Theories in More than Four Dimensions*

Abdel Pérez-Lorenzana†

Department of Physics, University of Maryland, College Park, Maryland 20742, USA
Departamento de Física, Centro de Investigación y de Estudios Avanzados del I.P.N.
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Abstract

Particle physics models where there are large hidden extra dimensions are currently on the focus of an intense activity. The main reason is that these large extra dimensions may come with a TeV scale for quantum gravity (or string theory) which leads to a plethora of new observable phenomena in colliders as well in other areas of particle physics. Those new dimensions could be as large as millimeters implying deviations of the Newton’s law of gravity at these scales. Intending to provide a basic introduction to this fast developing area, we present a general overview of theories with large extra dimensions. We center our discussion on models for neutrino masses, high dimensional extensions of the Standard Model and gauge coupling unification. We discuss the recently proposed technic of splitting fermion wave functions on a tick brane which may solve the problem of a fast proton decay and produce fermion mass hierarchies without invoking extra global symmetries. Randall-Sundrum model and some current trends are also commented.

I. INTRODUCTION

New extra dimensions beyond the four of our world could possible exist in Nature. This idea is as old as Kaluza and Klein’s work dating back to the 1920's [1]. In modern terms, the idea arose again with the advent of string theories. However, it was conventional to assume that such extra dimensions were compactified to manifolds of small radii with a size of the order of the inverse Planck scale, \( \ell_P = M_P^{-1} = G_N^{1/2} \) or so, such that they would remain hidden to low energy physics considerations. Thus, beyond the interest of a small


† email: aplorenz@glue.umd.edu
community, the study of theories with extra dimensions was almost far away from the scope of many particle physicists.

During the last two years of the XX century, the work on theories in more than four dimensions has increased almost exponentially. The intriguing fact that strongly motivates this renewed interest is the realization of the possibility that extra dimensions as large as millimeters [2] could exist and yet being hidden to the experiments [3–7], but with new effects not so far of being observed. Among the experimental signatures are deviation on the gravitational Newton’s law at small distances and a rich phenomenology for collider physics. One of the attractive features of these theories is that they may provide a natural solution to the hierarchy problem. In this scenarios, it is believed that we live on a hypersurface (3-brane) embedded in a higher dimensional world (the bulk). Although it is fair to say that similar ideas were proposed on the 80’s by several authors [8], they were missed by some time, until recent developments on string theory provided an independent realization of such models [9–11], given them certain credibility.

Our goal for the present notes is to give a general overview of this field. Among the scenarios presented until now in the literature, we will focus on studying mainly the case of large (millimetric) extra dimensions. This, in turn, will provide us the insight to extend our study to other more elaborated models.

To motivate the ideas we will start with the discussion of the origin of the long standing hierarchy problem and how extra dimensions offer a new way to understand it by providing a new low fundamental scale, \( M \), which, based on the phenomenological and experimental constrains, could be just at the TeV range. At this point we shall assume that the Standard Model particles are attached to the brane and that only gravity propagates on the bulk. As we will see, in such a theory the weakness of gravity is related to the large size of the volume of the extra space.

As it is clear, with a low fundamental scale all the particle physics phenomena that invoke high energy scales will not work any more. Then, problems as neutrino masses and mixings, gauge coupling unification and proton decay should be reviewed under the light of this new theories. This will be the central issue along our discussion, and in order to address it we will modify the above model accordingly.

First, after discussing some general aspects of the brane-bulk theories, we will explore neutrino oscillations phenomena. Here, we shall show how an isosinglet bulk neutrino that couples to the standard neutrinos may help to solve the neutrino puzzles [12–22]. Next we will analize the possibility that the standard model particles propagate in the extra dimensions and develop Kaluza-Klein excitations [23–26]. The contribution of this exited modes to standard processes will set bounds to the bulk radius and provide collider signatures [7]. Those modes will also modify the profile of the renormalization group equations that govern the running of the gauge coupling constants. The net effect is a power law running that accelerates their meeting, which now may occur at very low energies [26–32], even at the TeV scale. We comment on the accuracy of this low energy gauge coupling unification.

Next, we shall discuss a slightly modified scenario that may account for the explanation of fermion mass hierarchies and the suppression of proton decay by introducing a splitting of the wave functions along the extra dimension [33,34]. Finally, we will present the Randall-Sundrum model [35] which provides an alternative explanation for the hierarchy problem based on small extra dimensions and a non factorizable bulk geometry. We will also mention
some of the current trends on the study of this class of models [36–45].

II. HIERARCHY PROBLEM AND LARGE EXTRA DIMENSIONS

A. Radiative corrections and the Higgs mass

Our starting point is the scalar sector of the Standard Model (SM). This is perhaps the most peculiar sector in the theory. The scalar Higgs field, $H$, realizes the spontaneous symmetry breaking and gives masses to all other particles, fermions and gauge bosons, and yet, its own mass is introduced as a free parameter on the theory. The Higgs is the only particle of the SM which remains to be observed. It is also the only field that have self interactions that are only constrained by gauge invariance and the renormalizability condition which leave a $\lambda H^4$ term, with $\lambda$ a free parameter. And, what is more important for us, it is the only field for which the quantum corrections require a large fine tuning on the mass parameter. While the self energy diagrams of all other fields, evaluated in the $\overline{MS}$ scheme, develop logarithmic divergences that depend on their own bare mass, the scalar field develops quadratic divergences that are independent of its bare mass. For instance, at one loop order one gets

$$\delta m_H^2 = \frac{1}{8\pi^2} \left( \lambda_H^2 - \lambda_F^2 \right) \Lambda^2 + (\text{log. div.}) + \text{finite terms.} \quad (2.1)$$

where $\lambda_H$ is the self-couplings of $H$, $\lambda_F$ is the Yukawa coupling to fermions, and $\Lambda$ is the physical cut-off of the theory, which is usually believed to be the Planck scale, $M_{Pl} \sim 10^{19}$ GeV, or the GUT scale, $M_{GUT} \sim 10^{16}$ GeV.

Nevertheless, in order to keep the $WW$ scattering cross section from violating unitarity, the physical Higgs boson mass, $m_H$, must be less than about 1 TeV. Then we get the unpleasant result, $m_H^2 = m_{H,0}^2 + \delta m_H^2 + \text{counterterm}$, where the counterterm must be adjusted to a precision of roughly 1 part in $10^{15}$ in order to cancel the quadratically divergent contributions to $\delta m_H^2$. Moreover, this adjustment must be made at each order in perturbation theory. This large fine tuning is what is known as the hierarchy problem.

Of course, the quadratic divergence can be renormalized away in exactly the same manner as is done for logarithmic divergences, and in principle, there is nothing formally wrong with this fine tuning. In fact if this calculation is performed in the dimensional regularization scheme, $\overline{DR}$, one obtains only $1/\epsilon$ singularities which are absorbed into the definitions of the counterterms, as usual. Hence, the problem of quadratic divergences does not become apparent there. It arises only when one attempts to give a physical significance to the cut-off $\Lambda$. In other words, if the SM were a fundamental theory then the using of $\overline{DR}$ would be justified. However, most theorists believe that the final theory should also include gravity, then a cut-off must be introduced in the SM. Hence, we regard this fine tuning as unattractive.

B. A low fundamental scale from new dimensions

Explaining the hierarchy problem has been a leading motivations to explore new physics during the last twenty years. Supersymmetry [46] and compositeness [47] are two of the main
proposals that solve this problem. Supersymmetry predicts the existence of new particles, the super-partners, at the TeV scale. They belong to the same dimensional representation than those of the SM, but they differ each other on the spin. Their couplings are symmetric such that their contribution to the quadratic divergences get balanced and nullify each other. This has been, so far, the most popular extension of the SM. Compositeness, on the other hand, assumes that the Higgs is not a fundamental field, but a quantum condensate of other fields. Both theories reflect a common idea on the structure of physics beyond the SM: A new effective field theory will be revealed at the weak scale, \( m_{EW} \sim 1 \text{ TeV} \), stabilizing and perhaps explaining the origin of the hierarchy, \( m_{EW}/M_{P}\ell \), while, a desert among those scales will remain until the Planck scale, which is assumed to be the fundamental scale where gravity becomes as strong as the gauge interactions and where the quantum theory of gravity is revealed. Eventually the desert could be populated by other effective theories, responsible from explaining other phenomena (as fermion masses) or from triggering dynamical symmetry breakings, and a big deal of work has been dedicated to study those pictures.

It has been realized recently that if there are more than four dimensions, as it is suggested by string theory, the above scenario could be drastically modified. To explain this let us assume as in Ref. [2] that there are \( \delta \) extra space-like dimension which are compact. Compactification will be in principle a natural ingredient that would explain why we see only four dimensions. Let us also imaging that all (some) of these dimensions are large with a common radius \( R \). Then, if two test particles with masses \( m_1 \) and \( m_2 \) were separated each other by a distance \( r \gg R \), they would feel the usual gravitational potential

\[
U(r) = G_N \frac{m_1 m_2}{r}.
\]  

(2.2)

However, if \( r \ll R \), the potential between each other should be

\[
U(r) = G \frac{m_1 m_2}{r^{\delta+1}},
\]  

(2.3)

where \( G^{-1} = M^{\delta+2} \), is the coupling constant of gravity in \( \delta + 4 \) dimensions which defines the fundamental scale \( M \) where gravity becomes strong. Therefore, if we take the last relationship as the fundamental one, then Eq. (2.2) will imply that our effective four dimensional gravity scale is given by [2]

\[
M_{P\ell}^2 = M^{\delta+2} V_\delta,
\]  

(2.4)

where \( V_\delta \) is the volume of the extra space. It is worth saying that this same relationship arises if we dimensional reduce to four dimensions the gravity action in \( 4 + \delta \) dimensions, assuming the space-time to be \( \mathbb{R}^4 \times M_\delta \), where \( M_\delta \) is an \( \delta \) dimensional compact manifold of volume \( V_\delta \). If the volume were large enough, then the fundamental scale could be as low as \( m_{EW} \), and the hierarchy would be naturally removed. Of course, the price one has to pay is to explain why the extra dimensions are so large. For \( \delta = 2 \), we get for \( M \sim 1 \text{ TeV} \) that \( R \) is less than 1 millimeter. This case is highly interesting since it is on the current limits on the low distance gravity experiments [3] that should detect the predicted deviation of the inverse squared law of gravity at distances, \( r \sim V_\delta^{1/\delta} \). Nevertheless, more than two extra dimensions could be expected (strings predicts six more), and their size do not have to be
the same. More complex scenarios with a hierarchical distributions of the sizes could be natural. Then, for the investigation of the model, we will use a single large extra dimension, implicitly assuming other smaller dimensions.

