A model of discrete dynamics of entanglement of bipartite quantum state is considered. It involves a global unitary dynamics of the system and periodic actions of local bistochastic or decaying channel. For initially pure states the decay of entanglement is accompanied with an increase of von Neumann entropy of the system. We observe and discuss revivals of entanglement due to unitary interaction of both subsystems. For some mixed states having different marginal entropies of both subsystems (one larger than the global entropy and one smaller) we find an asymmetry in speed of entanglement decay. The entanglement of these states decreases faster, if the depolarizing channel acts on the "classical" subsystem, characterized by smaller marginal entropy.

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I. INTRODUCTION

Quantum entanglement may be considered as one of the most subtle and intriguing phenomena in nature [1,2]. Its potential usefulness has been demonstrated in various applications like quantum teleportation, quantum cryptography, quantum dense coding and quantum computation. On the other hand, quantum entanglement is a fragile feature, which can be destroyed by interaction with the environment. This effect, having its origin in decoherence [3], is the main obstacle for practical implementation of quantum computing. A model allowing to study the dynamics of entanglement in presence of interaction with the environment has been recently analyzed by Yi and Sun [4].

In this paper we investigate destruction of the entanglement in a proposed model of discrete dynamics. We consider a simple bipartite system consisting of two spin-$\frac{1}{2}$ particles. Only one of them is subjected to periodic actions of a quantum channel, which represents the interaction with environment. As the initial states we take an ensemble of pure separable states and the ensemble of maximally entangled pure states. We also investigate the time evolution of mixed states having some special property. The corresponding system is composed of two subsystems exhibiting different properties with respect to some entropy inequality which is satisfied by all classical systems. One of the subsystems (A) satisfies the inequality and may be considered "classical", while the other, "quantum" subsystem violates the inequality. We investigate an asymmetry in the process of destruction of entanglement with respect to the subsystem interacting with the environment. We demonstrate a possible presence of revivals of entanglement caused by the global unitary evolution entangling the subsystems between consecutive actions of the environment.

The paper is organized as follows. In section II we describe a simple model of discrete time evolution. In section III we derive bounds on the entropy increase under the action of the environment. Then in section IV we analyze the decrease of entanglement versus increase of the degree of mixing of the initially pure states. The asymmetry in the entanglement decay depending on the subsystem subjecting to influence of environment is described in section V for some initially mixed states. The entanglement revivals are studied in section VI and discussion is contained in section VII.

II. MODELS OF TIME EVOLUTION

In this paper we consider the bipartite state subjected sequential interactions with environment. They are modeled by quantum channels, defined as completely positive linear maps, preserving the trace of the state [5].

Let $\sigma$ be a density operator acting on a finite-dimensional Hilbert space $\mathcal{H}$. The most general form of the quantum channel is the following transformation $\sigma \rightarrow \sigma'$:

$$\sigma' = \Lambda(\sigma) = \sum_{i=1}^{K} V_i \sigma V_i^\dagger,$$

where

$$\sum_{i=1}^{K} V_i V_i^\dagger = I.$$

If in addition $\sum_{i=1}^{K} V_i V_i^\dagger = I$ holds then the channel is called bistochastic. Bistochastic channels are the only ones which do not decrease the von Neumann entropy of any state they act on. A particular example of the bistochastic channel is given by random external fields [6]. These quantum channels may be written as

$$\sigma' = \Lambda_R(\sigma) = \sum_{i=1}^{K} p_i A_i \sigma A_i^\dagger,$$

where $A_i, i = 1, 2, \ldots, K$ are unitary operators and the vector of probabilities $\vec{p} = [p_1, \ldots, p_K]$ is normalized

$$\sum_{i=1}^{K} p_i = 1, \quad p_i \geq 0.$$
Such random systems can be described in the formalism of quantum \textit{iterated function systems} \cite{7}. The so called Kraus form \eqref{1} can be reproduced setting \( V_i = \sqrt{p_i} A_i \). It is worth to note that in the case of the most elementary quantum system described on the Hilbert space \( \mathcal{H} = \mathbb{C}^2 \) the random external field form is the most general one. In short: for \( \mathcal{H} = \mathbb{C}^2 \) the channel is bistochastic if and only if it can be put in the form \eqref{2}, as shown in Ref. \cite{8}. Note that an unitary evolution of the system can be considered as the simplest case of the bistochastic quantum channel with \( K = 1 \).

