Constraining cosmological parameters with the clustering properties of galaxy clusters in optical and X-ray bands

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13 June 2001

\textbf{ABSTRACT}

We use a theoretical model to predict the clustering properties of galaxy clusters. Our technique accounts for past light-cone effects on the observed clustering and follows the non-linear evolution in redshift of the underlying dark matter correlation function and cluster bias factor. A linear treatment of redshift-space distortions is also included. We perform a maximum-likelihood analysis by comparing the theoretical predictions to a set of observational data, both in the optical (two different subsamples of the APM catalogue and the EDCC catalogue) and X-ray band (RASS1 Bright Sample, BCS, XBACs, REFLEX). In the framework of cold dark matter models, we compute the constraints on cosmological parameters, such as the matter density $\Omega_0$, the cosmological constant $\Omega_0$, the power-spectrum shape parameter $\Gamma$ and normalisation $\sigma_8$. Our results show that X-ray data are more powerful than optical ones, allowing smaller regions in the parameter space. If we fix $\Gamma$ and $\sigma_8$ to the values suggested by different observational datasets, we obtain strong constraints on the matter density parameter: $\Omega_0 \leq 0.5$ and $0.2 \leq \Omega_0 \leq 0.35$, for the optical and X-ray data, respectively. Allowing the shape parameter to vary, we find that the clustering properties of clusters are almost independent of the matter density parameter and of the presence of a cosmological constant, while they appear to be strongly dependent on the shape parameter. Using the X-ray data only, we obtain $\Gamma \sim 0.1$ and $0.4 \lesssim \sigma_8 \lesssim 1.1$ for the Einstein-de Sitter model, while $0.14 \lesssim \Gamma \lesssim 0.22$ and $0.6 \lesssim \sigma_8 \lesssim 1.3$ for open and flat models with $\Omega_0 = 0.3$. Finally, we use our model to make predictions on the correlation length of galaxy clusters expected in future surveys. In particular, we show the results for an optical catalogue with characteristics similar to the EIS project and for a very deep X-ray catalogue with the characteristics of the XMM/LSS survey. We find that clusters at high redshifts are expected to have larger a correlation length than local ones.

\textbf{Key words:} cosmology: theory – galaxies: clusters – large–scale structure of Universe – X-rays: galaxies – dark matter

\section{INTRODUCTION}

Clusters of galaxies are the largest collapsed objects with masses dominated by dark matter. Their properties are largely determined by the gravitational collapse in the cosmological density field, and so their distribution in the Universe depends both on cosmology and on the initial density perturbations. Therefore, the study of the clustering of clusters may provide important diagnostics for models of structure formation. One important advantage of using clusters to study the large-scale structure is that their formation in the cosmic density field is relatively easy to understand. Indeed, modern cosmological simulations show that clusters of galaxies may be identified as the most massive dark haloes produced by gravitational collapse, and so their clustering properties are relatively easy to interpret theoretically. In fact, based on an extension of the Press-Schechter formalism, one can obtain an analytical model in which
the dependence of the two-point correlation function of dark haloes on cosmology and initial power spectrum can be explicitly seen (Mo & White 1996; Catelan et al. 1998; Sheth, Mo & Tormen 2001).

It has been known for some time that the strong observed correlation of clusters on large scales is very difficult to reconcile with the standard cold dark matter (CDM) model (White et al. 1987; Dalton et al. 1992; Jing et al. 1993; Mo, Peacock & Xia 1993; Dalton et al. 1994; Borgani, Coles & Moscardini 1994; Croft & Efstathiou 1994; Borgani et al. 1995). More recently, analyses on the constraints by the cluster-cluster correlation function on current theoretical models of structure formation have been carried out by a number of authors (Eke et al. 1996; Mo, Jing & White 1996; Moscardini et al. 2000a,b; Robinson 2000; Colberg et al. 2000).

In this paper, we use current available observational data on the cluster two-point correlation function to constrain theoretical models of structure formation. We use analytical tools well tested by numerical simulations to make predictions for a large grid of models. Our analysis follows closely that of Mo et al. (1996) and Robinson (2000). However, there are several important differences. First, we consider new datasets; in particular, we use data from recent X-ray surveys (see also Moscardini et al. 2000b). Second, we use improved theoretical models (Matarrese et al. 1997). In particular we pay much attention to the redshift evolution in the clustering of clusters and to light-cone and selection effects. Third, we present theoretical predictions for some future surveys.

The plan of the paper is as follows. In Section 2 we list the observational cluster samples used in the following analysis. Section 3 is devoted to the presentation of our theoretical model to estimate the correlation of galaxy clusters both in the optical and in the X-ray bands. In Section 4 we present the results of a maximum-likelihood analysis performed on existing data and we discuss the constraints on the cosmological parameters. In Section 5 we show the predictions of the correlation length for possible future surveys. Conclusions are drawn in Section 6.

2 CLUSTER SAMPLES

In this section we present the observational data on the galaxy cluster correlation length that we will use in the following analysis to constrain the cosmological parameters. A summary of these data is presented in Table 1.

2.1 Optical data

As far as the optical band is concerned, we will use for our study the observational results from the analysis by Croft et al. (1997) on the APM cluster redshift survey and by Nichol et al. (1992) on the Edinburgh-Durham cluster catalogue (EDCC).