Now well, while submillimeter dimensions remain untested for gravity, the SM gauge forces have certainly been accurately measured up to weak scale distances. Therefore, the SM particles can not freely propagate in these large extra dimensions, but must be constrained to a four dimensional submanifold. Then the scenario we have in mind is one where we live in a four dimensional surface embedded in a higher dimensional space. This picture is similar to the D-brane models [48], as in the Horava-Witten theory [10]. We may also imagine our world as a domain wall of size \( M^{-1} \) where the SM fields are trapped by some dynamical mechanism [2]. As this framework solves the hierarchy problem, supersymmetry is no longer needed. However, we should notice that it may still be crucial for the self-consistency of the theory of quantum gravity above the \( M \) scale, although such theory is yet unknown. It could be superstring theory, but it may also be something yet to be discovered. In any case, supersymmetry may coexist with large extra dimensions.

C. Experimental bounds

There are a number of dramatic experimental consequences of large extra dimensions. First, as already mentioned, there is the deviation of the inverse squared law on gravity at submillimeter distances. The current experiments are just at this limit, so \( R < 0.2 \) mm [3]. Also, as gravity becomes comparable in strength to the gauge interactions at energies \( M \sim \) TeV, the nature of the quantum theory of gravity would become accessible to LHC and NLC. With gravity freely propagating on the bulk, the effect of the gravitational couplings will be mostly of two types: missing energy, that goes into the bulk, and corrections to the standard cross sections from graviton exchange. The Feynman rules were calculated from the linearized bulk gravity theory, and are given in Ref. [4]. From the effective four dimensional point of view, the graviton develops Kaluza Klein (KK) excitations of masses \( n/R \), with \( n \) an integer number (see next section). The coupling of each one of those modes to the SM particles is suppressed by the Planck scale, but the overall coupling that considers all the KK excitations become suppressed just by \( M \).

A long number of studies on this topic have appeared already [4,5], some nice and short reviews of collider signatures are given in [49]. We summarize some of the current bounds in tables 1 and 2. At \( e^+e^- \) colliders (LEP,LEPII, L3), the best signals would be the production of gravitons with \( Z, \gamma \) or fermion pairs \( \bar{f}f \). There is also the interesting monojet production [4] at hadron colliders (CDF, LHC) which is yet untested.

The virtual exchange of gravitons either leads to modifications of the SM cross sections and asymmetries, or to new processes not allowed in the SM at tree level. The amplitude for exchange of the entire tower naively diverges when \( \delta > 1 \) and has to be regularized, typically by a cut off at \( M \), which performs the replacement of the sum suppressed by \( M_{Pl} \) into the simple suppression by \( M \). An interesting channel is \( \gamma\gamma \) scattering, which appears at tree level, and may surpasses the SM background at \( s = 0.5 \) TeV for \( M = 4 \) TeV. Bi-boson productions of \( \gamma\gamma, WW \) and \( ZZ \) has been already analized [4,5]. Some experimental limits, most of them based on existing data, are given in Table 2. The upcoming experiments will easily overpass those limits.
Notice that all collider limits are about 1 TeV. A more stringent constraint comes from SN1987A and astrophysics [6], which gives $M > 30 - 100$ TeV.

### III. AN INTRODUCTION TO BRANE-BULK MODELS

Before getting into the discussion of models with large extra dimensions in more detail, let us make some useful remarks on the generical properties of brane-bulk models.

As we already mentioned, from the four dimensional effective theory point of view, a bulk field, as the graviton, will appear as an infinite tower of KK excitations. The appreciation of the impact of this KK excitations will depend in the relevant energy of the experiment, and on the compactification scale $\frac{1}{R}$. Roughly speaking, if $E \ll \frac{1}{R}$ the theory behaves purely four dimensional. However, for energies above $\frac{1}{R}$, a large number of KK excitations, $\sim (ER)^{\delta}$, becomes kinematically accessible making physics looks $4 + \delta$ dimensional. To clarify this matters let us consider a five dimensional model where the fifth dimension has been compactified on a circle of circumference $2\pi R$. The generalization of these results is straightforward. Let $\phi$ be a bulk scalar field for which the action has the form

$$S_{\phi} = \frac{1}{2} \int d^4x \, dy \left( \partial^A \phi \partial_A \phi - m^2 \phi^2 \right);$$

where $A = 1, \ldots, 5$, and $y$ denotes the fifth dimension. Demanding periodicity on the extra compact dimension under $y \rightarrow y + 2\pi R$, the field may be Fourier expanded as

$$\phi(x, y) = \frac{1}{\sqrt{2\pi R}} \phi_0(x) + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \left[ \phi_n(x) \cos \left( \frac{ny}{R} \right) + \phi^*_n(x) \sin \left( \frac{ny}{R} \right) \right].$$

Notice the different normalization of the excited modes, $\phi_n$ and $\hat{\phi}_n$, with respect to the zero mode, $\phi_0$. By introducing the last equation into the action and integrating over the extra dimension we get

$$S_{\phi} = \sum_{n=0}^{\infty} \frac{1}{2} \int d^4x \left( \partial^\mu \phi_n \partial_\mu \phi_n - m_n^2 \phi_n^2 \right) + \sum_{n=1}^{\infty} \frac{1}{2} \int d^4x \left( \partial^\mu \hat{\phi}_n \partial_\mu \hat{\phi}_n - m_n^2 \hat{\phi}_n^2 \right);$$

where the KK mass is given as $m_n^2 = m^2 + \frac{n^2}{R^2}$. Therefore, in the effective theory, the higher dimensional field looks like an infinite tower of fields with masses $m_n$. These are the so called KK modes. Notice that all these modes are fields with the same spin, and quantum numbers as $\phi$. But they differ in the KK number $n$, associated with the fifth component of the momentum. For $m = 0$ it is clear that for energies below $\frac{1}{R}$ only the massless zero mode will be kinematically accessible, making the theory looks four dimensional. As the energy increases, once it surpasses the threshold of the first exited level, the manifestation of the KK modes will evidence the higher dimensional nature of the theory.

The five dimensional field $\phi$ has mass dimension $\frac{3}{2}$, while all modes are dimension one. In general for $\delta$ extra dimensions we will get $[\phi] = d_4 + \frac{\delta}{2}$, where $d_4$ is the natural mass dimension of the field in four dimensions. Because this change on the dimensionality of $\phi$, the higher order operators (beyond the mass term) will all have dimensionful couplings. To keep them dimensionless a mass parameter should be introduced to correct the dimensions.
As usual, the natural choice for this parameter is the cut-off of the theory. For instance, let us consider the quartic couplings of $\phi$. Since all potential terms should be of dimension five, we should write down $\frac{1}{M} \phi^4$. After integrating the fifth dimension, this operator will generate quartic couplings among all KK modes. Four normalization factors containing $1/\sqrt{R}$ appear in the expansion of $\phi^4$. Two of them will be removed by the integration, thus, we left with the effective coupling $\lambda/MR$. By introducing Eq. (2.4) we observe that the effective couplings have the form

$$\lambda \left( \frac{M}{M_{Pl}} \right)^2 \phi_k \phi_l \phi_m \phi_{k+l+m}.$$  

(3.4)

where the indices are arranged to respect the conservation of the fifth momentum. From the last expression we conclude that in the low energy theory ($E < M$), the effective coupling appears suppressed. Thus, the effective four dimensional theory is weaker interacting compared with the bulk theory. Something similar happens to gravity on the bulk, where the coupling constant is stronger than in the brane, because it is also suppressed by the volume of the extra space as given in Eq. (2.4). By the naturalness principle, we must assume that all dimensionless coupling constants are of order one.

Let us now consider the brane fields represented by a fermion $\psi(x)$. The theory describing this brane fermion is purely four dimensional. Indeed, its action is localized by a delta function in the complete theory in the form

$$S_\psi = \int d^4x \ d y \mathcal{L}(\psi) \delta(y - y_0),$$  

(3.5)

where $y_0$ is the position of the brane. The coupling of $\psi$ to the bulk scalar field only may take place at the brane, where $\psi$ lives. For simplicity we will assume that the brane is located at the position $y_0 = 0$, which in the case of orbifolds corresponds to a fixed point. So, the part of the action that describes the brane-bulk coupling is

$$S_{int} = \int d^4x \ d y \mathcal{L}_{int}(\phi, \psi) \delta(y).$$  

(3.6)

Let's choose for instance the term

$$S_{\phi\psi} = \int d^4x \ d y \frac{f}{\sqrt{M}} \bar{\psi}(x)\psi(x)\phi(x, y = 0) \delta(y)$$

$$= \int d^4x \frac{M}{M_{Pl}} f \cdot \bar{\psi} \phi_0 + \sqrt{2} \sum_{n=1}^{\infty} \phi_n.$$

(3.7)

Here the Yukawa coupling constant $f$ is dimensionless. On the right hand side we have used the expansion (3.2) and Eq. (2.4). From here, we notice that the coupling of brane to bulk fields is generically suppressed by the ratio $\frac{M}{M_{Pl}}$. Also, notice that the modes $\phi_n$ decouple from the brane. Then, to get rid of those (harmless) fields in the effective theory we could assume that $\phi$ is an even field under the transformation $y \rightarrow -y$, thus, imposing a $Z_2$ symmetry on the theory, though the theory is consistent even without this extra assumption. That symmetry, on the other hand, is characteristic of the orbifolds and domain walls. By picking up an explicit parity for $\phi$ we are projecting out one half of the KK excitations, it means, the expansion (3.2) will involve either the cosine or the sine part, but not both.
Let us stress that the couplings in (3.7) do not conserve the KK number. This reflects the fact that the brane breaks the translational symmetry along the extra dimension. Nevertheless, it is worth noticing that the four dimensional theory is still Lorentz invariant. Physically this means that when the interactions among the brane fields reach enough energy to produce real emission of KK modes, part of the energy of the brane is released into the bulk. This is also the case of gravity.

Let us mention, parenthetically, that in general the linear perturbations of the metric lead to particles of spin two, one and zero; however, only the spin two graviton, $G_{\mu\nu}$, and one scalar field, $b$, the radion; couple to matter at the weak field limit. The Feynman rules are given in [4]. Briefly speaking, they come from the couplings to the energy momentum tensor

$$ \mathcal{L} = -\frac{1}{M_{pl}} \sum_n \left[ G^{(n)\mu\nu} - \frac{\kappa}{3} b^{(n)} \eta^{\mu\nu} \right] T_{\mu\nu}. $$

(3.8)

Here $\kappa$ is a parameter of order one. Notice that $G^{(0)\mu\nu}$ is massless. That is the source of long range four dimensional gravity. It is worth saying that $b^{(0)}$ is not actually massless, it get a mass from the stabilization mechanism. From supernova constrains such a mass should be larger than $10^{-3}$ eV. Experimental bounds for graviton production were given in the previous section.

