The hadroproduction of direct $J/\Psi$ in the framework of the $k_{\perp}$-factorization approach is studied. The color-singlet contribution is essentially larger than in the collinear approach but is still an order of magnitude below the data. The deficit may be well described by the color octet contribution with the value of the matrix element $\langle 0|G_{s}^{J/\Psi}(3S_{1})|0\rangle$ substantially decreased in comparison with the fits in the collinear factorization. This should lead to a reduction of the large transverse polarization, predicted in the collinear approach.

Recently we have considered the $\chi_{c}$ hadroproduction within the $k_{\perp}$ factorization [1]. The crucial element of the description of this process was the effective vertex for the production of $q\bar{q}$ pairs with finite invariant mass, which appeared in calculations of the next-to-leading order (NLO) corrections to the Balitsky-Fadin-Kuraev-Lipatov (BFKL) kernel [2].

The effective $q\bar{q}$-production vertex contains - apart from the standard term - also additional terms describing the $q\bar{q}$ production by means of the Reggeon-Reggeon-gluon vertex [3]. This additional part does not contribute, if the $q\bar{q}$ pair is produced in the color-singlet (CS) state. For color-octet $q\bar{q}$ states these additional terms contribute and lead to a $p_{\perp}$ dependence which is very different from the experimentally observed one. Thus we concluded that for $\chi_{c}$ hadroproduction our result suggest that the color octet mechanism (COM) is negligible. Specifically, a dominant color-singlet term and $k_{\perp}$-factorization provide a fair description of the data.

It is interesting to extend this approach to direct $J/\Psi$ production, which is known to be a long-standing puzzle (for review see e.g. [4]).

Because of the negative charge parity of the $J/\Psi$ the effective $q\bar{q}$ vertex can give only contribution via the COM. In the case of $J/\Psi$ production in the CS model, one needs to know the $q\bar{q}$ production vertex with an additional produced gluon (fig. 1). Such a $q\bar{q}g$ system having a finite invariant mass is called a cluster [2]. In the context of the BFKL approach this vertex corresponds to NNLLA corrections and is unknown at the moment. However, the unknown terms in this vertex (e.g. see figs. 1 c, 1 d and 1 e) should not contribute to the production of the CS state, in complete analogy with the $\chi_{c}$ case mentioned above [1]. The unknown part of the NNLLA vertex can include only diagrams where the additional gluon is emitted by the $t$-channel or the $s$-channel gluons. To produce a $J/\Psi$ in the CS model to LO in $\alpha_{S}$ three gluons have to be involved, one of them being emitted. Only those graphs contribute, in which the emitted gluon couples to a quark line (see fig. 1 a and b).

The emission vertex of this gluon is described by the usual vertex since the interaction of partons forming cluster with finite invariant mass is governed by usual QCD lagrangian [5]. Note that the colliding $t$-channel gluons are off-shell and longitudinally polarized.

FIG. 1.

In contrast to this the color-octet contribution requires the full effective NLLA vertex, as mentioned above. It is instructive to compare this situation with the standard collinear factorization case [7,8]. In that case, the emission of an additional hard gluon is required to balance the large transverse momentum of $J/\Psi$. The role of the color-octet states is to allow the fragmentation of high-$p_{\perp}$ gluons to $J/\Psi$.

In our case, the transverse momentum of colliding gluons allows for the production of high-$p_{\perp}$ color-octet state without emission of an additional hard gluon. The role of the gluon fragmentation (the production of $J/\Psi$ by a single gluon) is now played by the discussed additional term within the effective gluon vertex.

One should note the different role of singlet and octet contribution in comparison with $\chi_{c}$ production. While for $\chi_{c}$ the singlet and octet contributions appear from the same hard scattering subprocess, the situation is different for $J/\Psi$. In this case the CS subprocess requires an
are denoted by \( J/\Psi \) and \( \chi \) production than for \( \chi \_c \) production. This point of view is confirmed by our results.

The cross section for heavy quarkonium hadroproduction with an additional produced gluon in the \( k\_\perp \) -factorization approach is [9,10], [11]

\[
\sigma_{P_1 P_2 \rightarrow \Psi g x} = \frac{1}{16(2\pi)^4} \int \frac{d^3P}{P^+} \frac{d^3k}{k^+} \frac{d^2q_1}{q\_1\perp} d^2q_2 \frac{1}{(q\_1^2)^2} \left\{ \frac{\psi_{J/\Psi g}(x_1, q\_1\perp)}{(N_C - 1)^2} \right\} \frac{1}{(q\_2^2)^2} F(x_2, q_2\perp).
\]

The momenta of the \( J/\Psi \) and the produced gluon are denoted by \( P \) and \( k \). \( F(x_2, q_2\perp) \) is the unintegrated gluon distribution. The production amplitude \( \psi_{J/\Psi g}(x_1, x_2, q\_1\perp, q\_2\perp, P, k) \) is factorized in a hard part which describes the production of the quark-antiquark pair and the gluon and an amplitude describing the binding of this pair into a physical charmonium state. The explicit form of the hard part of the amplitude is given by the formulas from [1] supplemented with an additional gluon production vertex (figs. 1 a, 1b), and the formalisms used to describe the binding of the quark-antiquark pair into a bound state as well as the unintegrated gluon distribution are the same as in [1].

