Topological and confining properties of Abelian-projected SU(3)-QCD

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\textsuperscript{a}[1]

In this talk, we discuss several topics related to the Abelian-projected SU(3)-QCD. First of them is the Aharonov-Bohm effect emerging during the extension of this theory by the introduction of the \( \Theta \)-term. Another topic is devoted to various consequences of screening of electric and magnetic field correlators in the four-dimensional Abelian-projected SU(3)-QCD is studied. Finally, the bilocal correlator of electric field strengths in the three-dimensional gas of SU(3) Abelian-projected monopoles is discussed.

1. INTRODUCTION

In the present talk, we shall mostly discuss various nonperturbative properties of the effective low-energy theory of the SU(3)-QCD \([1]\), which models confinement of quarks as the dual Meissner effect \([2]\). The partition function of this \([U(1)]^2\) magnetically gauge-invariant theory reads

\[
Z = \int |\Phi_a| D |\Phi_a| D\theta_a DB_\mu \delta \left( \sum_{a=1}^{3} \theta_a \right) \times
\]

\[
\times \exp \left\{ - \int d^4x \left[ \frac{1}{4} \left( F_{\mu\nu} + F^{(c)}_{\mu\nu} \right)^2 + \right. \right.
\]

\[
+ \sum_{a=1}^{3} \left( \frac{1}{2} \left( \partial_\mu - 2ig_m e_a B_\mu \right) \Phi_a \right)^2
\]

\[
+ \lambda \left( |\Phi_a|^2 - \eta^2 \right)^2 \right\} . \tag{1}
\]

Here, \( g_m \) is the magnetic coupling constant, related to the QCD coupling constant \( g \) as \( g_m = \frac{4\pi}{g} \), and \( e_a \)'s are the root vectors of SU(3), whose explicit form is 
\[
e_1 = (1, 0), \quad e_2 = (-\frac{1}{2}, -\frac{\sqrt{3}}{2}), \quad e_3 = (-\frac{1}{2}, \frac{\sqrt{3}}{2}).
\]

Next, in Eq. (1), \( F_{\mu\nu} \) stands for the field strength tensor of the field \( B_\mu \) dual to the field \( A_\mu \equiv (A^3_\mu, A^8_\mu) \), and \( \Phi_a = |\Phi_a| e^{i\theta_a} \) are the dual Higgs fields describing the condensation of Cooper pairs of Abelian-projected monopoles.

Note that the phases \( \theta_a \)'s are related to each other by the constraint \( \sum_{a=1}^{3} \theta_a = 0 \), imposed by the respective \( \delta \)-function on the right-hand side of Eq. (1). This constraint reflects the dependence of the monopoles of three kinds of each other. Such a dependence is inspired by the fact that the monopole magnetic charges are distributed over the lattice defined by the root vectors, whose sum vanishes.

Finally, in Eq. (1) we have introduced the field strength tensor \( F^{(c)}_{\mu\nu} \) of an external quark of the colour \( c = R, B, G \) (red, blue, green, respectively), which obeys the equation \( \partial_\mu \hat{F}^{(c)}_{\mu\nu} = gQ^{(c)} J_\nu \). Here, \( \hat{D}_\mu \equiv \frac{1}{2} \varepsilon_{\mu\rho\lambda\sigma} \hat{O}_{\lambda\sigma} \hat{J}_\rho \), \( J_\mu(x) \equiv \oint C dx_\nu(\tau) \delta(x - x(\tau)) \), and \( Q^{(c)} \)'s are the charges of a quark of the colour \( c \) with respect to the Cartan subgroup of SU(3): 
\[
Q^{(R)} = \left( \frac{1}{2}, \frac{1}{2\sqrt{3}} \right), \quad Q^{(B)} = \left( 0, -\frac{1}{\sqrt{3}} \right), \quad Q^{(G)} = \left( 0, \frac{1}{\sqrt{3}} \right).
\]