Next, let us consider the scattering process among brane fermions: $\psi\psi \to \psi\psi$. In the present toy model this interaction is mediated by all the KK excitations of $\phi$, then the typical amplitude will receive the contribution

$$ \mathcal{M} = \hat{f}^2 \left( \frac{1}{q^2 - m^2} + 2 \sum_{n=1}^{N} \frac{1}{q^2 - m^2_n} \right) D(q^2), $$

(3.9)

where $\hat{f}$ represents the effective coupling, and $D(q^2)$ is an operator that depends on the Feynman rules and is usually independent of the index $n$, as a consequence of the universal coupling manifested on Eq. (3.7). In more than five dimensions the equivalent to the above sum usually diverges and has to be regularized by introducing a cut-off at the fundamental scale. Roughly speaking, at high energies, $qR \gg 1$, the overall factor becomes $\hat{f}^2 N$, where $N$ is the number of KK modes until the cut-off. It is $N = MR = M_{pl}^2 / M^2$. In the case of the graviton, $\hat{f}$ is about $1/M_{pl}$, and therefore, the overall coupling becomes just $1/M$ [4,5]. At low energies, on the other hand, by assuming that $q^2 \ll m^2 \ll 1/R^2$ we may integrate out the KK excitations, and at the first order we get

$$ \mathcal{M} = \frac{\hat{f}^2}{m^2} D(q^2) \left( 1 + \frac{\pi^2}{3} m^2 R^2 \right). $$

(3.10)

The last term between parenthesis is a typical correction produced by the KK modes exchange to the pure four dimensional result.

Let us now mention on some characteristics of other bulk fields. A massless bulk fermion field is defined as the solution of the higher dimensional Dirac Equation

$$ (i\partial^M \Gamma_M) \Psi = 0, $$

where the $\delta + 4$ Dirac matrices satisfy the algebra

$$ \{ \Gamma_M, \Gamma_N \} = 2g_{MN}. $$

In five dimensions we use the Weyl basis
\[
\Gamma_\mu = \gamma_\mu = \left(\begin{array}{c}
0 \\
\bar{\sigma}_\mu \\
0
\end{array}\right); \quad \text{and} \quad \Gamma_5 = \gamma_5 = -i \left(\begin{array}{cc}
1 & 0 \\
0 & -1
\end{array}\right). \tag{3.11}
\]

Therefore, a bulk fermion is necessarily a four component spinor. Thus, only vectorlike
theories are in principle possible. However, if the fifth dimension is orbifolded, the theory
on the brane may look as a chiral theory. In this basis, \(\Psi\) is conveniently decomposed as
\[
\Psi = \left(\begin{array}{c}
\nu_R \\
\nu_L
\end{array}\right); \tag{3.12}
\]
with each component having its own set of KK excitations. The free action for the massless
field may be expanded in terms of the KK modes, and it has the form
\[
S_\Psi = \int d^4x \ d y \ i \bar{\Psi} \Gamma_M \partial_M \Psi
= \int d^4x \ \left(i \bar{\nu}_L \bar{\sigma}^\mu \partial_\mu \nu_L + i \bar{\nu}_R \bar{\sigma}^\mu \partial_\mu \nu_R - \frac{n}{R} \bar{\nu}_R \nu_L + h.c\right); \tag{3.13}
\]
where a sum over \(n\) is to be understood. Notice that all KK masses are Dirac masses.

Two different Lorentz invariant fermion bilinears are possible in five dimensions: Dirac
mass terms \(\bar{\Psi} \Psi\) and Majorana masses \(\bar{\Psi} C_5 \Psi\), where \(C_5 = \gamma^0 \gamma^2 \gamma^5\). However, the Dirac
mass is an odd function under the orbifold symmetry, \(y \rightarrow -y\), under which \(\Psi \rightarrow \gamma^5 \Psi\). So,
if the theory is invariant under this symmetry that term will be zero.

Lets now consider the extension of a gauge field to five dimensions. For simplicity let us
consider only the case of a free gauge abelian theory. The Lagrangian in five dimensions is
given as
\[
L_{5D} = -\frac{1}{4} F_{MN} F^{MN}. \tag{3.14}
\]
Upon integration over the extra dimension one gets \[26\]
\[
L = -\frac{1}{4} \sum_{n=0}^{\infty} F_{\mu\nu}^{(n)} F^{(n)\mu\nu} - \frac{1}{2} \sum_{n=0}^{\infty} \left[ \partial_\mu A_5^{(n)} + \frac{n}{R} A_\mu^{(n)} \right]^2 \tag{3.15}
\]
The gauge invariance of the theory can be now expressed in terms of the (expanded) gauge
transformation of the KK modes
\[
A_\mu^{(n)} \rightarrow A_\mu^{(n)} + \partial_\mu \theta^{(n)} \quad \text{and} \quad A_5^{(n)} \rightarrow A_5^{(n)} - \frac{n}{R} \theta^{(n)}. \tag{3.16}
\]
Therefore, by fixing the gauge one may absorb the scalar field \(A_5\) into \(A_\mu\). Hence, the only
massless vector is the zero mode, all KK modes acquired mass by absorbing the Goldston
bosons, \(A_5^{(n)}\), associated to the spontaneous isometry breaking \[50\]. That keeps the gauge
symmetry on the four dimensional effective theory untouched. We must stress that this new
theory is essentially non renormalizable for the infinite number of fields that it involves.
However, the truncated theory that only consider a finite number of KK modes is renormal-
izable. The cut-off for the inclusion of the excited modes will be again the scale \(M\). Other
aspects of this theories will be discussed later on when we addressed the SM extensions.
IV. NEUTRINO MASSES

Several experiments have provided conclusive evidence for a deficit on the expected flux of atmospheric and solar neutrinos, and there is also the direct observation of $\bar{\nu}_e$ in the $\bar{\nu}_\mu$ beam of the LSND experiment. A simple explanation of these anomalies arises if neutrinos are massive and a large amount of work is devoted nowadays to explain the origin of their small squared mass differences [12]. However, most of the four dimensional mechanisms invoke high energy physics with scales about $10^{12}$ GeV or higher. Obviously, with a fundamental TeV scale, understanding the small neutrino masses poses a theoretical challenge to the new theories.

A second possible problem is the enhancement of all non renormalizable operators, now suppressed only by powers of $1/M$. Among them, there are those which produce a dangerous fast proton decay and the operator

$$\frac{(LH)^2}{M}$$

which produces a large neutrino mass of the order $\langle H \rangle^2 / M$. To exclude this operator one has to make the additional assumption that the theory respect lepton number symmetry (or more precisely $B - L$). Two class of models are then possible depending on whether $B - L$ is a global or local symmetry. We discuss both possibilities in this section.

A. Models with global $B - L$ symmetry

In the context of models that have a global $U(1)_{B - L}$ symmetry, one can get small neutrino masses by introducing isosinglet neutrinos in the bulk [13] which carry lepton number. As this is a sterile neutrino, it comes natural to assume that it may propagate into the bulk as well as gravity, while the SM particles remain attached to the brane. These models are interesting since they lead to small neutrino masses without any extra assumptions.

Let $\nu_B(x^\mu, y)$ be a bulk neutrino which we take to be massless since the Majorana mass violates conservation of Lepton number and the five dimensional Dirac mass is forbidden by the orbifold symmetry, which we assume. This neutrino couples to the standard lepton doublet, $L$, and to the Higgs field, $H$, via $\frac{h}{\sqrt{M}} LH \nu_{BR} \delta(y)$. Once the Higgs develops its vacuum, this coupling will generate the four dimensional Dirac mass terms

$$m\bar{\nu}_L \left( \nu_{0R} + \sqrt{2} \sum_{n=1}^{\infty} \nu_{nR} \right),$$

where the mass $m$ is given by [14]

$$m = h\nu \frac{M}{M_P} \sim 10^{-4} \text{eV} \times \frac{hM}{1 \text{TeV}}.$$  

Therefore, if $M \sim 1 \text{ TeV}$ we get just the right order of magnitude on the mass as required by the experiments. Moreover, even if the KK decouple for a small $R$, we will still get the same Dirac mass for $\nu_L$ and $\nu_{0R}$, as far as $M$ remains in the $\text{TeV}$ range. After including the KK masses from Eq. (3.13), we may write down all mass terms in the compact form [15]
\[
\begin{pmatrix}
\bar{\nu}_{eL}v_{\nu}^B \\
\end{pmatrix}
\begin{pmatrix}
m \\
0 \\
\end{pmatrix}
\begin{pmatrix}
\sqrt{2}m \\
\partial_5 \\
\end{pmatrix}
\begin{pmatrix}
\nu_0 \\
v_B \\
\end{pmatrix},
\tag{4.4}
\]

where the notation is as follows: \(\nu_0^B\) represents the KK excitations. The off diagonal term \(\sqrt{2}m\) is actually an infinite row vector of the form \(\sqrt{2}m(1,1,\cdots)\) and the operator \(\partial_5\) stands for the diagonal and infinite KK mass matrix whose \(n\)-th entrance is given by \(n/R\). Notice that the left handed zero mode, \(\nu_0^{BL}\), has decoupled from the spectra and will remain massless. Using this short hand notation it is straightforward to calculate the exact eigensystem for this mass matrix \([16]\). Simple algebra yields the characteristic equation \(2\lambda_n = \pi \xi^2 \cot(\pi \lambda_n)\), with \(\lambda_n = m_n R\), \(\xi = \sqrt{2}mR\), and where \(m_n\) is the mass eigenvalue \([13,14]\). The weak eigenstate is given in terms of the mass eigenstates, \(\tilde{\nu}_{nL}\), as

\[
\nu_L = \sum_{n=0}^{\infty} \frac{1}{N_n} \tilde{\nu}_{nL},
\tag{4.5}
\]

where the mixing \(N_n\) is given by \(N_n^2 = (\lambda_n^2 + f(\xi))/\xi^2\), with \(f(\xi) = \xi^2/2 + \pi^2 \xi^4/4\) \([16]\). Therefore, \(\nu_L\) is actually a coherent superposition of an infinite number of massive modes. As they evolve differently on time, the above equation will give rise to neutrino oscillations, \(\nu \rightarrow \nu_0^B\), even though there is only one single flavour. This is a totally new effect that arise the possibility of new ways of understanding the neutrino anomalies. An analysis of the implications of the mixing profile in these models for solar neutrino deficit was presented in \([14]\). Implications for atmospheric neutrinos were discussed in \([17]\), and some phenomenological bounds were given in \([17,18]\). A comprehensive analysis for three flavours is given in \([19]\). Here we summarize some of the main results.