\[ \langle \sigma \rangle \sim \frac{1}{\alpha_s^2} \frac{1}{\langle \sigma \rangle_{\text{LO}}} \]

Our results for the direct \( J/\Psi \) production cross section are shown in fig.2 in comparison to the Tevatron data [6] and the color singlet NLO QCD calculations in collinear factorization [7,8]. Although the color singlet contribution for \( k\_\perp \) factorization is substantially larger than the corresponding part in collinear factorization it still lies about a factor of 10 below the experimental results. In contrast to \( \chi_c \) production this leaves room for the color octet mechanism. Since in \( k\_\perp \) factorization there is no general need for an additional outgoing perturbative gluon in order to have some non-vanishing \( \sigma_{\text{collinear}} \), we include the color octet contribution using the NLLA effective \( q\_\perp \) production vertex.

A fit of the uncalculable color octet parameters \( \langle 0|O_8^{J/\Psi}(3S_1)|0 \rangle \), \( \langle 0|O_8^{J/\Psi}(3S_1)|0 \rangle \) and \( \langle 0|O_8^{J/\Psi}(3P_1)|0 \rangle \) which contribute in the lowest order of the NRQCD velocity expansion gives a good description of the data (see figure 2). As in the \( \chi_c \) production case the \( 3S_0^S \) part is smaller than for collinear factorization. This is again due to the flatter slope of the contribution which comes from the additional parts of the 3-gluon-vertex in the effective NLLA \( q\_\perp \) production vertex. Since the slopes of the \( 1S_0^S \) and \( 3P_0^S \) parts are very similar the fit cannot distinguish between these two, therefore only a certain linear combination

\[
M_8 = \frac{\langle 0|O_8^{J/\Psi}(3P_1)|0 \rangle}{m_c^2} + \frac{\langle 0|O_8^{J/\Psi}(3S_1)|0 \rangle}{R}
\]

of these parameters can be extracted [7,8]. In our case the factor \( R \) lies between 6 (low \( P_L \)) and 4.5 (high \( P_L \)). Taking only the \( 1S_0^S \) or the \( 3P_0^S \) contributions we obtain two slightly different results (for the color singlet contribution we take parameters from [12]):

<table>
<thead>
<tr>
<th>used in fit</th>
<th>( 1S_0^S )</th>
<th>( 3P_0^S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle 0</td>
<td>O_8^{J/\Psi}(3S_1)</td>
<td>0 \rangle )</td>
</tr>
<tr>
<td>( M_8 ) (( R = 5 ))</td>
<td>( 1.4(\pm 0.1) \cdot 10^{-2} )</td>
<td>( 1.2(\pm 0.1) \cdot 10^{-2} )</td>
</tr>
<tr>
<td>total ( \chi^2/\text{NDOF} )</td>
<td>0.42</td>
<td>0.53</td>
</tr>
</tbody>
</table>

The errors are only statistical.

On the basis of these results we conclude that it is impossible to describe the data for direct \( J/\Psi \) production entirely by the CS contribution, contrary to the case of \( \chi_c \). In our case the discrepancy between the CS contribution and the data is substantially smaller than for the NLO collinear factorization.

The results allow for a good description in the framework of the COM. We performed new fits of CO matrix elements. As a result, the value of \( \langle 0|O_8^{J/\Psi}(3S_1)|0 \rangle \) is reduced by a factor of \( \approx 30 \) in comparison with the analysis in the framework of collinear factorization. However, if this matrix element is put exactly to zero, the quality of the fit is much worse.

At the same time, the values of the matrix elements \( \langle 0|O_8^{J/\Psi}(1S_0)|0 \rangle \) and \( \langle 0|O_8^{J/\Psi}(3P_1)|0 \rangle \) stay about the same as in the collinear approximation. These matrix elements provide a dominant contribution to \( J/\Psi \) lepto-production, so that the longstanding problem of simultaneous description of lepto- and hadroproduction [13]...
probably persists also in $k_\perp$-factorization. This is the topic of a separate study.

The smallness of the color octet matrix element $\langle 0 | O^c_8 | (3^1S_1) | 0 \rangle$ is, however, quite promising from the point of view of $J/\Psi$ polarization. Please recall, that in the collinear factorization approach this matrix element provides the dominant contribution to the cross-section through the gluon fragmentation subprocess. As soon as the gluon is almost on-shell, it has a strong transverse polarization which should be transferred to the transverse polarization of $J/\Psi$, in disagreement with the experimental data [14,15]. Contrary to that, in our approach this fragmentation mechanism is substituted by the extra term in the effective vertex, whose convolution with the matrix element results in a wrong $P_\perp$ dependence and therefore this matrix element has to be small. Thus, the qualitative origin of the transverse polarization in the collinear approach is absent in the $k_\perp$-factorization approach. One should consistently calculate the polarization in the $k_\perp$-factorization approach to obtain quantitative statements. This study is in progress.

While this paper was finished we noticed an article by F. Yuan and K.-T. Chao [16] who studied $J/\Psi$ and $\Psi'$ production in the $k_\perp$ factorization approach in the CS model. They got very similar numerical results.

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