There exist, however, many quantum channels which are not bistochastic. We shall consider the following \textit{decaying channel}, sometimes called \cite{9} \textit{the amplitude damping channel}

\[
\sigma' = \Lambda_D(\sigma) = M_1 \sigma M_1 + M_2 \sigma M_2 \tag{4}
\]

where the matrices \( M_1 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{p} \end{bmatrix} \) and \( M_2 = \begin{bmatrix} 0 & \sqrt{1 - p} \\ 0 & 0 \end{bmatrix} \) are written in the standard basis.

Let \( \varrho \) denote a mixed state of a \( 2 \times 2 \) system i.e. the density operator defined on the Hilbert space \( \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B = \mathbb{C}^2 \otimes \mathbb{C}^2 \). The system consists of two subsystems \( A \) and \( B \) which can represent spin-\( \frac{1}{2} \) particles or two-level atoms. In our model the unitary dynamics is interrupted by periodic actions (cf. \cite{10}) of the environment as shown schematically in Fig. 1.

Discrete time evolution of the state \( \varrho \) reads in our model

\[
\varrho(n + 1) = U \varrho'(n) U^\dagger = U (\Lambda(\varrho(n))) U^\dagger \tag{5}
\]

where \( \Lambda = I \otimes \Lambda \) and the channel \( \Lambda \) is either bistochastic \eqref{2} or decaying \eqref{4}. Here \( U = e^{i \alpha \hat{H}} \) represents a unitary transformation which involves an interaction between both subsystems \( A \) and \( B \) described by the Hamiltonian \( \hat{H} \). We work in the dimensionless units and \( \alpha \) stands for a coupling parameter. In our work we take either \( \hat{H} = \sigma_x \otimes \sigma_y \equiv H \) or \( \hat{H} = \sigma_y \otimes \sigma_x \equiv H' \).

In general we shall use four types of dynamics defined by four different operators \( \Lambda \)-s in the formula \eqref{5}. Three of them will be random external fields \( \Lambda_R \) \eqref{2}, all defined by the same set of \( K = 4 \) unitary operators: \( A_1 = I, A_2 = \sigma_1, A_3 = \sigma_2, A_4 = \sigma_3 \) (where \( \sigma_i \) denote Pauli matrices), but with different vectors of probability: \eqref{3}:

\[
\begin{align*}
\bar{p}^{(1)} &= [1 - \epsilon, 0, 0, \epsilon], \\
\bar{p}^{(2)} &= [1 - \epsilon, 0, \epsilon, \epsilon], \\
\bar{p}^{(3)} &= [1 - \epsilon, \frac{\epsilon}{3}, \frac{\epsilon}{3}, \frac{2 \epsilon}{3}], \quad 0 \leq \epsilon \leq 1. 
\end{align*}
\tag{6}
\]

Each dynamics depends on two continuous parameters: \( \alpha \) contained in \( U = e^{i \alpha \hat{H}} \) governing the unitary dynamics and \( \epsilon \), included in the vector of probabilities, and describing the strength of the coupling with the environment. Additional discrete index \( j \) labels the different vectors of probability, \( \bar{p}^{(j)} \). For these three models of dynamics we shall use the compact notation \( \Theta_{\alpha, \epsilon} \). The fourth dynamics denoted by \( \Theta_{\alpha, p} \) is defined by putting in formula \eqref{5} the decaying channel \eqref{4}. Dynamics involving the operation \( U \) with reflected Hamiltonian, \( H' = \sigma_y \otimes \sigma_x \), will be denoted by the same symbols with only one change: \( \Theta \rightarrow \Theta \).