The present lattice data \([3]\) indicate that in the regime of the model (1) corresponding to the real QCD, the coupling constant \( \lambda \) is much larger than one, namely \( \lambda \simeq 65 \). This makes it reasonable to consider this model in the London limit, \( \lambda \to \infty \). In

\[
\text{\ldots} \]

\[
\text{\ldots} \]
Finally, in the definition of $\bar{\Sigma}$ has been evaluated in Ref. [7], which should be accounted for in this measure, on $K$ the propagator of these bosons, where from now on $K$’s stand for the modified Bessel functions. In Eq. (2), we have also introduced the notation $\Sigma_{\mu\nu} = \Sigma_{a\mu} - 2s_a^{(c)}\Sigma_{\mu\nu}$. In this expression, $s_a^{(c)}$’s stand for certain numbers equal to 0 and ±1, which obey the relation $Q^{(c)} = \frac{e}{3}s_a^{(c)}e_a$, and $\Sigma_{\mu\nu} = \int d\mu(x_1)\delta(x - x_1)$ is the vorticity tensor current defined on the closed dual string world sheet $\Sigma_a$ with $\xi \equiv (\xi_1, \xi_2)$.$^a$ Note that owing to the one-to-one correspondence existing between $\Sigma_{\mu\nu}$’s and the multivalued parts of $\theta_a$’s, the three vorticity tensor currents are subject to the constraint $\sum_{a=1}^{3} \Sigma_{\mu\nu} = 0$, which stems from the analogous constraint imposed on $\theta_a$’s. Finally, in the definition of $\Sigma_{\mu\nu}$, we have denoted by $\Sigma_{\mu\nu}$ the vorticity tensor current defined on an arbitrary surface $\Sigma$ bounded by the contour $C$, which is the world sheet of the open dual string, ending up at a quark and an antiquark.

The $\Sigma_{\mu\nu} \times \Sigma_{\mu\nu}$-interaction in the action (2) can be shown [6] to describe confinement of quarks, whereas the $j_\mu \times j_\nu$-interaction clearly describes their Yukawa interaction at small distances. Notice also that in what follows we shall be interested in the string effective actions, rather than the measure of integration over world-sheet coordinates $x_\alpha(\xi)$’s. The Jacobian emerging during the change of integration variables $\theta_a \rightarrow x_\alpha(\xi)$, which should be accounted for in this measure, has been evaluated in Ref. [7].

2. INCLUDING THE $\Theta$-TERM

Let us now add to the Lagrangian of the model (1) the following $\Theta$-term:

$$\Delta L = -\frac{i\Theta g_\mu^2}{4\pi^2} \left( F^{(c)}_{\mu\nu} + \tilde{F}^{(c)}_{\mu\nu} \right) \left( \tilde{F}^{(c)}_{\mu\nu} + F^{(c)}_{\mu\nu} \right).$$

The string representation of such an extended partition function then reads [8]

$$Z^{c}_{\Theta} = \exp \left[ -\frac{2}{3} \left( \frac{\pi^2}{g_m^2} + \frac{(\Theta g_m)^2}{\pi^2} \right) \times \right.$$}

$$\left. \times \int d\mu \int dy D_m^{(4)}(x - y) \right] \times$$

$$\int \left\langle \exp \left[ \frac{2i\Theta}{3} s_a^{(c)} \left( \int d^4x \int d^4y \bar{\Sigma}_{\mu\nu}(x)j_\nu(y) \times \right. \right.$$}

$$\left. \left. \partial_\mu D_m^{(4)}(x - y) - \hat{L}(\Sigma_a, C) \right) \right] \right\rangle_{\Sigma_a}. \tag{3}$$

Here,

$$\hat{L}(\Sigma_a, C) \equiv$$

$$\equiv \int d^4x \int d^4y \bar{\Sigma}_{\mu\nu}(x)j_\nu(y) \partial_\mu D_m^{(4)}(x - y)$$

is the 4D Gauss linking number of the contour $C$ with the closed world sheet $\Sigma_a$, where $D_m^{(4)}(x) = \frac{1}{4\pi x^2}$. The average $\langle \ldots \rangle_{\Sigma_a}$ is formally defined with respect to $S_c[\Sigma = 0]$, but its exact meaning will be discussed below.

The first exponential factor on the right-hand side of Eq. (3) shows that due to the $\Theta$-term quarks acquire a nonvanishing magnetic charge [9], i.e., become dyons. The first term in the second exponential factor on the right-hand side of Eq. (3) describes a short-ranged interaction of dyons with closed and open dual strings, whereas the last term describes the long-ranged interaction of dyons with closed strings. This long-ranged interaction can be viewed as a scattering of dyons by the dual strings, which thus
play the rôle of solenoids carrying electric flux. According to Eq. (3), such a scattering, which is nothing else but the four-dimensional analogue of the Aharonov-Bohm effect [10], takes place at $\Theta \neq 3\pi \times (\text{integer})$. Note that in the SU(2)-case, the respective critical values of $\Theta$ have been found [11] to be equal to $2\pi \times (\text{integer})$.