- To construct the APM cluster survey, an automated procedure has been used to select from the angular APM galaxy survey the clusters whose redshifts were later measured. An early sample, containing 364 clusters with richness \( R \geq 50 \), called Sample B (Dalton et al. 1994), has been created by fixing the limiting magnitude of the galaxy catalogue to \( b_j = 20.5 \). A new, richer sample was then obtained by Croft et al. (1997) by extending the limit to \( b_j = 21.0 \) and considering only clusters with richness \( R \geq 80 \). This catalogue (called Sample C) contains 165 objects with redshift \( cz \leq 55000 \) km s\(^{-1}\). Even if the clustering study made by Croft et al. (1997), using a maximum-likelihood technique, considered six different subsamples with decreasing cluster density, in our statistical analysis we will prefer to use the results only for the two whole samples B and C. This is done to reduce the possible problems coming from the assumption of independence of the different samples. This assumption is required by the way we write the likelihood (see equation 6). Our final results will not be affected by this choice, as shown by the fact that the confidence levels on the cosmological parameters obtained using sample B only are almost indistinguishable from those coming from the complete analysis of the optical datasets. The corresponding mean intracluster separations for samples B and C are 30 and 57 h\(^{-1}\) Mpc, respectively (\( h \) is the value of the local Hubble constant \( H_0 \) in units of 100 km s\(^{-1}\) Mpc\(^{-1}\)); the values of the correlation lengths \( r_0 \), of the slope \( \gamma \) of the correlation function, and their 1\( \sigma \) errorbars, obtained from a maximum-likelihood analysis, are reported in Table 1.

- The EDCC is a machine-based, objectively selected sample of galaxy clusters consisting of 737 objects of all richness, over 0.5 sr of sky centred on the South Galactic Pole (Lumsden et al. 1992). A sub-sample of 79 clusters, with at least 22 galaxies inside a radius corresponding to 1 h\(^{-1}\) Mpc, with magnitudes between the limits \( m_3 \) and \( m_3 + 2 \) (see Abell 1958 for the definitions) has been spectroscopically confirmed. The resulting catalogue, with a corresponding mean intracluster separation \( d_c = 46 h^{-1} \) Mpc, has been used by Nichol et al. (1992) to estimate the clustering properties. The values of \( r_0 \) and \( \gamma \) obtained from a least-squares fit are reported in Table 1.

2.2 X-ray data

In the following analysis we will consider four different catalogues of galaxy clusters selected in the X-ray band: the ROSAT All-Sky Survey 1 (RASS1) Bright Sample, the ROSAT Brightest Cluster Sample (BCS), the X-ray brightest Abell-type cluster sample (XBCs), the ROSAT – ESO Flux-limited X-ray sample (REFLEX).

Here we list some of their characteristics; more details can be found in the original papers.

- The RASS1 Bright Sample (De Grandi et al. 1999a) contains 130 clusters of galaxies selected from the ROSAT All-Sky Survey (RASS) data. The cata-
Table 1. The clustering data (in the optical and X-ray bands) used in the likelihood analysis. Column 1: catalogue name. Column 2: characteristics of the catalogue: for the optical catalogues, the richness \( R \) and the mean intrachannel distance \( d_c \); for the X-ray catalogues, the limiting luminosity \( L_x \) or the limiting flux \( S_{\text{lim}} \), unless the whole catalogue is analyzed. Column 3: reference for the clustering analysis. Column 4: number of clusters in the catalogue \( n_c \). Columns 5, 6 and 7: correlation length \( r_0 \) (in \( h^{-1} \text{Mpc} \)), the slope of the correlation function \( \gamma \) and the corresponding confidence levels of the quoted errorbars.

<table>
<thead>
<tr>
<th>Catalogue</th>
<th>Characteristics</th>
<th>Reference</th>
<th>( n_c )</th>
<th>( r_0 ) (( h^{-1} \text{Mpc} ))</th>
<th>( \gamma )</th>
<th>Errorbars</th>
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<tbody>
<tr>
<td>Optical band:</td>
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<tr>
<td>APM Sample B</td>
<td>( R \geq 50, d_c = 30 , h^{-1} \text{Mpc} )</td>
<td>Croft et al. (1997)</td>
<td>364</td>
<td>14.2^{+0.4}<em>{-0.9} , 2.13^{+0.09}</em>{-0.06} , 1\sigma</td>
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</tr>
<tr>
<td>APM Sample C</td>
<td>( R \geq 80, d_c = 57 , h^{-1} \text{Mpc} )</td>
<td><em>idem</em></td>
<td>110</td>
<td>18.4^{+2.2}<em>{-2.4} , 1.7^{+0.3}</em>{-0.3} , 1\sigma</td>
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<tr>
<td>EDCC</td>
<td>( d_c = 46 , h^{-1} \text{Mpc} )</td>
<td>Nichol et al. (1992)</td>
<td>79</td>
<td>16.4 \pm 4.0 , 2.1 \pm 0.3 , 1\sigma</td>
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<tr>
<td>X-ray band:</td>
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<tr>
<td>RASS1 Bright Sample</td>
<td>whole catalogue</td>
<td>Moscardini et al. (2000a)</td>
<td>130</td>
<td>21.5^{+3.4}<em>{-4.4} , 2.11^{+0.53}</em>{-0.56} , 2\sigma</td>
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<tr>
<td>BCS</td>
<td>( L_x \geq 0.24 \times 10^{44} \text{erg cm}^{-2} \text{s}^{-1} )</td>
<td>Lee &amp; Park (1999)</td>
<td>33</td>
<td>33.0^{+5.9}<em>{-2.3} , 1.82^{+0.49}</em>{-0.20} , 1\sigma</td>
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<tr>
<td>XBACs</td>
<td>whole catalogue</td>
<td>Borgani et al. (1999)</td>
<td>203</td>
<td>26.9 \pm 4.5 , 1.98^{+0.35}_{-0.15} , 2\sigma</td>
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<tr>
<td>REFLEX</td>
<td>( S_{\text{lim}} = 3 \times 10^{-12} \text{erg cm}^{-2} \text{s}^{-1} )</td>
<td>Collins et al. (2000)</td>
<td>449</td>
<td>18.8 \pm 0.9 , 1.83^{+0.10}_{-0.08} , 1\sigma</td>
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The catalogue has an effective flux limit in the \((0.5 - 2.0) \text{ keV}\) band between \(3.05 \times 10^{-12} \text{ erg cm}^{-2} \text{s}^{-1}\) over the selected area which covers a region of approximately \(2.5 \text{ sr}\) within the Southern Galactic Cap, i.e. \(\delta < 2.5^\circ\) and \(b < -20^\circ\). For our theoretical predictions we will use the exact sky map of the sample which is presented in Figure 2 of De Grandi et al. (1999a). The redshift distribution has a tail up to \(z \approx 0.3\) but the majority of the clusters have \(z < 0.1\). Moscardini et al. (2000a) found that the two-point correlation function of the whole sample is well fitted by a power-law with \(r_0 = 21.5^{+3.4}_{-4.4} \, h^{-1} \text{Mpc}\) and \(\gamma = 2.11^{+0.53}_{-0.56} \) (95.4 per cent confidence level with one fitting parameter).