\textbf{Remark.} If \( \alpha \) is equal to zero, then the unitary operation \( U \) in \eqref{5} is reduced to identity transformation. In particular, it can be seen that the dynamics \( \Theta_{\alpha, \epsilon} \) corresponds to periodic action of \textit{depolaringizing channel} \cite{12}.

Now the essence of our study is the following: we consider composite quantum systems subjected to the local interaction with the environment, which acts on one subsystem only. We investigate, how the decay of the entanglement in the system depends on the initial state and the type of the dynamics. In particular we analyze, to which extend the decrease of the mean entanglement is reflected by the evolution of von Neumann entropy of the system.

### III. Bounds on Entropy Increase Under Local Channel

We start establishing bounds for the increase of von Neumann entropy.

\textbf{Proposition} .- Under a local action of the quantum channel \( \varrho_{AB} \rightarrow (I \otimes \Lambda) \varrho_{AB} \), the increase of the von Neumann entropy \( \Delta S \) for a bipartite \( n \otimes m \) state is bounded by

\[
\Delta S \equiv S(\tilde{\varrho}'_{AB}) - S(\tilde{\varrho}{}_{AB}) \leq S(\tilde{\varrho}{}_{A}) - S(\tilde{\varrho}'_{AB}) + \log m, \tag{7}
\]

where \( S(\varrho_{AB}) \) denotes the entropy of the subsystem \( A \). In particular, if the system is separable then \( \Delta S \leq \log m \).

\textbf{Proof.} By the definition the local channel is trace preserving, hence it does not change the density matrix of the first subsystem. Thus \( \tilde{\varrho}{}_{AB} = \tilde{\varrho}{}_{A} \) and the same holds for the corresponding entropies. Then from subadditivity of the entropy we have

\[
S(\tilde{\varrho}'_{AB}) \leq S(\tilde{\varrho}{}_{A}) + S(\tilde{\varrho}'_{B}) \leq S(\tilde{\varrho}{}_{A}) + S(\tilde{\varrho}{}_{AB}) \leq S(\varrho_{AB}) + S^{B}_{\max} \equiv S(\varrho_{A}) + \log m. \tag{8}
\]

We get the first inequality in the Proposition by subtracting \( S(\tilde{\varrho}{}_{AB}) \) from both sides of the above inequality. To get the bound in the case of separable states we take the inequality \eqref{7}, we have already proved, and remember that for separable states \( \Delta S \equiv S(\tilde{\varrho}'_{AB}) - S(\tilde{\varrho}{}_{AB}) \geq 0 \) \cite{13}. This completes the proof of our Proposition.

Note that a sequence of quantum channels acting locally forms a quantum channel acting locally too. So the proposition works also for the dynamics \( \Theta_{\alpha, \epsilon} \) and \( \Theta_{\alpha, p} \). Moreover, from Eq. \eqref{7} we see that the entropy of initially pure separable state \( \varrho_{AB} \) cannot exceed \( \log m \).
In this section we study the time evolution of entanglement and compare it with the time evolution of von Neumann entropy. To characterize the degree of entanglement we use the entanglement of formation introduced by Bennett et al. [12]. For any $2 \times 2$ mixed state this quantity may be computed analytically as shown by Hill and Wootters [14]. In this case the entanglement of formation $E$ (or shorter, the entanglement) varies from zero (separable states) to $\ln 2$ (maximally entangled states), so in the figures we used the rescaled variable $E/\ln 2$.

Our results were obtained by averaging over ensembles of random initial states. They were generated according to natural measures on:

(i) 6 dimensional manifold of all pure states for $2 \times 2$ problem,
(ii) 3 dimensional manifold of maximally entangled pure states,
(iii) 4 dimensional manifold of separable pure states.

Numerical experiments have shown that the samples of 100 initial states, generated randomly as described in the appendix, were sufficient to receive reliable results.