\section*{B. New patterns of neutrino oscillations}

\textit{Small \(\xi\).} For \(\xi \ll 1\), the mixing in Eq. (4.5) is given by \(\tan \theta_n \sim \xi/n\) \([14]\). The masses are: \(m\) for \(n = 0\) and \(n/R\) otherwise. Therefore, as expected, the main component of \(\nu_L\) is the lightest mass eigenstate. The mixing, on the other hand, will induce a resonant conversion into bulk (sterile) modes. The survival probability has the form

\[
P_{\text{surv}}(L) = 1 - \frac{4}{\eta^2} \xi^2 \sum_{n=1}^{\infty} \frac{\sin^2 \left(\frac{n^2 L}{4 E R^2}\right)}{n^2} - \frac{2}{\eta^4} \xi^4 \sum_{k,n=1}^{\infty} \frac{\sin^2 \left(\frac{(n^2-k^2)L}{4 E R^2}\right)}{n^2 k^2},
\tag{4.6}
\]

where \(\eta = (1+\pi^2 \xi^2/6)^{1/2}\). Thus, the oscillation length is given by \(L_{\text{osc}} = 4\pi E R^2\). Clearly, at the low \(\xi\) order the above expression becomes \(P_{\text{surv}}(L) \approx 1 - 4\xi^2 \sin^2(L/4ER^2)\), which mimics the former small mixing angle case. Therefore, if \(1/R \sim 10^{-3} \text{ eV}\), which is just \(R \sim 0.2\text{ mm}\), we get an explanation for solar data by introducing MSW effects as described in \([14]\) that is consistent with other astrophysical constrains \([14]\), though it has been suggested that SN1987A may impose more stringent bounds: \(R < 1\text{ Å}\) \([17]\). A similar explanation for the atmospheric anomaly seems disfavored \([14,19]\), since it needs large mixing angle.

\textit{Large \(\xi\).} On the continuos approximation, the survival probability reads \([17]\)

\[
P_{\text{surv}}(z) = \left|1 - \text{erf}\left(\frac{\pi}{2} \xi^2 \sqrt{i z}\right)\right|^2
\tag{4.7}
\]
where $z = L/2ER^2$. Notice that now the probability has no oscillatory nature in terms of $L/E$, this feature should distinguish this models form the conventional two neutrino oscillations. Indeed, it is easy to check the physical origin of this effect from the exact solutions [19]. For $\xi \gtrsim 1$, the eigenvalues $\lambda_n$ start to deviate from the integral value $n$, and a large number ($N\xi \sim \pi^2 \xi^2/4$) of equally suppressed eigenstates contribute to (4.5). Then, once $\nu_L$ is released, the time evolution of the different components will most likely wash out the original coherent superposition and the initial $\nu_L$ will almost disappear, and the maximal probability will not be recovered away from the source [13]. The fast developing slope of $P_{\text{surv}}$ is governed by the single parameter $\xi^2/R$. As solar and atmospheric have $P_{\text{surv}} \approx 0.5$, in order to avoid a large deficit on $P_{\text{surv}}$, one must keep that parameter on a small range. For atmospheric one gets that $\xi^2/R \approx 10^{-2} \text{eV}$ [17,19]. That means $10^{-3} \text{eV} < 1/R < 10^{-2} \text{eV}$, and $1 < \xi^2 < 10$, and then, $R$ should remain in the submillimeter range. An explanation for solar data is, on the other hand, not possible in this limit [19].

C. Three flavour oscillations

The extension of this model to three brane generations, $\nu_{e,\mu,\tau}$, is straightforward. It was observed earlier [16] that to give masses to the three standard generations three bulk neutrinos are needed. This comes out from the fact that with a single bulk neutrino only one massless right handed neutrino is present (the zero mode), then, the coupling to brane fields will generate only one new massive Dirac neutrino. After introducing a rotation by an unitary matrix $U$ on the weak sector, the most general Dirac mass terms with three flavours and arbitrary Yukawa couplings may be written down as

$$-\mathcal{L} = \sum_{\alpha=1}^{3} \left[ m_\alpha \bar{\nu}_\alpha L \nu^\alpha_{BR}(y = 0) + \int dy \bar{\nu}^\alpha_{BL} \partial_5 \nu^\alpha_{BR} + h.c. \right] ,$$

(4.8)

where $\nu_{aL} = U_{a\alpha} \nu_{\alpha L}$, with $a = e, \mu, \tau$ and $\alpha = 1, 2, 3$. The mass parameters $m_\alpha$ are the eigenvalues of the Yukawa couplings matrix multiplied by the vacuum $v$, and as stated before are naturally of the order of $\text{eV}$ or less. This reduces the analysis to considering three sets of mixings given as in the previous case. Each set (tower) of mass eigenstates is characterized by its own parameter $\xi_\alpha \equiv \sqrt{2} m_\alpha R$. Now, each weak eigenstate can be expressed as a coherent superposition of these three different towers by

$$\nu_\alpha = \sum_{\alpha=1}^{3} \sum_{k=0}^{\infty} U_{a\alpha} \frac{1}{N_{ak}} \bar{\nu}_{ak};$$

(4.9)

with $\bar{\nu}_{ak}$ being the $k$-th mass eigenstates of the $\alpha$-th tower. It is now clear that the three flavour oscillations will correspond to the oscillations among the three towers. In this regards, if the KK do not decouple, the explanation to neutrino puzzles will not be any more described in terms of three single neutrinos. Now, the transition probability is given by $P_{ab} = \sum_{\alpha\beta} U^*_{a\alpha} U_{b\alpha} U_{a\beta}^* U_{b\beta}^* p_{a\beta}$, where we have introduced the partial transition probabilities $p_{a\beta}$ defined as $p_{a\beta}(L) \equiv \langle \bar{\nu}_a(L) | \nu_\alpha(0) \rangle \langle \nu_\beta(L) | \nu_\beta(0) \rangle$. The diagonal $p_{aa}$ is interpreted as the survival probability of (the non standard) $\nu_\alpha$, and it has the form of (4.6) and (4.7) for small and large $\xi_\alpha$ respectively.
The analysis of Ref. [19] shows that there is not simultaneous explanation for solar, atmospheric and LSND data within this minimal model. Without LSND, we have the following possible scenarios: (i) $\xi_{1,2,3} \ll 1$; then, $R$ is smaller than 1 $\mu$m. KK modes decouple. Solar and atmospheric neutrino data are understood as in the case of four dimensional models. (ii) $\xi_{1,2} \ll 1 \lesssim \xi_3$; solar neutrino data is provided as in the four dimensional models but atmospheric data is explained by $\nu_\mu$ to $\nu_{\text{bulk}}$ oscillation as from Eq. (4.7). $R$ is on the submillimeter range. (iii) $\xi_1 \ll 1 \sim \xi_2, 3$. Both, solar and atmospheric data are explained by $\nu \to \nu_{\text{bulk}}$ oscillations. $1/R^2 \sim \Delta m^2_{\text{sol}} \sim 10^{-3}$ eV with matter effects for solar and $\xi_2, 3 \sim 10$. Finally, (iv) for $\xi_{1,2,3} \gg 1$ there is no explanation for solar neutrino data. This case is therefore ruled out.

D. Models for Majorana masses

Some extended scenarios that consider the generation of Majorana masses from the breaking of lepton number either on the bulk or on a distant brane have been considered in Refs. [20,21] (the breaking of global symmetries at distant branes was first proposed in Ref. [51]). In these models a bulk scalar field $\chi$ that carries lepton number (2) is introduced. It develops a small vacuum and gives mass to the neutrinos which are generically of the form

$$m_\nu \sim \frac{\langle H \rangle^2 \langle \chi \rangle_B}{M M^{3/2}},$$

then, with $M$ of the order of 10 TeV, we need $\langle \chi \rangle_B \sim (10 \text{ MeV})^{3/2}$. Such a small vacuum is possible in both classes of models, though it usually needs a small mass for $\chi$. Obviously, with Majorana masses, a bulk neutrino may not be needed but new physics must be invoked. We should notice the there is also a Majoron field associated to the spontaneous breaking of the lepton number symmetry. Its phenomenology depends on the details of the specific model. In the simplest scenario, the coupling $(LH)^2 \chi$ is the one responsible for generating Majorana masses. It also gives an important contribution for neutrinoless double beta decay which is just right at the current experimental bounds [21].

E. Models with local $B-L$ symmetry

We now proceed to consider the second class of models. For the case where $B-L$ is local, anomaly cancellation requires that right handed neutrinos must be present in the brane as in the models discussed in Ref. [15,16]. The simplest gauge model where this scenario is realized is the left-right symmetric model where the right handed symmetry is broken by the Higgs doublet $\chi_R(1,2,1)$, where the number inside the parenthesis correspond to the quantum numbers under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The model then contains the left and right handed brane leptons and a blind bulk neutrino. The relevant terms of the action for one generation are [15]

$$S = \int d^4x \left[ \kappa \bar{L}_L \nu_B (y = 0) + \kappa \bar{R}_R \nu_B (y = 0) + h \bar{L} \phi R \right] + \int d^4x dy \bar{\nu}_B \Gamma^5 \partial_5 \nu_B + h.c.$$

By setting $\langle \chi_R \rangle = v_R$ and $\langle \chi_L \rangle = 0$, the following Dirac neutrino mixing matrix is obtained
\[
\left( \begin{array}{c}
\bar{\nu}_{eL} \\
\bar{\nu}_{0BL}
\end{array} \right)
\left( \begin{array}{ccc}
\hbar v & 0 \\
\kappa v_R & 0 \\
\sqrt{2}\kappa v_R & \partial_5
\end{array} \right)
\left( \begin{array}{c}
\nu_{eR} \\
\nu_{BR}
\end{array} \right).
\]

(4.11)

Note that in general the effect of \(\langle \chi^0_L \rangle = 0\) may also be produced if the bulk neutrino breaks explicitly the parity symmetry [22]. Now, a massless field, \(\nu_0\), which is predominantly the electron neutrino, provided that \(\kappa v_R \gg \hbar v \simeq \text{few MeV}\), appears. Since \(\kappa \simeq \frac{M}{M_{pl}}\), this constraint implies that \(M\) must be as large as \(10^8\) GeV or so, however \(R\) may remain in the submillimeters. Oscillations into bulk neutrino will now result. The profile of the oscillations in the present case is quite different from the case of models with global \(B-L\).

Now, the mass eigenstates obey the characteristic equation \(\lambda_n = \pi \kappa^2 v_R^2 R^2 \cot(\pi \lambda_n)\). For the weak eigenstate we found \(\nu_e = \cos \theta \nu_0 + \sin \theta \tilde{\nu}_0\), where \(\tan \theta = \frac{\hbar v}{\kappa v_R}\); and \(\tilde{\nu}_0\) is given in terms of the mass eigenstates as \(\tilde{\nu}_0(t) = \sum \frac{1}{\eta_n} \nu_n\), with the mixing factors given by \(\eta_n^2 = 2(\lambda_n^2/\zeta^2)(\lambda_n^2 + f(\zeta))\), with \(\zeta = \sqrt{2} \kappa v_R R\) and \(f(\zeta)\) as before. Thus, in this case the KK contributions enter in \(\nu_e\) trough the universal mixing angle \(\theta\), in contrast with Eq. (4.5).