- The BCS catalogue (Ebeling et al. 1998) is an X-ray selected, flux-limited sample of 201 galaxy clusters with \(z \leq 0.3\) drawn from the RASS data in the northern hemisphere (\(\delta \geq 0^\circ\)) and at high Galactic latitude (\(|b| \geq 20^\circ\)). The limiting flux is \(S_{\text{lim}} = 4.45 \times 10^{-12} \text{ erg cm}^{-2} \text{s}^{-1}\) in the \((0.1 - 2.4) \text{ keV}\) band. Since the sky-coverage \(\Omega_{\text{sky}}(S)\) of BCS is not available, we will use \(\Omega_{\text{sky}}(S) = \text{const} \approx 4.13 \text{ sr}\) for fluxes larger than \(S_{\text{lim}}\). Lee & Park (1999) analyzed the clustering properties of this catalogue using four different volume-limited subsamples. For simplicity, in the following analysis we will consider only the catalogue with \(L_x \geq 0.24 \times 10^{44} \text{ erg cm}^{-2} \text{s}^{-1}\) and a limiting redshift of \(z = 0.07\). The correlation function in this case is fitted by a power-law with \(r_0 = 33.0^{+6.2}_{-5.9} \, h^{-1} \text{Mpc}\) and \(\gamma = 1.82^{+0.49}_{-0.50} \) (68.3 per cent confidence level with one fitting parameter). We checked that the inclusion of the deeper subsamples produced very small changes in our likelihood analysis.

- The XBACs catalogue (Ebeling et al. 1996) is an all-sky X-ray sample of 242 Abell galaxy clusters extracted from the RASS data. Being optically selected, it is not a complete flux-limited catalogue. The sample covers high Galactic latitudes (\(|b| \geq 20^\circ\)). The adopted limiting flux is \(S_{\text{lim}} = 5 \times 10^{-12} \text{ erg cm}^{-2} \text{s}^{-1}\) in the \((0.1 - 2.4) \text{ keV}\) band. Also in this case, since the actual sky coverage is not available, we will adopt \(\Omega_{\text{sky}}(S) = \text{const} \approx 8.27 \text{ sr}\) for fluxes larger than \(S_{\text{lim}}\). The above mentioned selection effects produce a luminosity function for XBACs which is much lower in the faint part than that obtained from other catalogues (e.g. Ebeling et al. 1997; Rosati et al. 1998; De Grandi et al. 1999b). We take into account this incompleteness in our model following the same method described in Moscardini et al. (2000b).

The clustering properties of this catalogue have been studied by different authors. Abadi, Lambas & Muriel (1998) found for the whole catalogue a correlation length \(r_0 = 21.1^{+1.6}_{-1.2} \, h^{-1} \text{Mpc}\) (1\,sr errorbar) and a slope \(\gamma = -1.92\). Borgani, Plionis & Kolokotronis (1999) analyzed the same sample finding a somewhat larger correlation amplitude: \(r_0 = 26.0^{+4.1}_{-4.4} \, h^{-1} \text{Mpc}\) and \(\gamma = 1.98^{+0.35}_{-0.53} \) (95.4 per cent confidence level with one fitting parameter). The difference between these two estimates is probably due to a different assumption on the errors. Since the maximum-likelihood approach is more robust than the quasi-Poisson assumption made by Abadi et al. (1998), we prefer to use in the following analysis the results obtained by Borgani et al. (1999).

- The REFLEX survey (Böhringer et al. 1998) is a large sample of optically confirmed X-ray clusters selected from RASS. The sample includes 452 X-ray selected clusters in the southern hemisphere, at high Galactic latitude (\(|b| \geq 20^\circ\)). For our computations, we use the actual sky coverage given in Figure 1 of Collins et al. (2000). Using a catalogue with a limiting flux of \(3 \times 10^{-12} \text{ erg cm}^{-2} \text{s}^{-1}\) (defined in the 0.1 - 2.4 keV energy band), where the sky coverage falls to 97.4 per cent of the whole surveyed region (4.24 steradians), Collins et al. (2000) found that the two-point correlation function is fitted by a power-law with \(r_0 = 18.8 \pm 0.9 \, h^{-1} \text{Mpc}\).
and \( \gamma = 1.83_{-0.08}^{+0.15} \) (68.3 per cent confidence level from a maximum-likelihood analysis). As for the APM subsamples at higher richness, in our analysis we prefer not to use the clustering results for the REFLEX subsamples at higher luminosities, also discussed in Collins et al. (2000).