### A. Bistochastic channels

As shown in [15,16] the mean entanglement of mixed states decreases monotonically with increasing degree of mixing. Due to interaction with the environment the initially pure states become mixed and their von Neumann entropy, $S(\rho) = -\text{Tr}(\rho \ln \rho)$, grows in time. Thus one should expect a corresponding monotonous decay of the mean entanglement. This is indeed the case, as shown in Fig. 2 in absence of the unitary dynamics, $(\alpha = 0)$. Initial states were taken randomly from the entire space of pure states, so in accordance to [16], the initial mean entanglement is close to $(\ln 2)/2$. Parallel processes of decay of the entanglement and increase of the entropy are accelerated, if the parameter $\epsilon$ describing the interaction with environment increases.

For initially maximally entangled pure states (case (ii)) a similar dependence is represented by circles in Fig. 3. Here $\langle E(0) \rangle = \ln 2$. The picture changes when unitary evolution is involved. The latter leads to oscillations of entanglement of formation, reflected in the time evolution of entropy. The frequency of oscillations is proportional to $\alpha$. The larger this parameter, the faster the unitary evolution $U$ rotates the states $\rho$ from and into the convex set of separable states. In the case of entropy, oscillations are only due to changes of the second derivative i.e. entropy is still monotonically decreasing. This is not the case for entanglement $E$, which can also be seen in Fig. 4 for several individual initial states (without averaging). For short times the curve for $\alpha = 0$ (no unitary evolution) seems to constitute an envelope for all other curves.
FIG. 2. As in Fig. 2 for a sample of 100 maximally entangled states \(E(0) = \ln(2)\) with \(\varepsilon = 0.01\); \(\alpha = 0.0(\circ)\) and \(\alpha = 0.1(\triangle)\) for channels described by a) \(\vec{p}^{(1)}\) and b) \(\vec{p}^{(2)}\). Observe how the influence of the unitary dynamics depends on the kind of the channel.

It is worth to emphasize an important difference between \(\Theta_{\alpha,\varepsilon}^1\) (Fig. 3.a) and \(\Theta_{\alpha,\varepsilon}^2\) (Fig. 3.b). In the former case the presence of unitary evolution can accelerate the process of entropy increase. In the latter, on the contrary, switching on unitary evolution results in slower increase of the mean entropy. The iteration of the channel \(\Theta_{\alpha,\varepsilon}^1\) preserves both the number and the position of the nonzero component in \(\vec{p}\). It is not the case for \(\Theta_{\alpha,\varepsilon}^2\), for which two Pauli matrices generate the whole algebra of unitary matrices \(A_i\) involved.

FIG. 3. Dependence of entanglement of formation on time for several randomly chosen maximally entangled pure states. The unitary dynamics \(U = \exp(\alpha \vec{H})\) is governed by the parameter \(\alpha\). Here \(\varepsilon = 0.01\) in \(\vec{p}^{(2)}\), and \(\alpha = 0.1\) (narrow lines). Reference bold line represents the case of no unitary dynamics \((\alpha = 0)\), for which the dynamics of entanglement does not depend on the initial state.

Consider now the case (iii), of initially separable states, presented in Fig. 5. The presence of the unitary evolution may increase the mean entanglement, initially equal to zero. However, there is one difference more; for both dynamics \(\Theta_{\alpha,\varepsilon}^1\) (Fig. 5.a) and \(\Theta_{\alpha,\varepsilon}^2\) (Fig. 5.b) presence of the unitary dynamics accelerates the process of increase of entropy. In absence of the unitary dynamics \((\alpha = 0)\) the entropy does not exceed the value \(\ln 2\), in accordance to our proposition proved in section III.

Obtained results show that the oscillations of mean entanglement \(E\) are anti-correlated with the oscillations of the entropy \(S\). It was also checked that if \(\alpha\) is kept constant, but the Hamiltonian is chosen randomly then the oscillations of entanglement are smeared out. It means that effects of quantum coherence are destroyed and the destruction of entanglement occurs faster.
FIG. 4. As in Fig. 3 for a sample of 100 initially separable pure states ($E(0) = 0$). In absence of unitary dynamics, ($\alpha = 0$), the entanglement equals to zero.