Now, the survival probability after the neutrino traverses a distance \(L\) in vacuum reads
\[P_{\text{surv}}(L) = \cos^4\theta + \sin^4\theta |\langle \tilde{\nu}_0|\tilde{\nu}_0(L)\rangle|^2 + 2 \cos^2\theta \sin^2\theta \text{Re}\langle \tilde{\nu}_0|\tilde{\nu}_0(L)\rangle.\]
The averaged probability has the form
\[P_{\text{surv}} = \cos^4\theta + \frac{2}{3} \sin^4\theta,\]
which is smaller than the two neutrino case with the same mixing angle, though, for small mixing it approaches the former result.

Further analysis extending the present model to three brane generations was presented in Ref. [16]. There, seesaw Majorana terms were included, and a single bulk neutrino plays the role of a sterile neutrino with its lightness associated to the geometry of the extra dimension. The possible scenario that explain the neutrino anomalies could be as follows: solar data is given by \(\nu_e \to \nu_B\) oscillations and matter effects, this implies a submillimeter radius. Atmospheric data, is provided by the usual \(\nu_\mu \to \nu_\tau\) oscillations thanks to a natural decoupling of this sector from the KK modes. Finally, LSND is explained by a small \(\nu_e - \nu_\mu\) mixing.

V. STANDARD MODEL IN EXTRA DIMENSIONS

Considering the possibility that the SM particles propagate on the extra dimensions drives the models back to the former KK theories, nevertheless, besides the new possibility of having large extra dimensions, the fact that some particles could be still attached to the branes makes this scenario quite different from the former one. Before considering any possible case one must notice that the conservation of the charges, that is, the consistency of the local gauge symmetry, implies that the first natural candidates to propagate in the bulk are the gauge bosons. Once they are promoted to be bulk fields we will think on the SM fields as the zero modes of higher dimensional fields. However, as there is not experimental evidence of light copies of \(Z, W, \text{etc.}\), we lead to the conclusion that this models can not have a too large extra dimension. The current experimental data provide lower bounds for the size of \(R\) just at the TeV scale, suggesting that these extra dimensions may show up in the near future at the colliders. The whole scenario could be as follows: There are several extra dimensions. The SM particles are free to propagate within one (or more) \(p\)-brane(s), where \(p > 3\) and where the largest extra compact dimensions are about TeV’s, while gravity lives in a higher dimensional bulk with some large (millimetric) extra dimensions. It is worth
mentioning that this scenario could in fact be realized from string theory. To simplify the discussion of this models we will follow Ref. [23], although we recommend the reader to see also the important early works, some of which are given in Refs. [24–27].

A. Theoretical setup

Let us consider again five dimensions, with an orbifolded fifth dimension. Then, let us assume that the SM gauge fields live in the bulk, while fermions, \( \psi \), and Higgs doublets \( H_i \) can either live in the bulk or on the \( y = 0 \) brane. The analysis will follow for two Higgs doublets to make a possible extension to supersymmetry obvious. The case with only one scalar doublet is straightforward. The lagrangian reads

\[
\mathcal{L}_{5D} = -\frac{1}{4} F^2_{MN} + \sum_i \left[ (1 - \varepsilon_i H_i)|D_M H_i|^2 + (1 - \varepsilon_i \psi_i) \bar{\psi}_i \Gamma^M D_M \psi_i \right] \\
+ \sum_i \left[ \varepsilon_i H_i |D_\mu H_i|^2 + \varepsilon_i \psi_i \bar{\psi}_i \sigma_\mu D_\mu \psi_i \right] \delta(y),
\]

(5.1)

where \( \varepsilon^F = 1 \ (0) \) when the \( F \)-field lives on the boundary (bulk); \( D_M = \partial_M + ig_5 V_M \); \( V_M = V^a_M T^a \), with \( T^a \) the group generators and \( g_5 \) the 5D gauge coupling. Clearly a \( g'_5 \) should be introduced for \( U(1)_Y \). Notice that the effective four dimensional couplings obeys \( g = g_5 / \sqrt{\pi R} \), thus the gauge sector should be strongly coupled on the bulk. Gauge and Higgs bosons living in the 5D bulk are assumed to be even under the \( Z_2 \) symmetry. We will choose the even assignment for the \( \psi_L \) (\( \psi_R \)) components of fermions \( \psi \) which are doublets (singlets) under \( SU(2)_L \), this is in order to recover the low energy SM spectra.

At intermediate energies, below the compactification scale, \( M_c \), the impact of this theory on standard (boson exchange) processes may be studied by integrating out the KK modes, and summing up the diagrams as we did for Eq. (3.10). Thus, let us introduce the useful small parameter

\[
X = \sum_{n=1}^{\infty} \frac{2 m_{2n}^Z}{n^2 M_c^2} = \frac{\pi^2 m_{2n}^Z}{3 M_c^2},
\]

(5.2)

and do all the analysis at first order on \( X \). Also, it is useful to introduce the effective mixing angle \( s_\alpha^2 = \sin^2 \alpha = \varepsilon^{H_2} s_\beta^2 + \varepsilon^{H_1} c_\beta^2 \) where, as usual, \( \tan \beta = \langle H_2 \rangle / \langle H_1 \rangle \), with \( v^2 \equiv \langle H_1 \rangle^2 + \langle H_2 \rangle^2 \simeq (174 \text{ GeV})^2 \). In these terms the charged sector in the four dimensional effective theory and in the unitary gauge has the form [23]

\[
\mathcal{L}_{\text{eff}}^{\text{cc}} = \frac{1}{2} M_W^2 W \cdot W - g W \cdot \left[ J - s_\alpha^2 c_\theta^2 X J^{KK} \right] - \frac{g^2}{2 m_Z^2} X J^{KK} \cdot J^{KK},
\]

(5.3)

where \( M_W^2 = \left[ 1 - s_\alpha^4 c_\theta^2 X \right] m_W^2 \); being \( m_W^2 = g^2 v^2 / 2 \) and \( \theta \) the usual electroweak mixing angle. \( J_\mu \) and \( J^{KK}_\mu \) are the fermion currents of the zero and excited modes respectively. The last term in (5.3) is an effective four points interaction induced by the exchange of the heavy \( W^{KK} \). Note that the tree level mass of the \( W \) receive a contribution from its mixing with the excited modes if the Higgs is a brane field (\( \varepsilon^H = 1 \)). This lagrangian induce a tree level correction to the Fermi constant from the \( \mu \) decay, that reads
\[ \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \left[ 1 + \varepsilon \ell c_{2\alpha} c_\theta^2 X \right]. \] (5.4)

On the other hand, for the neutral currents we get at the same limit
\[ L_{\text{eff}}^{\text{nc}} = \frac{1}{2} M_Z^2 Z \cdot Z - \frac{e}{s_\theta c_\theta} Z \cdot \left[ J_Z - s_\alpha^2 X J_{ZKK} \right] - eA \cdot J_{em} \]
\[ - \frac{1}{2 M_Z^2} \varepsilon_\ell^2 \lambda s_\alpha^2 X J_{ZKK} \cdot J_{ZKK} - \frac{e^2}{2 M_Z^2} X J_{em} \cdot J_{em}^\ast, \] (5.5)

where \( M_Z^2 = [1 - s_\alpha^4 X] m_Z \). As before the \( J \)'s represent the usual four dimensional currents and \( J_{ZKK} \) that corresponding to the matter KK excitations, if they exist. Note that the zero mode of the photon \((A)\) remain massless. From the \( W \) and \( Z \) masses combined with Eq. (5.4), one may relates the weak mixing angle to the \( G_F \) as
\[ s_\alpha^2 c_\theta^2 = \frac{\pi \alpha}{\sqrt{2 G_F M_Z^2}} (1 + \Delta), \] (5.6)

where the parameter \( \Delta = \left[ \varepsilon \ell c_{2\alpha} c_\theta^2 - s_\alpha^4 s_\theta^2 \right] X; \) and \( \alpha \) is the fine structure constant.

Another ingredient that may be reinstalled on the theory is supersymmetry. Although it is not necessary to be considered, it is an interesting extension. After all, it seems plausible to exist if the high energy theory is string theory. If the theory is supersymmetric, the bulk fields come in \( N = 2 \) supermultiplets [25,24]. The on-shell field content of the gauge supermultiplet is \( V = (V_\mu, V_5, \lambda^i, \Sigma) \) where \( \lambda^i (i = 1, 2) \) is a simplectic Majorana spinor and \( \Sigma \) a real scalar in the adjoint representation; \((V_\mu, \lambda^1)\) is even under \( Z_2 \) and \((V_5, \Sigma, \lambda^2)\) is odd. Matter and Higgs fields are arranged in \( N = 2 \) hypermultiplets that consist of chiral and antichiral \( N = 1 \) supermultiplets. The chiral \( N = 1 \) supermultiplets are even under \( Z_2 \) and contain massless states. These will correspond to the SM fermions and Higgses.

Supersymmetry must be broken by some mechanisms that gives masses to all the superpartners which we may assume are of order \( M_c \) [24]. For some possible mechanism see Ref. [25]. In contrast with the case of four dimensional susy, where no extra effects appear at tree level after integrating out the superpartners, in the present case integrating out the scalar field \( \Sigma \) induces a tree-level contribution to \( M_W \) and \( \Delta \) parameters. In Ref. [23] they were calculated to be given by \( M_W^2 = [1 - \epsilon H_1 \varepsilon H_2 s_\beta^2 c_\theta^2 X] m_W^2 \); and \( \Delta = [\varepsilon \ell c_{2\alpha} c_\theta^2 - s_\alpha^4 + \varepsilon H_1 \varepsilon H_2 s_\alpha^2 c_\theta] X \) respectively. The form of the low energy lagrangian remains.

**B. Experimental constrains**

There are two important effects of gauge KK boson states on collider experiments. (i) Mixing effects and (ii) real production of KK modes.