3 THEORETICAL MODELS FOR THE CORRELATION FUNCTION

Our theoretical predictions for the spatial two-point correlation function of galaxy clusters in different cosmological models have been here obtained by using an updated version of the method presented in Moscardini et al. (2000a,b), where the application was limited to X-ray selected clusters. Here we will give only a short description of the method and we refer to those papers for a more detailed discussion.

3.1 Clustering in the past-light cone

Matarrese et al. (1997; see also Moscardini et al. 1998 and Yamamoto & Suto 1999 and references therein) developed an algorithm to describe the clustering in our past light-cone taking into account both the non-linear dynamics of the dark matter distribution and the redshift evolution of the bias factor. The final expression of the bias factor. The final expression for the observed spatial correlation function \( \xi_{\text{obs}} \) in a given redshift interval \( Z \) is

\[
\xi_{\text{obs}}(r) = \frac{\int d r_{1} d z_{2} N(z_{1}) N(z_{2}) \xi_{\text{obj}}(r, z_{1}, z_{2})}{\left[ \int d z_{1} N(z_{1}) \right]^{2}},
\]

where \( N(z) \equiv N(z)/r(z) \) and \( N(z) \) is the actual redshift distribution of the catalogue. In the previous formula \( \xi_{\text{obj}}(r, z_{1}, z_{2}) \) represents the correlation function of pairs of objects at redshifts \( z_{1} \) and \( z_{2} \) with comoving separation \( r \). An accurate approximation for it is given by

\[
\xi_{\text{obj}}(r, z_{1}, z_{2}) \approx b_{\text{eff}}(z_{1}) b_{\text{eff}}(z_{2}) \xi_{m}(r, z_{\text{ave}}),
\]

where \( \xi_{m} \) is the dark matter covariance function and \( z_{\text{ave}} \) is a suitably defined intermediate redshift.

Another important ingredient entering the previous equation is the effective bias \( b_{\text{eff}} \) which can be expressed as a weighted average of the ‘monochromatic’ bias factor \( b(M, z) \) of objects of some given intrinsic property \( M \) (like mass, luminosity, etc):

\[
b_{\text{eff}}(z) = N(z)^{-1} \int_{M} d M' b(M', z) N(z, M'),
\]

where \( N(z, M) \) is the number of objects actually present in the catalogue with redshift in the range \( z, z + dz \) and \( M \) in the range \( M, M + dM \), whose integral over \( \ln M \) is \( N(z) \).

As galaxy clusters are expected to form by the hierarchical merging of smaller mass units, one can fully characterize their properties by the mass \( M \) of their hosting dark matter haloes at each redshift \( z \). One can then estimate their comoving mass function \( \dot{n}(z, M) \) by the Press-Schechter (1974) formula and adopt the Mo & White (1996) relation for the monochromatic bias. Actually, we use the relations recently introduced by Sheth & Tormen (1999) and Sheth, Mo & Tormen (2001), which have been shown to produce a more accurate fit of the distribution of the halo populations in numerical simulations (Jenkins et al. 2001).

To complete our model we need a technique to compute the redshift evolution of the dark matter covariance function \( \xi_{m} \). For this, we use the fitting formula by Peacock & Dodds (1996), which allows to evolve \( \xi_{m} \) into the fully non-linear regime.

3.2 From the catalogue characteristics to the halo mass

In order to predict the abundance and clustering of galaxy clusters in the different catalogues described above (both in the optical and X-ray bands) we need to relate the sample characteristics to a corresponding halo mass at each redshift.

For the optical catalogues, the various samples of APM and EDCC considered by Croft et al. (1997) and Nichol et al. (1992) respectively, are characterized by a different cluster number density (see Table 1). In this case, for each cluster subsample, we fix the minimum mass (which in our model represents the lower limit of the integral in equation 3) of the hosting dark matter haloes in such a way that the comoving cumulative mass function reproduces the number density of clusters in the range of redshift sampled by the original catalogues.