B. Decaying channel

Figure 6 presents results obtained for the amplitude damping channel (4). In the absence of the unitary evolution ($\alpha = 0$) the mean entropy, $\langle S \rangle$, averaged over the entire manifold of pure states (case (i)), does not tend monotonically to its maximal value. At $t_n \sim 20$ the entropy reaches its maximum and then decreases to its limiting value about 0.3 (see full circles in Fig. 6b). This is due to the fact that for the decaying channel the entropy of the system may decrease.

Numerical data received by averaging over the set of maximally mixed states (case (ii), diamonds) and the set of separable pure states (case (iii), squares) are shown in Fig. 6a. Observe that the steady state limiting values of the von Neumann entropy, $\langle S \rangle$, represents the initial average entanglement $\langle E \rangle$. Indeed, in absence of the unitary evolution the perturbed subsystem is eventually dumped to the ground state. So finally the system is in the product state consisting of the ground state of the affected subsystem and the reduced density matrix of the unperturbed subsystem. Thus, after the averaging procedure, one gets the averaged von Neumann entropy of the subsystem not subjected to action of the channel.

A random choice of initially pure states of the composite system induces a certain measure in the space of the reduced density matrices. Our numerical calculations indicate that the natural rotationally invariant measure on the space of $N = 4$ pure states induces a uniform measure in the Bloch ball representing the density matrices for $N = 2$. Denoting the spectrum of reduced matrices by $\{1/2 - r, 1/2 + r\}$ we may write more formally, $P(r) = 24r^2$ for $r \in [0, 1/2]$. The mean von Neumann entropy, averaged according to this measure equals 1/3, in accordance to the numerical data presented in Fig. 6.

FIG. 5. As in Fig. 2 for samples of 100 initially separable pure states, a) ($\bigcirc$), maximally entangled pure states, a) ($\diamond$), and a sample of all pure states, b) ($\circ$), subjected to the Kraus channel (4) with $p = 0.05$ and $\alpha = 0.0$. The case (i) with unitary evolution, $\alpha = 0.1$, is denoted by ($\triangle$) in panel b).

For non-zero values of $\alpha$ we observe the oscillations of the mean entanglement, caused by the unitary evolution. It is interesting, however, that the presence of
V. ASYMMETRY OF ENTANGLEMENT DECAY

We shall consider here dynamics of mixed states having a very intriguing property. Namely we choose a quantum bipartite system, which violates some entropy inequality only with respect to one of both subsystems. Let us recall first that the information gain resulting from the measurement of any of subsystems of a quantum state with classical correlations is not greater than the gain obtained from measurement performed on the entire system. This classical feature is characteristic of quantum separable states. They do satisfy the following two inequalities concerning von Neumann entropy [13,17]:

\[ S(\rho_{AB}) \geq S(\rho_A), \]

and

\[ S(\rho_{AB}) \geq S(\rho_B), \]

where \( \rho_A \) and \( \rho_B \) denote the reduced density matrices, e.g. \( \rho_A \equiv \text{Tr}_B(\rho_{AB}) \). Now we shall focus on the following family of states introduced in [18]. They can be written as \( \rho^{(i)} := q|\Psi_1\rangle\langle\Psi_1| + (1-q)|\Psi_2\rangle\langle\Psi_2|, 0 < q < 1 \), with normalized pure state vectors \( |\Psi_1\rangle = q|00\rangle + \sqrt{1-a^2}|11\rangle \) and \( |\Psi_2\rangle = a|10\rangle + \sqrt{1-a^2}|01\rangle \). In the standard basis, \(|00\rangle, |01\rangle, |10\rangle, |11\rangle\), the corresponding density matrix takes the form

\[
\rho^{(i)} = \begin{bmatrix}
  qa^2 & 0 & 0 & qa\sqrt{1-a^2} \\
  0 & (1-q)(1-a^2) & (1-q)a\sqrt{1-a^2} & 0 \\
  0 & (1-q)a\sqrt{1-a^2} & (1-q)a^2 & 0 \\
  qa\sqrt{1-a^2} & 0 & 0 & q(1-a^2)
\end{bmatrix}
\]