Let us now carry out the average $\langle \ldots \rangle_{\Sigma_a}$, taking into account that at zero temperature dual closed strings with opposite winding numbers are known to form virtual bound states, called vortex loops [12]. The summation over the grand canonical ensemble of these objects has been performed in Ref. [13] and yields an effective sine-Gordon theory of two antisymmetric spin-1 tensor fields. In the dilute gas approximation, which is relevant to the reality since vortex loops are only virtual (and therefore small-sized) objects, such a theory enables one to calculate correlators of loops. Averaging the $\Sigma_a$-dependent exponential factor on the right-hand side of Eq. (3) in the sense of this effective theory by making use of the cumulant expansion and accounting in this average only for the contribution of the dominant, bilocal, irreducible correlator of vortex loops, we get

$$Z_{\Theta}^c = \exp\left\{ -\frac{8i\Theta}{3} \int d^4x \int d^4y \Sigma_{\mu\nu}(x)j_\mu(y) \times\right.$$ 

$$\times \partial_\mu D^{(4)}_m(x-y) - \frac{2}{\sqrt{\pi}} \int dx_\mu \int dy_\mu \times$$

$$\times \left[ \frac{\pi^2}{g^2} D^{(4)}_m(x-y) + \frac{(\Theta g_m)^2}{\pi^2 (g_H^2 + g_D^2)} \times \right.$$ 

$$\times \left( \frac{g_H^2}{M_0^2}(x-y) + \frac{g_D^2}{M_0^2}(x-y) \right) \right\}. \quad (4)$$

Here, $\Lambda = \sqrt{\frac{2\pi}{\pi}}$ is the ultraviolet momentum cutoff with $L$ standing for the typical distance between the neighbours in the gas of vortex loops and $a$ denoting the typical size of the loop, $a \ll L$.

Next, $\zeta \propto e^{-S_0}$ is the Boltzmann factor of a single vortex loop with the action $S_0$ equal to the string tension times the characteristic area of the loop. Furthermore in Eq. (4), $g_D = \frac{2\pi\sqrt{\zeta}}{a}$ denotes the contribution to the magnetic charge of the dual vector bosons, stemming from the Debye screening of those in the gas of electric vortex loops. Due to this effect, the mass $m$ of the dual vector bosons enhances. Since owing to the constraint $\sum_{\alpha=1}^3 \Sigma^{a}_{\mu\nu} = 0$ there exists the second independent type of vortex loops, there respectively appears also the second value of the Debye charge, equal to $g_D\sqrt{3}$. The two full masses then read $M_1 = \eta \sqrt{g_H^2 + g_D^2}$ and $M_2 = \eta \sqrt{g_H^2 + 3g_D^2}$.

The first term in the argument of the exponent on the right-hand side of Eq. (4) is contained in an explicit form in Eq. (3). As far as the second term is concerned, it tells us that the above-mentioned screening changes drastically the magnetic part of dyonic interaction. Namely, besides the modification of the massive propagator due to the additional contribution to the mass of the dual vector bosons, there also appears a novel massless interaction. The origin of the latter one is the absence of mass in the Aharonov-Bohm interaction on the right-hand side of Eq. (3). It is also worth noting that, as it should be, when the effect of screening is disregarded, i.e. $g_D \ll g_H$, one recovers the classical result given by the first exponential factor on the right-hand side of Eq. (3).

3. APPLICATIONS TO THE STOCHASTIC VACUUM MODEL

Let us now discuss in more details the confining properties of the model (1) in the London limit. Namely, let us consider the bilocal correlator of electric field strengths, $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, which plays the major rôle in the so-called stochastic vacuum model [14] (see Ref. [15] for reviews). Within this model, such a correlator is parametrized by the two coefficient functions as follows:

$$\left\langle f_{\mu\nu}(x)f_{\lambda\rho}(0) \right\rangle_{\Lambda, j^\mu_{\lambda\rho}} = \delta^{ij} \left\{ (\delta_{\mu\lambda} \delta_{\nu\rho} - \delta_{\mu\rho} \delta_{\nu\lambda}) \times \right.$$ 

$$\times \left( x^2 + \frac{1}{2} \partial_\mu (x_\lambda \delta_{\nu\rho} - x_\rho \delta_{\nu\lambda}) + \right.$$
In the infrared limit, \( |x| \gg M_1^{-1} \), whereas in the ultraviolet limit, \( D \ll D_1 \).