First, the mixings between the zero and the KK modes of gauge bosons modify the SM observables, affecting the Electroweak precision tests [7,23,49]. Lets consider for instance a specific non supersymmetric model that fixes fermions and one Higgs doublet to the boundary, while gauge bosons and another Higgs doublet propagates on the bulk. That fixes \( s_\alpha = s_\beta \) in our expressions above. Then the model has two more parameters than the
SM, given by $s_\beta$ and $X$, or something equivalent. All observables will be expressed explicitly or implicitly in terms of these and the usual SM parameters. For example, Rizzo and Wells in Ref. [7], introduce the new parameter $V = \frac{M_W}{m_Z}$ and the effective interaction couplings $g_W = g[1 - s_\beta^2 V]$ and $g_Z = g[1 - s_\beta^2 V/c_\theta^2]$ to get

$$G_F(\mu \, \text{decay}) = \frac{\sqrt{2}g_W^2}{8M_W^2}[1 + V], \quad \Gamma(Z \to f\bar{f}) = \frac{N_\nu M_Z}{12\pi} \left( \frac{g_Z}{2c_\theta} \right)^2 \left[ v_f^2 + a_f^2 \right],$$

$$Q_W = \frac{1}{M_Z^2} \left\{ \frac{g^2(1 - s_\beta^2 V/c_\theta^2)^2 + g^2V}{c_\theta^2} \right\} a_e \left[ v_u(2Z + N) + v_d(2N + Z) \right],$$

$$R = \frac{\sigma_{NC} - \sigma_{NC}^p}{\sigma_{CC} - \sigma_{CC}^p} = \left[ \frac{g_Z^2}{c_\theta^2 M_Z^2} + \frac{g^2V}{c_\theta^2 M_Z^2} \right] \left[ \frac{g_W^2}{M_W^2} - \frac{g^2V}{M_W^2} \right]^{-1} \left( \frac{1}{2} - s_\theta^2 \right), \quad (5.7)$$

$$A_f = \frac{2v_f a_f}{v_f^2 + a_f^2}, \quad \sin^2 \theta_{\text{eff}} = x + \frac{x(1 - x)}{1 - 2x} V \left[ c_\beta^4 - \frac{s_\beta^4}{1 - x} \right],$$

$$M^2_W = M_Z^2(1 - x) \left\{ 1 + V \left[ 1 - 2s_\beta^2 - \frac{c_\beta^4(1 - x) - s_\beta^4}{1 - 2x} \right] \right\},$$

where $Q_W$ is a measure of atomic parity violation, $x$ is the solution to the equation $x(1 - x) = \pi\alpha/\sqrt{2}G_F m_Z^2$; $v_f \equiv T_{3f} - 2Q fs_\theta^2$ and $a_f \equiv T_{3f}$. Overall, the limit on $M_e$ using the precision data measurements [52] is just about $M_e \gtrsim 3.3 - 3.8$ TeV [49].

Future colliders may be able to observe resonances due to KK modes if the compactification scale turns out to be on the TeV range. This needs a collider energy $\sqrt{s} \gtrsim M_c$. In hadron colliders (TEVATRON, LHC) the KK excitations might be directly produced in Drell-Yang processes $pp(p\bar{p}) \to \ell^- \ell^+ X$ where the lepton pairs ($\ell = e, \mu, \tau$) are produced via the subprocess $q\bar{q} \to \ell^+ \ell^+ X$. This is the more useful mode to search for $Z^{(n)}/\gamma^{(n)}$ even $W^{(n)}$. Current search for $Z'$ on this channels (CDF) impose $M_c > 510$ GeV. Future bounds could be raised up to 650 GeV in TEVATRON and 4.5 TeV in LHC, which with 100 fb$^{-1}$ of luminosity can discover modes up to $M_c \approx 6$ TeV.

Deviations on the cross sections due to virtual exchange of KK modes may be observed in both, hadron and lepton colliders. With a 20 fb$^{-1}$ of luminosity, TEVATRONII may observe signals up to $M_c \approx 1.3$ TeV. LEPIII with a maximal luminosity of 200 fb$^{-1}$ could impose the bound at 1.9 TeV, while NLC may go up to 13 TeV, which slightly improve the bounds coming from precision test.

VI. GAUGE COUPLING UNIFICATION

Once we have assumed a low fundamental scale for quantum gravity, the natural question is whether the former picture of a Gran Unified Theory [53] should be abandoned and with it a possible gauge theory understanding of the quark lepton symmetry and gauge hierarchy.
On the other hand, if string theory were the right theory above $M$ an unique fundamental coupling constant would be expect, while the SM contains three gauge coupling constants. Then, it seems clear that, in any case, a sort of low energy gauge coupling unification is required. As pointed out in Ref. [26] and later explored in [27–31], if the SM particles live in higher dimensions, as in the model discused above, such a low GUT scale could be realized.

For comparison let us mention how one leds to gauge unification in four dimensions. Key ingredient in our discussion are the renormalization group equations (RGE) for the gauge coupling parameters that at one loop, in the $\overline{MS}$ scheme, read

$$\frac{d\alpha_i}{dt} = \frac{1}{2\pi} b_i \alpha_i^2$$  \hspace{1cm} (6.1)

where $t = \ln \mu$. $\alpha_i = g_i^2/4\pi$; $i = 1, 2, 3$, are the coupling constants of the SM factor groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ respectively. The coefficient $b_i$ receives contributions from the gauge part and the matter including Higgs field and its completely determinated by $4\pi b_i = \frac{11}{3} C_i(vectors) - \frac{2}{3} C_i(fermions) - \frac{1}{3} C_i(scalars)$, where $C_i(\cdots)$ is the index of the representation to which the $(\cdots)$ particles are assigned, and where we are considering Weyl fermion and complex scalar fields. Fixing the normalization of the $U(1)$ generator as in the $SU(5)$ model, we get for the SM $(b_1, b_2, b_3) = (4/10, -19/6, -7)$ and for the Minimal Supersymmetric SM (MSSM) $(33/5, 1, -3)$. Using Eq. (6.1) to extrapolate the values measured at the $M_Z$ scale [52]: $\alpha^{-1}_1(M_Z) = 58.97 \pm .05$; $\alpha^{-1}_2(M_Z) = 29.61 \pm .05$; and $\alpha^{-1}_3(M_Z) = 8.47 \pm .22$, (where we have taken for the strong coupling constant the global average), one finds that only in the MSSM the three couplings merge together at the scale $M_{GUT} \sim 10^{16}$ GeV. This high scale naturally explains the long live of the proton and in the minimal $SO(10)$ framework one gets a very compelling and predictive scenario [54].

A different possibility for unification that does not involve supersymmetry is the existence of an intermediate left-right model [53] that breaks down to the SM symmetry at $10^{11–13}$ GeV. It is worth mentioning that a non canonical normalization of the gauge coupling may, however, substantially change the above figures, predicting a different unification scale. Such a different normalization may arise either in some no minimal unified models, or in string theories where the SM group factors are realized on non trivial Kac-Moody levels [55,56]. Such scenarios are in general more complicated than the minimal $SU(5)$ or $SO(10)$ models since they introduce new exotic particles.

### A. Power law running

It is clear that the presence of KK excitations will affect the evolution of couplings in gauge theories and may alter the whole picture of unification of couplings. This question was first studied by Dienes, Dudas and Gherghetta (DDG) [26] on the base of the effective theory approach at one loop level. They found that above the compactification scale $M_c$ one gets

$$\alpha^{-1}_i(M_c) = \alpha^{-1}_i(\Lambda) + \frac{b_i - \tilde{b}_i}{2\pi} \ln \left( \frac{\Lambda}{M_c} \right) + \frac{\tilde{b}_i}{4\pi} \int_{r^M_c}^{\Lambda} dt \left\{ \vartheta_3 \left( \frac{it}{\pi R^2} \right) \right\}^\delta,$$

with $\Lambda$ as the ultraviolet cut-off and $\delta$ the number of extra dimensions. The Jacobi theta function $\vartheta(\tau) = \sum_{-\infty}^\infty e^{i\pi n^2}$ reflects the sum over the complete tower. Here $b_i$ are the beta
functions of the theory below $M_c$, and $\tilde{b}_i$ are the contribution to the beta functions of the KK states at each excitation level. The numerical factor $r$ depends on the renormalization scheme. For practical purposes, we may approximate the above result by decoupling all the excited states with masses above $\Lambda$, and assuming that the number of KK states below a certain energy $\mu$ between $M_c$ and $\Lambda$ is well approximated by the volume of a $\delta$-dimensional sphere of radius $\frac{\mu}{M_c}$ given by $N(\mu, M_c) = X_\delta \left(\frac{\mu}{M_c}\right)^\delta$; with $X_\delta = \pi^{\delta/2}/\Gamma(1 + \delta/2)$. The result is a power law behaviour of the gauge coupling constants [32]:

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_c) - \frac{b_i}{2\pi} \ln \left(\frac{\mu}{M_c}\right) - \frac{\tilde{b}_i}{2\pi} \cdot \frac{X_\delta}{\delta} \left[\left(\frac{\mu}{M_c}\right)^\delta - 1\right],$$

(6.3)

which accelerates the meeting of the $\alpha_i$'s. In the MSSM the energy range between $M_c$ and $\Lambda$ –identified as the unification (string) scale $M$– is relatively small due to the steep behaviour in the evolution of the couplings [26,28]. For instance, for a single extra dimension the ratio $\Lambda/M_c$ has an upper limit of the order of 30, and it substantially decreases for larger $\delta$.

This same relation can be understood on the basis of a step by step approximation [29] as follows. We take the SM gauge couplings and extrapolate their values up to $M_c$ then we add to the beta functions the contribution of the first KK levels, then we run the couplings upwards up to just below the next consecutive level where we stop and add the next KK contributions, and so on, until the energy $\mu$. Despite the complexity of the spectra, the degeneracy of each level is always computable and performing a level by level approach of the gauge coupling running is possible. Above the $N$-th level the running receives contributions from $b_i$ and of all the KK excited states in the levels below, in total $f_\delta(N) = \sum_{n=1}^{N} g_\delta(n)$, where $g_\delta(n)$ represent the total degeneracy of the level $n$. Running for all the first $N$ levels leads to

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_c) - \frac{b_i}{2\pi} \ln \left(\frac{\mu}{M_c}\right) - \frac{\tilde{b}_i}{2\pi} \left[ f_\delta(N) \ln \left(\frac{\mu}{M_c}\right) - \frac{1}{2} \sum_{n=1}^{N} g_\delta(n) \ln n \right].$$

(6.4)

A numerical comparison of this expression with the power law running shows the accuracy of that approximation. Indeed, in the continuous limit the last relation reduces into Eq. (6.3). Thus, gauge coupling unification may now happen at TeV scales [26]. Next, we will discuss how accurate this unification is.