Let us take \( a^2 > q > \frac{1}{2} \). Then the first inequality (9) is violated, while the second one (10) is not. Thus the composite system can be called 'quantum' with respect to the subsystem A and 'classical' with respect to the subsystem B. One may then expect that the bipartite system will loose entanglement in different ways, depending on whether the environment interacts with classical or quantum subsystem. Intuitively one could guess that the entanglement should be more robust if the noise affects the classical subsystem.

For concreteness we studied the system \( \rho^{(1)} \) for \( q = 3/5 \) and \( a^2 = 3/4 \). Then von Neumann entropy of the entire system, \( S = s(2/5) \approx 0.673 \) is greater than the entropy of the classical subsystem B for which \( S_B = s(1/4) \approx 0.562 \), and smaller than the entropy of the quantum subsystem, \( S_A = s(9/20) \approx 0.688 \). Here \( s \) stands for the Shannon entropy of a partition consisting of two elements, \( s(x) := -x \ln x - (1-x) \ln(1-x) \). We analyzed the time evolution of this quantum system in presence of a depolarizing channel \( \Theta^{(1)}_{\epsilon,\bar{\epsilon}} \) given by (2). In the theory of error correcting codes it is one of the most popular models of environment induced noise. The evolution of entanglement for the state \( \rho^{(1)} \) is represented by stars in Fig.7. In this case the bistochastic channel \( A \) acts on the 'classical' subsystem B. To investigate a possible asymmetry of the entanglement decay we consider the state \( \rho^{(2)} \), for which both subsystems are exchanged. More precisely, all elements of both density matrices are equal, apart of \( \rho_{23}^{(2)} = \rho_{32}^{(1)} \) and \( \rho_{32}^{(2)} = \rho_{23}^{(1)} \). The corresponding dynamics of \( \rho^{(2)} \) is denoted by crosses in Fig. 7. In this case the noise interacts with the 'quantum' subsystem A. The magnification in the inset reveals the asymmetry in the time evolution. Observe that our naive guess concerning the robustness is false. The attack on the 'classical' part of the system is more harmful to the entanglement properties of the system. This counter intuitive effect links quantum and classical features of the state from information-theoretical point of view. We propose to call it anomalous entanglement decay, (AED).

![FIG. 6. Comparison of the dependence of the entanglement formation for the state \( \rho^{(1)} \) with \( a^2 = 3/4; q = 3/5 \) (stars) and \( \rho^{(2)} \) (crosses). The bistochastic channel \( \tilde{\rho}_3 \) with \( \epsilon = 0.01 \) interacts with the 'classical' subsystem B in the former case, and with the 'quantum' subsystem A in the latter case. Solid line represents the behavior of a maximally entangled state \( \rho_{max} \). Magnification of the initial dependence provided in the inset reveals the asymmetry of the entanglement decay.](image)

Let us recall that any \( 2 \times 2 \) system may be described by two Bloch vectors, representing locally both subsystems, and a correlation matrix \( T \), which represents the projection of the composite system onto the family of mixtures of maximally entangled states (see [19]). A possible explanation of AED should take into account the fact that the local action of environment changes both the Bloch vectors, the correlation matrix, as well as their relationship. A depolarizing channel may affect in a similar way both local parameters, but it may distinguish, (in sense of the destruction of the entanglement), the correlation parameters with respect to the side of the action.

It should be noted that, regardless which part is sub-
Let us consider two cases:

(a) the noise parameter \(\epsilon\) is much less than the parameter \(\alpha\) characterizing the unitary interaction,

(b) both parameters are of the same order of magnitude.

