plane

Fig. 3 shows the 3d coordinate system for the calculations of the conductor positioning in the end regions together with the inclination angles and positions of each coil block which have to be given as input. They are the radius of the winding mandrel, the inclination and positioning angle in the 2d cross-section, the inclination angle in the YZ plane, the ellipticity of the upper edge of the conductor and an additional straight section.

Fig. 4 shows the cut in the YZ plane of the coil end. Between the turns additional wedges are placed and the conductors are aligned on the winding mandrel. Fig. 5 shows the cut in the YZ plane of a coil end with conductors aligned on the outer diameter of the end spacers and with additional shelves to support the coil blocks.

From a given \( b_0 \) and a given inclination angle \( \beta_0 \) one can derive with \( \gamma = \arctan\left(\frac{w_i - w_o}{2h}\right) \) and \( \gamma_2 = \beta_0 + 2 \cdot \gamma \) the conductor positions in the YZ plane by repeating the calculations for the z position

\[
\begin{align*}
  z_4 &= b_0 + z_0 \\
  z_1 &= z_4 + \sqrt{h^2 + \left(\frac{w_i - w_o}{2}\right)^2} \cdot \cos \beta_0 \\
  z_2 &= z_1 + w_i \cdot \cos(90 - \beta_0 - \gamma) \\
  z_3 &= z_4 + w_o \cdot \cos(90 - \beta_0 - \gamma)
\end{align*}
\]
Fig. 3: Coordinate system of the coil-end

Fig. 4: YZ plane of coil end with inter-turn wedges and conductors aligned on the mandrel

Fig. 5: YZ plane of coil end with conductors aligned on outer diameter of the wedges

and for the y position

\[ y_1 = 0 \]  \hspace{2cm} (5)

\[ y_2 = w_i \cdot \sin(90 - \beta_0 - \gamma) \]  \hspace{2cm} (6)
\[ y_4 = \sqrt{h^2 + \left( \frac{w_i - w_o}{2} \right)^2} \cdot \sin \beta_0 \]  \hspace{1cm} (7) 

\[ y_3 = y_4 + w_o \cdot \sin(90 - \beta_0 - \gamma) \]  \hspace{1cm} (8) 

in case of the ID alignment. For the OD alignment we get (c.f. fig. 6):

\[ y_3 = r_i + h \]  \hspace{1cm} (9) 

\[ y_4 = y_3 - w_o \cdot \sin(90 - \beta_0 - \gamma) \]  \hspace{1cm} (10) 

\[ y_1 = y_4 - \sqrt{h^2 + \left( \frac{w_i - w_o}{2} \right)^2} \cdot \cos \beta_0 \]  \hspace{1cm} (11) 

\[ y_2 = y_1 + w_i \cdot \sin(90 - \beta_0 - \gamma) \]  \hspace{1cm} (12) 

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Fig. 6: Position of the conductors in the yz plane, left: ID alignment, right: OD alignment

3 Conductor positioning in the sz plane

The upper and lower edges of the conductors are defined by their position in the xy cross section which yield \( a_i \) and \( a_o \) the half axis of the ellipse in the sz plane. Each turn can have an additional straight section \( z_0 \). Together with the position \( z_i \) in the yz plane and the ellipticity of the upper edge \( b_0 \) the ellipticity of the inner edge \( b_i \) can be calculated from the constant perimeter condition.

3.1 Elliptical shapes

In the sz-plane we assume the upper edges of the conductor as being of elliptical shape cf. fig. X.

\[ \frac{s^2}{a^2} + \frac{z^2}{b^2} = 1 \]  \hspace{1cm} (13)
Fig. 7: Upper and lower edges of the conductors in the sz plane

Fig. 8: SZ plane of coil end with elliptical shape of conductor edges

The positions of the lower edge is assumed as being of constant perimeter thus resulting in a straight section $l_i$ and an ellipse with a smaller half axis $b_i$. The perimeter of the ellipse (starting at $z = z_0$) of the upper edge can be approximated by

$$P_o = \frac{1}{2} \pi (a_i + b_i) \left( 1 + \frac{\lambda^2}{4} + \frac{\lambda^4}{64} \right)$$  \hspace{1cm} (14)

with

$$\lambda = \frac{a_o - b_o}{a_o + b_o}$$ \hspace{1cm} (15)