**B. One step unification**

Many features of unification can be studied without bothering about the detailed subtleties of the running. Consider the generic form for the evolution equation

$$\alpha_i^{-1}(M_Z) = \alpha^{-1} + \frac{b_i}{2\pi} \ln \left(\frac{M}{M_Z}\right) + \frac{\tilde{b}_i}{2\pi} F_\delta \left(\frac{M}{M_c}\right),$$

where we have changed $\Lambda$ to $M$ to keep our former notation. Above, $\alpha$ is the unified coupling and $F_\delta$ is given by the expression between parenthesis in Eq. (6.4) or its correspondent limit in Eq. (6.3). Note that the information that comes from the bulk is being separated into two independent parts: all the structure of the KK spectra $M_c$ and $M$ are completely
embedded into the $F_\delta$ function, and their contribution is actually model independent. The only (gauge) model dependence comes in the beta functions, $\bar{b}_i$. Indeed, Eq. (6.5) is similar to that of the two step unification model where a new gauge symmetry appears at an intermediate energy scale. Such models are very constrained by the one step unification in the MSSM. The argument goes as follows: let us define the vectors: $b = (b_1, b_2, b_3); \ b = (\bar{b}_1, \bar{b}_2, \bar{b}_3); \ a = (\alpha_1^{-1}(M_Z), \alpha_2^{-1}(M_Z), \alpha_3^{-1}(M_Z))$ and $u = (1, 1, 1)$, and construct the unification barometer [29] $\Delta \alpha \equiv (u \times b) \cdot a$. For single step unification models the unification condition amounts to the condition $\Delta \alpha = 0$. As a matter of fact, for the SM $\Delta \alpha = 41.13 \pm 0.655$, while for the MSSM $\Delta \alpha = 0.928 \pm 0.517$, leading to unification within two standard deviations. In this notation Eq. (6.5) leads to

$$\Delta \alpha = [(u \times b) \cdot \bar{b}] \frac{1}{2\pi} F_\delta. \quad (6.6)$$

Therefore, for the MSSM, we get the constrain [57]

$$\ (7\bar{b}_3 - 12\bar{b}_2 + 5\bar{b}_1) F_\delta = 0. \quad (6.7)$$

There are two solutions to the this equation: (a) $F_\delta(M/M_c) = 0$, which means $M = M_c$, bringing us back to the MSSM by pushing up the compactification scale to the unification scale. (b) Assume that the beta coefficients $\bar{b}$ conspire to eliminate the term between brackets: $(7\bar{b}_3 - 12\bar{b}_2 + 5\bar{b}_1) = 0$, or equivalently [26]

$$\frac{B_{12}}{B_{13}} = \frac{B_{13}}{B_{23}} = 1; \quad \text{where} \quad B_{ij} = \frac{\bar{b}_i - \bar{b}_j}{\bar{b}_i - \bar{b}_j}. \quad (6.8)$$

The immediate consequence of last possibility is the indeterminacy of $F_\delta$, which means that we may put $M_c$ as a free parameter in the theory. For instance we could choose $M_c \sim 10$ TeV to maximize the phenomenological impact of such models. It is compelling to stress that this conclusion is independent of the explicit form of $F_\delta$. Nevertheless, the minimal model where all the MSSM particles propagate on the bulk does not satisfy that constrain [26,28]. Indeed, in this case we have $(7\bar{b}_3 - 12\bar{b}_2 + 5\bar{b}_1) = -3$, which implies a higher prediction for $\alpha_s$ at low $M_c$. As lower the compactification scale, as higher the prediction for $\alpha_s$. However, as discussed in Ref. [28] there are some scenarios where the MSSM fields are distributed in a nontrivial way among the bulk and the boundaries which lead to unification. There is also the obvious possibility of adding matter to the MSSM to correct the accuracy on $\alpha_s$.

The SM case has similar complications. Now Eq. (6.5) turns out to be a system of three equation with three variables, then, within the experimental accuracy on $\alpha_i$, specific predictions for $M, M_c$ and $\alpha$ will arise. As $\Delta \alpha \neq 0$, the above constrain does not aply, instead the matter content should satisfy the consistency conditions [29]

$$\text{Sign}(\Delta \alpha) = \text{Sign}[(u \times b) \cdot \bar{b}] = -\text{Sign}(\bar{\Delta} \alpha) ; \quad (6.9)$$

where $\bar{\Delta} \alpha \equiv (u \times \bar{b}) \cdot a$. However, in the minimal model where all SM fields are assumed to have KK excitations one gets $\bar{\Delta} \alpha = 38.973 \pm 0.625$; and $(u \times b) \cdot \bar{b}^{SM} = 1/15$. Hence, the constraint (6.9) is not fulfilled and unification does not occur. Extra matter could of course improve this situation [26,28,29]. Models with non canonical normalization may also modify this conclusion [29]. A particularly interesting outcome in this case is that there are some
cases where, without introducing extra matter at the SM level, the unification scale comes out to be around $10^{11}$ GeV (for instance $SU(5) \times SU(5)$, $[SU(3)]^4$ and $[SU(6)]^4$). These models fit nicely into the new intermediate string scale models recently proposed in [58], and also with the expected scale in models with local $B-L$ symmetry. High order corrections has been considered in Ref. [30]. The analysis for the running of other coupling constants could be found in [26,31]. Two step models were also studied in [29].

Now well, with a TeV unification scale a extremely fast proton decay mediated by new gauge interactions may occur. There are two possible solutions to this problem. The obvious one is invoking an unified group that keeps the proton stable. A less trivial possibility was suggested in [26]. If the gauge bosons that mediate proton decay are odd under the $\mathbb{Z}_2$ symmetry of the orbifold, then their coupling to the quarks (fixed at the brane) is forbidden, and the proton remains stable. On the context of string theories it may also happen that gauge coupling unification occurs without the appearance of any extra gauge symmetry at the string scale. We should note, however, that this last mechanism does not remove the danger of having a fast proton decay induced by high order operators.

**VII. SPLITTING WAVE FUNCTIONS ON THICK WALLS**

As we already mentioned some mechanism is needed in this theories to forbid dangerous higher dimension operators which lead to proton decay, large neutrino masses, etc., since they are now suppressed just by $M$. Without the knowledge of the theory above $M$ it is difficult just to assume that such operators are not being induced. One might of course invoke global symmetries again. However, an interesting mechanism that explain how proton decay could get suppressed at the proper level appeared in [33]. It relays on the idea that the branes are being formed from an effective mechanism that traps the SM particles in it, resulting in a wall with thickness $L \sim M^{-1}$, where the fermions are stuck at different points. Then, fermion-fermion couplings get suppressed due to the exponentially small overlaps of their wave functions. This provides a framework for understanding both the fermion mass hierarchy and proton stability without imposing extra symmetries, but rather in terms of a higher dimensional geography [34]. Note that the dimension where the gauge fields propagate does not need to be orthogonal to the millimetric dimensions, but gauge fields may be restricted to live in a smaller part of that extra dimensions. Here we briefly summarize those ideas.

**A. Localizing wave functions on the brane**

Let us start by assuming that the translational invariance along the fith dimension is being broken by a bulk scalar field $\Phi$ which develops a spatially varying expectation value $\langle \Phi(y) \rangle$. We assume that this expectation value have the shape of a domain wall transverse to the extra dimension and is centered at $y = 0$. With this background a bulk fermion will have a zero mode that is stuck at the zero of $\langle \Phi(y) \rangle$. To see this let us consider the action

$$S = \int d^4x \ d y \overline{\Psi} \left[ i \Gamma^M \partial_M + \langle \Phi(y) \rangle \right] \Psi,$$

(7.1)

in the chiral basis, as before. By introducing the expansions
\[
\Psi_L(x, y) = \sum_n f_n(y) \psi_n L(x); \quad \text{and} \quad \Psi_R(x, y) = \sum_n g_n(y) \psi_n R(x); \quad (7.2)
\]

where \(\psi_n\) are four dimensional spinors, we get for the \(y\)-dependent functions \(f_n\) and \(g_n\) the equations

\[
(\partial_5 + \langle \Phi \rangle) f_n + \mu_n g_n = 0; \quad \text{and} \quad (-\partial_5 + \langle \Phi \rangle) g_n + \mu_n f_n = 0. \quad (7.3)
\]

Therefore, the zero modes have the profiles [33]

\[
f_0(y) \sim \exp \left[-\int_0^y ds \langle \Phi \rangle(s) \right] \quad \text{and} \quad g_0(y) \sim \exp \left[\int_0^y ds \langle \Phi \rangle(s) \right]; \quad (7.4)
\]

up to normalization factors. Notice that when the extra space is supposed to be finite, both modes are normalizable. For the special choice \(\langle \Phi \rangle(y) = 2\mu^2 y\), we get \(f_0\) centered at \(y = 0\) with the gaussian form

\[
f_0(y) = \frac{\mu^{1/2}}{(\pi/2)^{1/4}} \exp \left[-\mu^2 y^2 \right]. \quad (7.5)
\]

The other mode has been projected out from our brane by being pushed away to the end of the space. Thus, our theory in the wall is a chiral theory. Notice that a negative coupling among \(\Psi\) and \(\phi\) will instead project out the left handed part.

The generalization of this technique to the case of several fermions is straightforward. The action (7.1) is generalized to

\[
S = \int d^5 x \sum_{i,j} \bar{\Psi}_i [i \Gamma^M \partial_M + \lambda \langle \Phi \rangle - m]_{ij} \Psi_j , \quad (7.6)
\]

where general Yukawa couplings \(\lambda\) and other possible five dimensional masses \(m_{ij}\) have been considered. For simplicity we will assume both terms diagonal. The effect of these new parameters is a shifting of the wave functions, which now are centered around the zeros of \(\lambda_i \langle \Phi \rangle - m_i\). Taking \(\lambda_i = 1\) with the same profile for the vacuum leads to gaussian distributions centered at \(y_i = m_i / 2\mu^2\). Thus, at low energies, the above action will describe a set of non interacting four dimensional chiral fermions localized at different positions in the fifth dimension.

Localization of gauge and Higgs bosons needs extra assumptions. The explanation of this phenomena is close related with the actual way the brane was formed. A field-theoretic mechanism for localizing gauge fields was proposed by Dvali and Shifman and was later extended and applied in [2]. There, the idea is to arrange for the gauge group to confine outside the wall; the flux lines of any electric sources turned on inside the wall will then be repelled by the confining regions outside and forced to propagate only inside the wall. This traps a massless gauge field on the wall. Since the gauge field is prevented to enter the confined region, the thickness \(L\) of the wall acts effectively as the size of the extra dimension in which the gauge fields can propagate. In this picture, the gauge couplings will exhibit power law running above the scale \(L^{-1}\).
B. Fermion mass hierarchies and proton decay

Let us consider the Yukawa coupling among the Higgs field and the leptons: \( \kappa H L^T E^c \); where the massless zero mode \( l \) from \( L \) is localized at \( y = 0 \) while \( e \) from \( E^c \) is localized at \( y = r \). Let us also assume that the Higgs zero mode is delocalized inside the wall. Then the zero modes term of this coupling will generate the effective Yukawa action

\[
S_{Yuk} = \int d^4 x \kappa h(x)l(x)e^c(x) \int dy \phi_l(y) \phi_{e^c}(y),
\]

where \( \phi_l \) and \( \phi_{e^c} \) represent the gaussian profile of the fermionic modes. Last integral gives the overlap of the wave functions, which is exponentially suppressed \cite{33} as

\[
\int dy \phi_l(y) \phi_{e^c}(y) = e^{-\mu r^2/2}.
\]

This is a generic feature of this models. The effective coupling of any two fermion fields is exponentially suppressed in terms of their separation in the extra space. Thus, the explanation for the mass hierarchies becomes a problem of the cartography on the extra dimension. A more detailed analysis was presented in \cite{34}.