For X-ray selected clusters, we use a different approach (see Moscardini et al. 2000a,b and Suto et al. 2000). In this case it is quite easy to relate the limiting flux (and/or eventually the limiting luminosity, as required for instance by the BCS subsample that we consider) to the minimum halo mass. In fact the flux \( S \) in a given band corresponds to an X-ray luminosity \( L_{X} = 4 \pi d_{L}^{2} S \) in the same band, where \( d_{L} \) is the luminosity distance. The quantity \( L_{X} \) can be converted into the total luminosity \( L_{\text{bol}} \) by performing band and bolometric corrections (we assume an overall ICM metallicity of 0.3 times solar). Local observations suggest that the cluster bolometric luminosity is related to the temperature by a simple relation: \( T = A L_{\text{bol}}^{\frac{1}{2}} \), where the temperature is expressed in keV and \( L_{\text{bol}} \) is in units of \( 10^{44} h^{-2} \) erg s\(^{-1}\). We assume \( A = 4.2 \) and \( B = 1/3 \), which are a good representation of the data with \( T_{\text{bol}} \gtrsim 1 \) keV (e.g. Markevitch 1998 and references therein). Below this temperature (i.e. for galaxy groups) the \( L_{\text{bol}} - T \) relation has much steeper a slope: for this reason we set a minimum temperature at \( T = 1 \) keV. Recent analyses of cluster temperature data at higher redshifts (Mushotzky & Scharf 1997; Donahue et al. 1999) are consistent with no evolution in the \( L_{\text{bol}} - T \) relation out to \( z \approx 0.4 \). Therefore, we can safely assume that the previous relation holds true in the redshift range sampled by the considered catalogues. Moscardini et al. (2000b), who allowed a moderate redshift evolution of the \( L_{\text{bol}} - T \) to reproduce the observed \( \log N \)-\( \log S \) relation, showed that the clustering properties are only slightly sensitive to this assumption. To convert the cluster temperature
into the mass of the hosting dark matter halo, we assume virial isothermal gas distribution and spherical collapse: $T \propto M^{2/3} E^{2/3}(z) \Delta_{v_0}(z)^{1/3}$, where $\Delta_{v_0}$ represents the mean density of the virialized halo in units of the critical density at that redshift and $E(z)$ is the ratio between the value of the Hubble constant at redshift $z$ and today. Once the relation between observed flux and halo mass at each redshift is established we have to account for the catalogue sky coverage $\Omega_{sky}(S)$ (when available) to predict the redshift distribution.

3.3 Redshift distortions

We include the effect of redshift-space distortions using linear theory and the distant-observer approximation (Kaiser 1987). Under these assumptions the enhancement of the redshift-space averaged power spectrum is approximately $1 + 2\beta / 3 + 3\beta^2 / 5$, where $\beta \simeq \Omega_{c0}^{6} / h_{0.7}$. We find that the total effect of redshift distortions on the value of the correlation length is always smaller than 10 per cent.

4 RESULTS

4.1 Cosmological models

In what follows we use a maximum-likelihood analysis to constrain the main parameters defining a cosmological model. We will consider a set of structure formation models all belonging to the general class of cold dark model. We will consider a set of structure formation models all belonging to the general class of cold dark matter (CDM) ones, for which the linear power-spectrum reads $P_{lin}(k, 0) \propto k^n T^2(k)$, with $T(k)$ the CDM transfer function (Bardeen et al. 1986):

$$T(q) = \frac{\ln(1 + 2.34q)}{2.34q} \times [1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{-1/4},$$

where $q = k / h \Gamma$. The shape parameter $\Gamma$ depends on the Hubble parameter $h$, on the matter density $\Omega_{c0}$ and on the baryon density $\Omega_{b0}$ (Sugiyama 1995):

$$\Gamma = \Omega_{b0} h \exp(-\Omega_{c0} - \sqrt{h / 0.5} \Omega_{b0} / \Omega_{c0}).$$

We fix the spectral index $n$ to unity and we allow $\Gamma$ to vary in the range 0.05–0.5, while $\Omega_{c0}$ ranges from 0.1 to 1 in the framework of both open and flat models, with a cosmological constant contributing to the total density with $\Omega_{c0} = 1 - \Omega_{b0}$. Finally, we use different normalisations of the primordial power-spectrum, parameterized by $\sigma_8$ (the r.m.s. fluctuation amplitude in a sphere of $8h^{-1}$ Mpc) in the range $0.2 \leq \sigma_8 \leq 2$.

In summary, the cosmological models we consider are defined by four parameters: $\Omega_{c0}$, $\Omega_{c0}$, $\Gamma$ and $\sigma_8$.

4.2 Maximum likelihood analysis

Confidence levels for the cosmological parameters are obtained through a maximum likelihood analysis. The likelihood is $L \propto \exp(-\chi^2 / 2)$, where

$$\chi^2 = \sum_{i=1}^{N_{data}} \frac{[r_0(i) - r_0(i; \Omega_{c0}, \Omega_{c0}, \Gamma, \sigma_8)]^2}{\sigma^2_{r_0}(i)}.$$

The sum runs over the observational dataset described in Section 2, i.e. $N_{data} = 3$ and $N_{data} = 4$ for the optical and X-ray bands, respectively. The quantities $r_0(i)$ and $\sigma_{r_0}(i)$ represent the values of the correlation length and its 1σ error bar for each catalogue, as reported in Table 1; $r_0(i; \Omega_{c0}, \Omega_{c0}, \Gamma, \sigma_8)$ is the corresponding theoretical prediction obtained with a given choice of cosmological parameters. Note that the observed correlation lengths reported in Table 1 have been obtained using different fitting methods (least-squares or maximum-likelihood analyses) and over different spatial scales in ways which could be inconsistent with each other. Moreover, the quoted values of $r_0$ come from fits of the data where also the values of the slope are marginalized and it is known that there is significant covariance between $r_0$ and $\gamma$. In spite of this, we prefer not to estimate $r_0$ through the standard power-law fit procedure with two parameters, because in the case of the maximum-likelihood analyses the comparison would not be appropriate. For these reasons we define the predicted correlation length as the distance where the two-point correlation function is unity. However, in all cases we also checked that the values of the slope $\gamma$ obtained from the power-law fit are well inside the observed 1σ error bar. This ensures that the effect of the different definition of $r_0$ is small.

Finally, we note that equation 6 assumes that the different data are uncorrelated. This is expected to be a fair assumption, especially for the X-ray data, which come from different catalogues. In the case of the optical datasets, some larger correlation is probably present between the APM samples, due to the presence of common objects in the two catalogues. However, we will show that we obtain almost identical conclusions about the confidence levels for the cosmological parameters considering the APM sample B only or the whole set of optical catalogues. This shows that the assumption of independence which underlies our likelihood analysis is not essential for the results.