$z_i$ is given from the position of the lower edge in the yz plane. It yields:

$$P_i + 2 \cdot l_i = P_o$$ \hspace{1cm} (16)

$$l_i + b_i = z_i - z_0$$ \hspace{1cm} (17)
\( b_i \) and therefore \( l_i \) can be calculated recursively with the equation

\[
b_{i+1}^n = \frac{P_i - 2(z_i - z_0) - \frac{1}{2\pi}a_i(1 + \frac{\nu^2}{\pi})}{\frac{1}{2\pi}(1 + \frac{\nu^2}{\pi} + \frac{\nu^4}{64}) - 2}
\]

with

\[
\nu = \frac{a_i - b_i^n}{a_i + b_i^n}
\]

### 3.2 Hyper-elliptical shapes

The radius of curvature of the ellipse for small values of \( z \) is

\[
R = \frac{b^2}{a}
\]

An alternative with a \( 0 \) curvature at the onset of the bend is the hyper-elliptical shape:

\[
\frac{s^2}{a^2} + \frac{z^3}{b^3} = 1
\]

![Fig. 9: SZ plane of coil end with hyper-elliptical shape of the edges](image)

The calculation of the lower edge follows the reasoning for the ellipse, however, there is no closed form available for the calculation of the perimeter of the hyper-ellipse. Therefore a polynomial approximation for the perimeter is used:

\[
P = 3.868 + 1.552\left(\frac{a}{b}\right) + 1.601\left(\frac{a}{b}\right)^2 - 0.489\left(\frac{a}{b}\right)^3
\]

Note however that this approximation only holds for \( a/b \) ratios between 0.3 and 1.
4 Radius of curvature

One aim of the coil end design is to keep the minimum radius of curvature in each coil block within certain limits. The curvature is calculated with a finite difference approach as:

\[ R = \frac{1}{k} = \frac{1}{\sqrt{(\frac{d^2 x}{ds^2})^2 + (\frac{d^2 y}{ds^2})^2 + (\frac{d^2 z}{ds^2})^2}} \]  

(23)

with

\[ \frac{d^2 x}{ds^2} \approx \frac{x_{i+1} - 2x_i + x_{i-1}}{\Delta s^2} \]  

(24)

\[ \frac{d^2 y}{ds^2} \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta s^2} \]  

(25)

\[ \frac{d^2 z}{ds^2} \approx \frac{z_{i+1} - 2z_i + z_{i-1}}{\Delta s^2} \]  

(26)

For equidistant spacing of the conductors. If the 3d bricks are not equal in length the equations reads:

\[ \frac{d^2 f}{ds^2} \approx \frac{\Delta s_2(f_{i+1} - f_i) - \Delta s_1(f_i - f_{i-1})}{\Delta s_1 \Delta s_2 \cdot 0.5(\Delta s_1 + \Delta s_2)} \]  

(27)

5 End-spacer manufacture

The shape of the end-spacers is determined by the shape and position of the coil blocks as found in the field optimization process. The surfaces to be machined are described by 9 polygons, which are transferred into a CAM system, e.g., CATIA, for the calculation and emulation of the cutter movements for machining the piece. As an interface an ASCII file and a DXF file is available. The spacers are machined by means of a 5-axis CNC machine from glass-epoxy tubes (G11). Because of the abrasive nature of the glass dust, diamond tools must be used.

Fig. 10 shows the polygons describing the spacer surfaces for piece No. 2 in a set of connection side end-spacers as shown in Fig. 11. 3 different ways of sectioning of these polygons are provided.

- Equiangular slicing of the ellipses in the sz plane
- Equidistant slicing of the ellipses. This makes necessary an iterative procedure for the determination of the ellipse perimeter and thus takes considerably more computing time
- Equitangential slicing of upper and lower ellipse thus allowing for the usage of cylindrical cutters on 5 axis CNC machines.

Fig. 11 shows an artists view of the end-spacers (outer layer coil) for a dipole without shelf (conductor blocks aligned on the mandrel) together with a view of the coil windings.
Fig. 10: Display of polygones for the CNC machining of the end-spacers

Fig. 11: Connection side of dipole coil together with its endspacers