It is also worth noting that when the screening is disregarded, i.e. \( g_D \ll g_H \), the correlation length of the vacuum goes over to \( m \), and the classical expressions [4] (see Ref. [22] for the \( SU(2) \)-case) for the functions \( D \) and \( D_1 \) recover. In particular, the nonperturbative screening-motivated \( 1/|x|^2 \)-contribution to the function \( D_1 \) vanishes in this limit.

Besides the above-discussed modifications of the classical expressions for the correlators of electric field strengths inspired by screening, this effect changes also the classical expression for the propagator of the dual vector bosons. Indeed, the latter one reads

\[
\langle B^a_\mu(x)B^b_\nu(0)\rangle = \delta^{ab}\delta_{\mu\nu}D^{(4)}_m(x)
\]

with \( B^a_\mu = e_\mu B_\mu \), whereas with the account for screening it changes to [16]

\[
\delta_{\mu\nu} \left\{ \frac{1}{3} D^{(4)}_m(x) + \frac{g_D^2}{2} \left[ \frac{2}{(g_H^2 + 3g_D^2)} \left( g_H + g_D^2 \right) \right] \times D^{(4)}_0(x) + \frac{g_D^2}{g_H^2 + g_D^2} D^{(4)}_{M_1}(x) \right\}
\]

for \( a = b \) and

\[
\delta_{\mu\nu} \left\{ \frac{1}{3} D^{(4)}_m(x) + \frac{g_D^2}{2} \left[ \frac{2g_D^2}{(g_H^2 + 3g_D^2)(g_H^2 + g_D^2)} \right] \times D^{(4)}_0(x) + \frac{g_D^2}{g_H^2 + g_D^2} D^{(4)}_{M_1}(x) \right\}
\]

for \( a \neq b \). We see that the screening makes the propagator of the dual vector bosons nonvanishing even for \( a \neq b \). As it should be, in the limit when the screening is disregarded, \( g_D \ll g_H \),

\[
+ \partial_\nu (\rho_\mu \delta_{\mu\nu} - \chi_{\alpha \mu} \delta_{\mu\nu}) D_1 (x^2) \}
\]

In this formula, \( \langle \ldots \rangle_A^\mu \) stands for the average over free diagonal gluons, and \( \langle \ldots \rangle_J^\mu \) denotes a certain average over monopoles, which provides the condensation of their Cooper pairs. It is the coupling of the \( B_\mu \)-field, dual to the field \( A_\mu \), to \( J^\mu \)'s, which makes both functions \( D \) and \( D_1 \) nontrivial and finally adequate to those of the real QCD. Namely, these functions turn out to have the following form [16]:

\[
D = \frac{m^2 M_1 K_1(M_1|x|)}{4\pi^2 |x|},
\]

\[
D_1 = \frac{g_D^2}{\pi^2 (g_H^2 + g_D^2) |x|^4} + \frac{m^2}{2\pi^2 M_1 |x|^4 \times}
\]

\[
\left\{ \frac{K_1(M_1|x|)}{|x|} + \frac{M_1}{2} \left( K_0(M_1|x|) + K_2(M_1|x|) \right) \right\}.
\]

One can see that according to the above-presented asymptotics, the correlation length of the vacuum in the model under study is equal to \( M_1^{-1} \). Besides that, owing to the screening, the function \( D_1 \) contains the novel nonperturbative \( 1/|x|^2 \)-term, which might be important for modelling the Lüscher term [17] in the quark-antiquark potential [18]. Subtracting from the function \( D_1 \) this contribution, which has the same functional form as the pure perturbative contribution in the real QCD, we see that the remained part of the asymptotics (7) together with the asymptotics (6) are in a good agreement with the QCD lattice measurements of the functions \( D \) and \( D_1 \) [19] (see Ref. [20] for recent developments and Ref. [21] for reviews). Namely, due to the preexponential behaviour, \( D \gg D_1 \) in the infrared limit, \( |x| \gg M_1^{-1} \), whereas in the ultraviolet limit, \( D \ll D_1 \).
these off-diagonal components of the propagator given by Eq. (10) vanish, whereas the diagonal ones given by Eq. (9) go over to the classical expression, so that Eq. (8) recovers.