Let us now show how a fast proton decay is evaded in these models. Assume, for instance, that all quark fields are localized at \( y = 0 \) whereas the leptons are at \( y = r \). Then, lets consider the following baryon and lepton number violating operator

\[
S \sim \int d^5 x \frac{(Q^T C_5 L^\dagger)(U^c C_5 D^c)}{M^3}.
\]

In the four dimensional effective theory, once we have introduced the zero mode wave functions, we get the suppressed action \cite{33}

\[
S \sim \int d^4 x \lambda \frac{(q l^\dagger)(u^c d^c)}{M^2}
\]

where \( \lambda \sim \int dy \left[ e^{-\mu^2 y^2/3} e^{-\mu^2(y-r)^2} \right] \sim e^{-3/4 \mu^2 r^2} \). Then, for a separation of \( \mu r = 10 \) we obtain \( \lambda \sim 10^{-33} \) which renders these operators completely safe even for \( M \sim 1 \) TeV. Therefore, we may imagine a picture where quarks and leptons are localized near opposite ends of the wall so that \( r \sim L \). This mechanism, however, does not work for suppressing the other dangerous operator \( (L H)^2/M \) responsible of a large neutrino masses.

VIII. RANDALL SUNDRUM MODEL AND OTHER CURRENT TRENDS

To close our present discussion allow us to mention another important direction of research in this area. It was motivated by a seminal work of Randall and Sundrum \cite{35}, who proposed a drastic change on our present point of view of the way the bulk enters in the explanation of the hierarchy problem. Here we summarize some aspects of this model and some further trends \cite{36–44} that may give a rough idea of the way this area is going.
A. Mass Hierarchy from a Small Extra Dimension

Let's consider the following setup. A five dimensional space with an orbifolded fifth dimension. Consider two branes at the fixed points \( y = 0, \pi \); with tensions \( \sigma \) and \( -\sigma \) respectively. Assume that the bulk has a cosmological constant \( \Lambda \). Contrary to our previous philosophy, here let us assume that all parameters are of the order of the Planck scale. Moreover, we will no longer assume that the bulk has a flat metric, instead we will consider a non factorizable geometry induced by the (explicitly) broken translational invariance. Thus, the more general metric that respects four dimensional Poincare invariance on the brane has the form:

\[
ds^2 = G_{AB} dx^A dx^B = e^{-2\beta(y)} g_{\mu\nu}(x) dx^\mu dx^\nu - r^2 dy^2.
\]

By solving the Einstein equations with this ansatz for the metric, it turns out that \( \sigma \) and \( \Lambda \) need to be related by the (fine tuning) condition

\[
\Lambda = -\frac{\sigma^2}{6M_3};
\]

that is equivalent to the exact cancellation of the effective four dimensional cosmological constant. On the other hand, one gets \( \beta(y) = kr|y| \), where \( k^2 = \Lambda/6M^3 \). The effective Planck scale is then given by

\[
M_{Pl}^2 = \frac{M^3}{k} \left(1 - e^{-2kr\pi}\right).
\]

The effect of this metric on the brane fields parameters is non trivial. Let's consider for instance the Higgs action for the brane at the end of the space, where we assume all SM fields are fixed, it is given by

\[
S_H = \int d^4x \sqrt{-g} e^{-4kr\pi} \left[ e^{2kr\pi} g^{\mu\nu} \partial_\mu H \partial_\nu H - \lambda \left(H^2 - \hat{v}_0^2\right)^2 \right].
\]

After introducing the normalization \( H \rightarrow e^{kr\pi} H \) that recovers the canonical kinetic term, the above action becomes

\[
S_H = \int d^4x \sqrt{-g} \left[ g^{\mu\nu} \partial_\mu H \partial_\nu H - \lambda \left(H^2 - v^2\right)^2 \right],
\]

where the vacuum \( v = e^{-kr\pi} \hat{v}_0 \). Therefore, by choosing \( kr \sim 12 \), the physical mass of the Higgs field, its vacuum, and, thus all the SM masses appear at the TeV scale without needness or a large hierarchy on the radius, even though the original mass parameter \( \hat{v}_0 \sim M \sim M_{Pl} \). Notice that on the contrary, any field located on the other brane will get a mass of the order of \( M \). Moreover, it also implies that no new particles exist in our world with masses larger than TeV.
B. Some about the Phenomenology

Despite this impressive property of the model, which has attracted a lot of attention. It also turns out that the effective cut-off of the theory is also at the TeV scale. Indeed, all high order operators get now suppressed just by \( m_0 \sim e^{-k r \pi} M \), that is at the TeV range. The reason is as follows. Any operator of dimension \( n, \Theta_n \), is originally suppressed by the fundamental scale \( M \sim M_{Pl} \). However, under the change on the normalization of the fields, \( \Theta_n \rightarrow e^{-n k r \pi} \Theta_n \). Therefore, on the effective theory we get the large enhancement \( M \rightarrow m_0 \). So far no definite solution to the dangerous presence of those operators exist, tough one might impose global symmetries again. On the other hand, this scenario has not yet been realized on the context of string theory.

Kaluza Klein modes in this model also get masses at the TeV scale, and the zero modes remain massless. They may get mass from brane contributions. The radion, however, will pick up a mass from the stabilization potential [36,37]. The intriguing possibility that the extra space could be actually infinite was presented in [38]. Some phenomenological bounds on the effect of KK gravitons are found in [39]. Neutrino masses were analyzed in [40]. Here, only the zero modes get tiny masses. Since the KK modes are heavy, they do not participate on the mixings. Thus, there is not a clear experimental signature for this models in the neutrino sector. A further application of this model, and its extensions, could be a possible resolution of the cosmological constant problem [41].

Cosmology on this models has received a lot of attention. It was early observed [42] that the Hubble parameter \( H \) is proportional to the density on the brane, \( \rho \), instead of the usual \( H \sim \sqrt{\rho} \) of the standard big bang cosmology. This could be disastrous for late cosmology and BBN. Nevertheless, at low energies the leading order on \( H \) has the right behaviour [37,43]. A big deal of work has been devoted to further study those ideas [44,45].

C. Radion stabilization

Other interesting aspect of the RS framework is the recent considerations of the stabilization mechanism provided by a bulk scalar field. The idea is as follows. Consider the bulk scalar action

\[
S_{\phi} = \frac{1}{2} \int d^5 x \sqrt{-G} \left( G^{AB} \partial_A \phi \partial_B \phi - m^2 \phi^2 \right). \tag{8.6}
\]

Let us assume that the scalar field satisfies certain boundary conditions associated to its couplings to the visible and hidden branes (at \( y = 0 \) and \( y = \pi \) respectively). For instance

\[
S_{h,v} = - \int d^4 x \sqrt{-g_{h,v}} \lambda_{h,v} \left( \phi^2 - v_{h,v}^2 \right). \tag{8.7}
\]

Those terms cause \( \phi \) to develop a \( y \)-dependent vacuum which is determined classically by solving the equation of motion. Inserting this solution into \( S_{\phi} \) and integrating over \( y \) yields an effective potential for the radius of the form [36]

\[
V_{\text{eff}}(r) = 4k e^{-4k r \pi} \left( v_v - v_h e^{-k r \pi} \right)^2; \tag{8.8}
\]

where \( \epsilon = m^2/4k^2 \ll 1 \). This has a minimum at
\[ kr = \left( \frac{4}{\pi} \right) \frac{k^2}{m^2} \ln \left[ \frac{v_h}{v_v} \right]. \] \hspace{1cm} (8.9)

With the logarithmic part of the order of one, we just need \( k^2/m^2 \) of order 10 to get the right order in \( kr \). Notice that despite the fact that we are only passing the small hierarchy on \( kr \) to the ratio among \( k \) and \( m \), this model provides a stabilization potential that also gives a mass for the radion in the TeV range or so \([36,37]\).

**IX. CONCLUDING REMARKS**

The wave induced by the seminal works in \([2,35]\) has generated a big industry that is still growing. Several aspects of this higher dimensional models have been investigated in the recent years and more is yet to come. As any new field, the study of physics on large extra dimensions is still confronting several criticism that eventually have posed serious challenges. So far the field have survived to many of these test, though several open questions remain. Several ingenious applications have attracted the attention of the community and created new directions of research. We mentioned along these notes what we believe are some of the more interesting applications of the idea. However we must stress that some of them are still disconnected of the other parts. It is fair to say that yet a definite and comprehensive model of our world in this framework does not exist. So far, only separate pieces of the puzzle have been analyzed. There are many unsolved problems not just of technical nature but of fundamental nature. There is the problem of stabilization of the large extra dimensions; connecting all the phenomenology (neutrino masses, proton decay, etc.) in a single picture; and studying the impact of this scenario in other well established areas of physics. There is for instance the realization of Randall Sundrum and other models from string theory.

Besides the hope of observing deviations in the Newton law at small distances in the near future, neutrino physics on this models seems very promising too. On the other hand, it is possible that the collider experiments may only increase the lower bounds on the fundamental scale (as for other models of new physics), although we could also discover the first signals of extra dimensions on the next colliders. Besides, let us mention that while in these theories it seems possible to maintain gauge coupling unification, it is not yet clear whether it leads to a compelling scenario that may provide any light on the origin of other SM parameters. Certainly it seems clear that several other mechanisms must be invoked in contrast with the progress that it has been made on four dimensional theories \([54]\). There is, however, the hope that the years to come helps us either on conforming a more accurate picture or, perhaps on ruling out these ideas. Surely, the coming years will see a lot more on this topic, and perhaps new ideas and results could set it on the side of the well established world of physical theories. Meanwhile, most of the present results remain speculative, although well motivated. As always, Nature has the last word.

The present notes have been prepared intending to be a first introductive guide to the newborn field of models in large (and short) extra dimensions. Unable to make reference to all existing works in the area, we have tried to collect those we believed are relevant for our goal, although some important works could have been omitted. We advise the interested reader to consult the more extended literature that exist already.
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<td>$e^+e^- \rightarrow ZG$</td>
<td>$e^+e^- \rightarrow Z\nu \bar{\nu}$</td>
<td>${515 \text{ GeV} }$</td>
<td>LEPII</td>
</tr>
<tr>
<td>$Z \rightarrow \bar{f}f G$</td>
<td>$Z \rightarrow \bar{f}f \bar{\nu} \nu$</td>
<td>$0.4 \text{ TeV}$</td>
<td>L3</td>
</tr>
</tbody>
</table>

TABLE I. Collider limits for the fundamental scale $M$. Graviton Production.

<table>
<thead>
<tr>
<th>Process</th>
<th>M limit</th>
<th>Collider</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow \gamma \gamma, WW, ZZ$</td>
<td>${0.7 - 1 \text{ TeV} }$</td>
<td>LEP</td>
</tr>
<tr>
<td>All above</td>
<td>1 TeV</td>
<td>L3</td>
</tr>
<tr>
<td>Bhabha scattering</td>
<td>1.4 TeV</td>
<td>LEP</td>
</tr>
<tr>
<td>$gq \rightarrow \gamma \gamma }$</td>
<td>0.9 TeV</td>
<td>CDF</td>
</tr>
</tbody>
</table>

TABLE II. Collider limits for the fundamental scale $M$. Virtual Graviton exchange