Finally, the best-fit cosmological parameters are obtained by maximizing $L$, i.e. by minimizing $\chi^2$. The 95.4 and 99.73 per cent confidence levels for the parameters are computed by finding the region corresponding to an increase $\Delta \chi^2$ with respect to the minimum value of $\chi^2$: the exact value of $\Delta \chi^2$ depends on the given number of degrees of freedom $\nu_f$.

A similar analysis has been very recently performed by Robinson (2000), who used a simplified version of the above model and considered only APM data (all six subsamples). Wherever the comparison is possible, the results are qualitatively similar, but some quantitative differences are present. These differences can be ascribed to a different definition of the effective bias and, above all, to the absence of any account of the past-light cone effects in his analysis.

We start our analysis by considering as a free parameter $\Omega_{c0}$ only. We fix $\Gamma = 0.2$, which is in the range suggested by different works (see e.g. Peacock & Dodds 1996), and $\sigma_8$ to reproduce the cluster abundance. For the normalization we adopt the fitting formula by Viana & Liddle (1999), who revised the Henry & Arnaud (1991) dataset of cluster X-ray temperatures.
and included a treatment of measurement errors. Their formula reads
\[ \sigma_8 = 0.56 \Omega_{0m}^{1/2}, \]
where \( C = 0.34 \) and \( C = 0.47 \) for open and flat models, respectively. The claimed accuracy of this expression is better than 3 per cent for \( \Omega_{0m} \) between 0.1 and 1. The results of our maximum likelihood analysis obtained by using the complete set of catalogues are shown for flat models by heavy solid lines in Figure 1. The corresponding results for open models (not shown here) are very similar. The left panel refer to the analysis of the optical catalogues. In this case the minimum of \( \chi^2 \) is in the range \( \Omega_{0m} = 0.2 - 0.3 \) and we find \( \Omega_{0m} \gtrsim 0.5 \) and \( \Omega_{0m} \lesssim 0.7 \) at 95.4 and 99.73 per cent levels, respectively. The results for the X-ray catalogues are shown in the right panel. Also in this case the most likely value for \( \Omega_{0m} \) is 0.2-0.3 but the allowed regions are much smaller, i.e. the X-ray data on the correlation length give tighter constraints: \( 0.2 \lesssim \Omega_{0m} \lesssim 0.35 \) and \( 0.2 \lesssim \Omega_{0m} \lesssim 0.45 \) at 95.4 and 99.73 per cent levels, respectively. This result is in qualitative agreement with our previous analyses, where we found that an Einstein-de Sitter model can be rejected because it predicts too low a correlation function for the RASS1 and XBACs catalogues (Moscardini et al. 2000a,b).

In order to understand which subsamples contribute to the constraints, we repeat the same analysis using in both the optical and X-ray cases the largest catalogues, i.e. APM sample B and REFLEX. The corresponding curves are shown in the relative panels as light dotted lines. Unlike the case of the optical catalogues where very small differences are found, the constraints obtained using REFLEX only are less tight: 0.2 \( \lesssim \Omega_{0m} \lesssim 0.5 \) and 0.2 \( \lesssim \Omega_{0m} \lesssim 0.6 \) at 95.4 and 99.73 per cent levels, respectively. This shows the importance of including in the analysis the subsamples at higher X-ray luminosities.

In Figure 2 we show the results of our maximum likelihood analysis when we only fix the model normalisation to reproduce the cluster abundance, leaving two free parameters: \( \Omega_{0m} \) and \( \Gamma \).

The upper panels refer to the analysis of the optical catalogues. Confidence levels are shown separately for flat (left panels) and open (right panels) models. Here \( \nu_f = 2 \), i.e. \( \Delta \chi^2 = 6.17 \) and 11.8, for 95.4 and 99.73 per cent levels, respectively. The allowed regions, at least for values of the density parameter \( \Omega_{0m} \gtrsim 0.4 \), appear to strongly depend only on \( \Gamma \). The data suggest \( \Gamma \sim 0.15 \pm 0.05 \) (errorbars are at 3\( \sigma \) level), both for open and flat models. At smaller \( \Omega_{0m} \), the resulting values of \( \Gamma \) are larger, especially for models with non-zero cosmological constant. For instance, for \( \Omega_{0m} = 0.2 \) we find \( \Gamma \sim 0.24^{+0.10}_{-0.08} \) for flat models, and \( \Gamma \sim 0.20^{+0.06}_{-0.05} \) for open ones. The centre of the allowed region gives a relation between \( \Gamma \) and \( \Omega_{0m} \), namely \( \Gamma = 0.382-0.797\Omega_{0m}+1.029\Omega_{0m}^2-0.478\Omega_{0m}^3 \) for the flat models and \( \Gamma = 0.215-0.079\Omega_{0m}-0.064\Omega_{0m}^2+0.069\Omega_{0m}^3 \) for the open models.