Numerical results obtained in the weak noise case (a) are presented in Fig. 8. and 9. The revivals of the entanglement, caused by the unitary interaction, are manifestly visible, since the strength of the interaction with the environment \(\epsilon = 0.002\) is much less than the parameter \(\alpha = 0.06\) governing the unitary dynamics. Note the characteristic entanglement plateaus, if the analyzed state travels across the set of the separable states and the entanglement attains its minimal value equal to zero.

The effect of anomalous entanglement decay is clearly visible in Fig. 8.a, where the entanglement decays faster if the environment interacts with the classical subsystem. This contrasts the situation shown in Fig. 8.b, for which the unitary evolution is due to the reflected Hamiltonian \(H'\) and the exposure of the ‘quantum’ subsystem to the action of the environment action is more damaging for the entanglement.

It is instructive to analyze the same system with the unitary evolution reversed in time. Such a case, obtained...
by a change of the parameter $\alpha \to -\alpha$, is presented in Fig. 9. The general character of the evolution is kept. The significant difference is that here the entanglement is amplified at the very beginning which may have practical consequences if we are interested in short times of the process. Note that the figures 8.a. and 9.a reflected along the vertical line at $t_n = 0$ (respectively, 8.b and reflected 9.b) exhibit some kind of symmetry with respect to the initial moment.

What happens if we allow the strength of the coupling with the environment to be comparable with the parameter of the unitary interaction? This situation, corresponding to the case (b), is illustrated in Fig. 10. Here some interesting qualitative changes occur. The AED effect is present in the case shown in Fig. 10a; at the beginning the entanglement disappears faster when the ‘classical’ part of the system is affected by the environment. Moreover, in this case the entanglement disappears completely and never revives. If the ‘quantum’ subsystem interacts with the environment, a single entanglement revival occurs.

In the complementary case, for which $\rho^{(1)}$ interacts with the reflected Hamiltonian $H'$ (see 10b), we observe a special kind of competition: for short times the entanglement is smaller, if the quantum subsystem is perturbed. For longer times, the roles are interchanged, and the oscillations of the entanglement are damped faster, if the classical subsystem interacts with the environment.

In general one can see that the pictures corresponding to the cases (a) and (b) are qualitatively different depending on the ratio $\epsilon/\alpha$. This fact may be related with the observation concerning the processes of decoherence. Depending on the relation between two coupling parameters the so called pointer basis is determined either by the internal self-Hamiltonian of the system or by the Hamiltonian of the interaction with environment [3].

VII. DISCUSSION

We investigated the behavior of entanglement of bipartite spin-$\frac{1}{2}$ system subjected to periodic action of the environment. The process of destruction of entanglement of initially pure states is accompanied by increasing of von Neumann entropy. The asymptotic value of the entropy depends on the form of the interaction with the environment. For strongly mixing bistochastic channels, (e.g. $\Theta^2$ and $\Theta^3$) the entropy achieves the maximal value $\ln 4$. If the decaying channel is involved, the entropy gets its maximum after which it monotonically decays to the asymptotic value, which reveals the initial entanglement of the system.

If the internal unitary evolution entangling the system is present, the decay of the entropy due to the decaying channel can be replaced by the process of mixing the state more and more. The general feature of the time evolution is that the entanglement decreases as the system becomes more mixed. This corresponds to the results recently obtained in [15,16], where it was shown that the mean entanglement of quantum states, averaged over a sample of mixed states with the same von Neumann entropy, decreases with the degree of mixing. The presence of the internal unitary evolution leads to the revivals of the entanglement and to suppression (or acceleration) of the entanglement decay.

Perhaps the most intriguing is the character of asymmetry of the time evolution of the entanglement. For some initial mixed states consisting of two non-equivalent subsystems, the entanglement decays faster, if the environment interacts with the ‘classical’ subsystem, which satisfies the entropic inequality. Many years ago Schrödinger considered entanglement of pure state as a property of having both subsystems less informative for the observer, than the composite system. Mixed states (11) considered here exhibit this property only with respect to one subsystem [18]. Our results show that the action of environment to the ‘classical’ subsystem is sometimes more harmful to the entanglement. In this case one can thus say that the quantum entanglement runs away faster through the classical door.