Contrary to the four-dimensional case, in three dimensions the present lattice data allow one to assume that Abelian-projected monopoles form a gas. Such a gas of SU(3)-monopoles has for the first time been considered in Ref. [23], and its partition function reads

\[
Z = 1 + \sum_{N=1}^{\infty} \frac{\zeta^N}{N!} \left( \prod_{n=1}^{N} \int d^3 z_n \sum_{a_n=\pm 1, \pm 2, \pm 3} \right) \times \exp \left[ -\frac{g^2}{4\pi} \sum_{n<k} e_{a_n} e_{a_k} \frac{1}{|z_n - z_k|} \right].
\]

(11)

In this formula, \( \zeta \propto \exp \left( -\frac{\text{const}}{g^2} \right) \) stands for the Boltzmann factor of a single monopole, and \( e_{-a} = -e_a \). The string representation of the Wilson loop in this gas, constructed in Ref. [24], turned out to be alternative to the one of the SU(2)-case, found in Ref. [25]. (See Ref. [26] for the discussion of Polyakov loops and their correlators in the SU(2) monopole gas.) By virtue of this representation in the approximation when the monopole gas is so dilute that its density is much less than \( \zeta \), one can deduce the respective expressions for the functions \( D \) and \( D_1 \). In the model under study, those are defined by Eq. (5) with the average \( \langle ... \rangle_{j^m} \) replaced by the average with respect to the partition function (11). Together with the contribution of the free diagonal gluons to the function \( D_1 \), these two functions read

\[
D = 12\pi \zeta e^{-\frac{m|x|}{|x|}},
\]

(12)

\[
D_1 = \frac{24\pi \zeta}{(m|x|)^2} \left( m + \frac{1}{|x|} \right) e^{-\frac{m|x|}{|x|}}.
\]

(13)

Here, \( m = g_m \sqrt{3\zeta} \) is the Debye mass of the two scalar bosons, dual to the diagonal gluons. Similarly to the above-considered four-dimensional case, we see that Eqs. (12) and (13) well agree with the lattice calculations in the real QCD [19], [20], [21]. In particular, the correlation length of the vacuum is now equal to \( m^{-1} \), and at the distances larger than this length, \( D \gg D_1 \) due to the preexponential behaviour.

4. CONCLUSIONS

In the present talk, we have discussed various nonperturbative properties of Abelian-projected SU(3)-QCD in four and three dimensions. In the four-dimensional case, we have firstly considered the respective dual Abelian Higgs type model extended by the introduction of the \( \Theta \)-term. In this way, the string representation of such an extended model has been derived, which has in particular demonstrated how the \( \Theta \)-term leads to the appearance of the magnetic charge of external quarks, making out of them dyons. The critical values of \( \Theta \), at which the Aharonov-Bohm scattering of dyons over the closed dual strings disappears, have been found. Next, the effect of the Debye screening of the dual vector bosons by virtual electric vortex loops, built out of the closed dual strings, has been taken into account. It turned out to lead to the significant modification of the magnetic part of the dyon-dyon interaction.

Then, the confining properties of the four- and three-dimensional SU(3) Abelian-projected theories within the stochastic vacuum model have been addressed. In particular, in the four-dimensional case the rôle of the Debye screening has been discussed, and it has been shown that owing to this effect there appears a novel nonperturbative long-ranged \( \frac{1}{m^4} \)-term in the bilocal correlator of electric field strengths. The influence of screening to the propagators of the dual vector bosons has also been considered.

In conclusion, the performed investigations have shown that Abelian-projected theories are not only adequate to the description of confinement of quarks in QCD, but possess also a lot of interesting nonperturbative properties themselves.
5. ACKNOWLEDGMENTS

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6. DISCUSSION

1. Dr. N. Brambilla (University of Heidelberg): You have mentioned that the London limit is in the agreement with the lattice results, but I would like to point out that a lot of the most recent lattice data show that the QCD vacuum looks like a dual superconductor at the border of type-I and type-II. Could you please comment on this issue.

D. Antonov: My statement was based on the lattice data of Ref. [3], which demonstrated that in the QCD-relevant regime of the effective dual theory considered, the coupling constant of the dual Higgs potential should be much larger than unity.

REFERENCES

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