The lower panels present the results obtained for the X-ray catalogues. Even if the allowed regions are qualitatively compatible with those displayed by the optical analysis, we notice some differences. First, the 2- and 3\( \sigma \) regions are narrower, i.e. the X-ray data on the correlation length give tighter constraints on the cosmological parameters. Once again, the \( \Omega_{0m} \) depen-
Figure 2. Confidence contours (95.4 and 99.73 per cent confidence levels; light and heavy solid lines, respectively) of $\Gamma$ and $\Omega_{0m}$ for CDM models with normalization reproducing the cluster abundance. The left panels refer to flat cosmological models with varying cosmological constant $\Omega_{0\Lambda} = 1 - \Omega_{0m}$; the right ones to open models with vanishing $\Omega_{0\Lambda}$. The upper row shows the results obtained using the optical dataset, while the lower one refers to the dataset in the X-ray band. The results obtained using only the largest subsamples (APM sample B for the optical case, REFLEX for the X-ray band one) are shown by light and heavy dotted lines (95.4 and 99.73 per cent confidence levels, respectively).
dence is small for high-density models: we find $\Gamma$ in the range 0.1-0.15 ($3\sigma$ confidence levels) for $\Omega_m \gtrsim 0.5$, with no dependence on the presence of a cosmological constant. Again, models with low matter density favour larger values of $\Gamma$, between 0.2 and 0.3. The relations describing the centre of the allowed region are $\Gamma = 0.487 - 1.342\Omega_m + 1.687\Omega_m^2 - 0.73\Omega_m^3$ for the flat models and $\Gamma = 0.394 - 0.933\Omega_m + 1.084\Omega_m^2 - 0.44\Omega_m^3$ for the open models.

Note that, combining the optical and X-ray data altogether, the allowed regions in the parameter space (not reported in the Figure) are very similar to those obtained in the analysis of the X-ray catalogues only, showing once again that these data have larger discriminating power.

Also in this case we checked the robustness of the results when only the largest catalogues are used in the analysis. The results, presented by dotted lines, show that the confidence regions obtained using the APM sample B are very similar to those obtained using all three optical catalogues. On the contrary, when we use the REFLEX catalogue only, the resulting allowed regions, even if compatible with those obtained using also the catalogues at higher X-ray luminosity, appear to be smaller. For instance, values of $\Gamma$ as large as 0.4 cannot be excluded if the matter density parameter is small ($\Omega_m \lesssim 0.2$).

In Figure 3 we present the constraints in the $\Gamma - \sigma_8$ plane, once the values for $\Omega_m$ and $\Omega_b$ are fixed (i.e. $\nu_f = 2$). We consider an Einstein-de Sitter model, an open model with $\Omega_m = 0.3$ and a flat model, always with $\Omega_m = 0.3$. Again the upper panels refer to the optical data. The confidence limits are quite similar for the three cosmological models, confirming once again the weak dependence on the density parameter. These results can be directly compared to the analysis made by Robinson (2000; its Figure 3): the agreement is good, even if at a given value of $\sigma_8$ slightly larger values of $\Gamma$ are allowed in our analysis.

The lower panels refer to the X-ray data and show the strongest constraints coming from these data. For example, for the Einstein-de Sitter model only a small region with values of $\Gamma$ quite close to 0.1 and 0.42 $\lesssim \sigma_8 \lesssim 1.1$ is allowed. The confidence limits obtained in the case of open and flat models with $\Omega_m = 0.3$ are similar and are consistent with the analysis of the optical data, but the allowed region is narrower: the $2\sigma$ region has $\Gamma$ in the range 0.14-0.22 and $\sigma_8$ between 0.6 and 1.3 for the open model, while $0.16 \lesssim \Gamma \lesssim 0.22$ and $0.7 \lesssim \sigma_8 \lesssim 1.3$ for the flat model.

Again the combination of optical and X-ray catalogues produces results almost indistinguishable from those obtained by the X-ray analysis only. The main difference is the further reduction of the allowed region in the case of the Einstein-de Sitter model: in this case we find $\Gamma \sim 0.1$ and $0.45 \lesssim \sigma_8 \lesssim 0.6$.

When the analysis is limited to the X-ray catalogues, again no significant differences are found between the confidence regions allowed by the APM sample B and the complete set of optical catalogues. On the contrary, the results for the X-ray catalogues show that the inclusion in the analysis of the shallower catalogues allows to reduce the confidence regions, excluding high values of $\Gamma$ and $\sigma_8$.

In conclusions, the previous results show that the constraints coming from the X-ray datasets are in general tighter than those obtained from the optical data. This is also evident from the analysis limited to the largest corresponding catalogues (APM sample B and REFLEX). At first glance this is unexpected. In fact the errorbars of the correlation length $r_0$ for the APM sample B are smaller than the REFLEX ones (see Table 1) and consequently their weights in the $\chi^2$ estimates are larger. However, the galaxy clusters belonging to the X-ray datasets typically have a mass larger than the optical ones. In fact the density of the present optical clusters requires a minimum mass smaller than that obtained from the limiting fluxes of the X-ray catalogues here considered. Since the bias factor entering in the correlation estimates is strongly dependent on the mass (see, for example, the discussion in Moscardini et al. 2000b), this difference in mass produces a large spread of the $r_0$ values predicted for different cosmological models, increasing the constraining ability of the results. For a similar reason, the inclusion in our analysis of other X-ray datasets besides REFLEX, even if with a smaller number of objects and with larger errorbars, helps in reducing the allowed regions in the parameter space. In fact these catalogues have a larger limiting flux than REFLEX and sample the cluster population at higher mass.

5 PREDICTIONS FOR FUTURE SURVEYS

5.1 The catalogues

In this section we will use our model to predict the correlation length expected in possible future surveys both in the optical and X-ray bands. Thanks to their depth, in these surveys it would be possible to obtain information on the high-redshift behaviour of the cluster two-point function. For this reason we present our predictions dividing the data in two different redshift bins ($z \leq 0.3$ and $z > 0.3$), to allow a discussion of the redshift evolution of cluster clustering.