In the context of the above discussion some general questions emerge. Consider a quantum entangled state $\rho$...
with, say, $S(\hat{\rho}_A) > S(\hat{\rho}_B)$, not necessarily violating the inequality (9). Under which conditions the entanglement is less robust to the environment action on the less informative subsystem $B$? How is it related to the possible violation of the von Neumann entropy inequality by subsystem $A$? What happens if instead of the inequalities (9,10) one applies the generalized α-entropies inequalities (17,18,19) satisfied for classical systems? All these questions seem to be important for deeper understanding of the dynamics of quantum entanglement.

It would be also interesting to analyze the role of entropic asymmetric states like (11) in context of quantum communication. In fact these states have only one coherent information positive [21,22] (see also [23]). For the corresponding quantum channels this might imply an asymmetry in the transfer of quantum information with the corresponding quantum channels this might imply an inherent information positive [21,22] (see also [23]). For communication. In fact these states have only one co-
tropic asymmetric states like (11) in context of quantum

entanglement [24].

In this appendix we present algorithms allowing one to
In this appendix we present algorithms allowing one to
generate random quantum states distributed uniformly
On the two sphere $P$ taken in a nonuniform way, with the probability density

Any $2\times2$ pure separable state may be written as $|\Psi_1\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, where $|\psi_1\rangle$ and $|\psi_2\rangle$ are $N = 2$, one-particle pure states. The 4 dimensional manifold of separable states has thus a simple structure of a Cartesian product $\mathbb{C}P^1 \times \mathbb{C}P^1$. A uniform measure on this manifold is obtained be taking both states $|\psi_1\rangle$ distributed uniformly (and independently) at the Bloch sphere, $\mathbb{C}P^1 \sim S^2$.

Working in the standard basis,

$$ |\Psi_1\rangle = U_1 \otimes U_2 |(1, 0, 0, 0)\rangle, \quad (A3) $$

where $U_1$ and $U_2$ denote two independent random unitary matrices distributed uniformly on $SU(2)$. This parametrization describes the entire 4D manifold of the separable pure states.

3. Random maximally entangled states

In an analogous way we may represent the maximally entangled states as

$$ |\Psi_\varphi\rangle = I \otimes U_1 |(0, 1, 1, 0)/\sqrt{2}\rangle. \quad (A4) $$

It is easy to see that for this states the reduced density matrix is proportional to identity matrix, and the entropy of entanglement achieves its maximum $\ln 2$. The states obtained by a symmetric operations $U_1 \otimes I$ are also maximally entangled. Using the standard representation of $U_1$ we parametrize maximally entangled states by [27]

$$ |\Psi_{\varphi_1}\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos \varphi_1 e^{i\varphi_2} \\ \sin \varphi_1 e^{-i\varphi_2} \\ -\sin \varphi_1 e^{i\varphi_2} \\ \cos \varphi_1 e^{-i\varphi_2} \end{bmatrix}. \quad (A5) $$

The angles $\varphi_1$ are distributed uniformly in $[0, 2\pi)$, whereas according to (A1) $P(\vartheta) = \sin(2\vartheta)$ for $\vartheta \in [0, \pi/2]$. Note that the standard element of the volume on the two sphere $dS = \sin \theta d\theta d\varphi$ is written in a rescaled
variable $\theta = 2\vartheta$. Given maximally entangled state corresponds to a single unitary matrix $U_1$ pertaining to $SU(2)$. Thus the 3-D manifold of maximally entangled states has the topology of the hyper-sphere $S^3$. 

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FIG. 10. Discrete model of periodic dynamics (5) (cf. Fig. 8.1, 8.2 of Ref. [11]). Interaction with the environment $\hat{\Lambda}$ transforms the state $\rho_n$ into $\rho'_n$ and then the unitary transformation $U$ maps it into $\rho_{n+1}$.
\[ \frac{\langle E \rangle}{\ln 2} \]

\[ \langle S \rangle \]

\( t_n \)

(a) and (b)