In the optical band, a large improvement of our knowledge on the properties of the large-scale structure as traced by galaxy clusters will be obtained when the Sloan Digital Sky Survey (SDSS; York et al. 2000) will be completed. This survey, which covers an area three times larger than the APM, will contain redshifts for approximately one million galaxies. The expected number of galaxy clusters having at least 100 redshift measurements is approximately 1000. Moreover, the availability of $5 \times 10^7$ galaxies in the photometric data (complete to a magnitude limit of $r' \simeq 22$) will allow the application of automated cluster-finding algorithms (e.g. the matched-filter approach), to extend the cluster catalogue to higher redshifts. In its final form the SDSS is expected to be nearly as deep as two existing catalogues, the Palomar Distant Cluster Survey (PDCS; Postman et al. 1996; Holden et al. 1999) and the ESO Imaging Survey (EIS; Olsen et al. 1999; Scoddig et al. 2000).
Figure 3. Confidence contours (95.4 and 99.73 per cent confidence levels; light and heavy solid lines, respectively) of $\Gamma$ and $\sigma_8$ for CDM models with $(\Omega_0m, \Omega_0\Lambda) = (1, 0)$ (left panels), $(0.3, 0)$ (central panels) and $(0.3, 0.7)$ (right panels). The upper row shows the results obtained using the optical dataset, while the lower one refers to the X-ray dataset. The results obtained using only the largest subsamples (APM sample B for the optical case, REFLEX for the X-ray band one) are shown by light and heavy dotted lines (95.4 and 99.73 per cent confidence levels, respectively).
which however are much smaller, covering only 5.1 and 14.4 square degrees, respectively. These two catalogues have been built to find good distant candidates for successive more detailed observations and cannot be considered complete and well-defined samples suitable for statistical studies. Nevertheless, considering the extended versions of these catalogues, the number density of candidates and the corresponding estimated redshift distributions are compatible. For these reasons we decided to use their characteristics to define the properties of a possible future cluster survey in the optical band. In particular, we use the number density measured for the EIS catalogue, which consists of 304 objects in the redshift range $0.2 \lesssim z \lesssim 1.3$, with a median redshift of $z \sim 0.5$. The redshifts have been estimated by applying the matched-filter algorithm (Postman et al. 1996) and have an intrinsic uncertainty of at least $\Delta z = 0.1$. A large effort began to validate these cluster candidates. Preliminary results (da Costa et al. 2000; Ramella et al. 2000) show that more than 65 per cent of the studied candidates have strong evidence of being real physical associations. Moreover, first direct spectroscopic determinations of redshifts are in reasonable agreement with those derived from the matched-filter algorithm, with a possible systematic difference of $\Delta z \sim 0.1$. We will compute the predictions of the clustering properties of this catalogue by fixing the minimum mass needed to reproduce the EIS cluster density for $z \leq 0.3$ and $z > 0.3$. This choice automatically takes into account the possibility that false candidates are included in the EIS catalogue by reducing the required minimum mass. As a consequence, the estimated correlation function will refer to the objects targeted as candidates and not to the real clusters which will be actually validated using spectroscopic techniques. The underlying assumption is that the failure of the technique used to select the candidates comes from the inclusion in the catalogue of high-luminosity systems with a mass just below the minimum mass of the true EIS clusters.

In the X-ray band, the existing cluster clustering studies were expected to be largely overcome by the data that the ABRIXAS satellite (Trümper, Hasinger & Staubert 1998) was expected to collect, starting from mid 1999. Unluckily, problems with energy supply caused the untimely loss of the satellite at the end of April 1999. In the plans, the ABRIXAS catalogue would have covered an area of 8.27 steradians up to a limiting flux of $S_{\text{lim}} = 5 \times 10^{-13}$ erg cm$^{-2}$ s$^{-1}$ in the 0.5–2 keV band. These characteristics have been used by Moscardini et al. (2000b) to make predictions of the cluster two-point correlation function.

More recently, it has been proposed to use the very high sensitivity and good point-spread function of the XMM/Newton satellite, successfully launched in December 1999, to build a very deep large-scale structure survey. The idea is to cover a region of 64 square degrees at high galactic latitude using $24 \times 24$ 10ks XMM/EPIC pointings separated by 20 arcmin offsets. The expected limiting flux will be approximately $S_{\text{lim}} = 5 \times 10^{-15}$ erg cm$^{-2}$ s$^{-1}$ in the 0.5–2 keV band, which is 500 times more sensitive than the REFLEX one (see Pierre 2000 for more details). We predict the clustering properties of this sample using a constant sky coverage (in absence of real estimates) and assuming the previous limiting flux. Hereafter this sample will be called XMM/LSS.

5.2 Results

In Figure 4 we show the predicted value of the correlation length for future surveys in the optical band (EIS) and in the X-ray band (XMM/LSS) as a function of the present matter density content $\Omega_{\text{m0}}$. Here we consider only CDM models with $\sigma_8$ fixed to reproduce the cluster abundances (see equation 7) and we allow the $\Gamma$ parameter to assume values from 0.1 to 0.5. The results are shown separately for clusters having $z \leq 0.3$ and $z > 0.3$. As a general result, we find that the presence of a cosmological constant increases the correlation length by a factor always smaller than 15 per cent. Once again we find that the dependence on the shape parameter $\Gamma$ is strong: the higher $\Gamma$, the lower the predicted correlation length. On the contrary varying the matter density parameter, once $\Gamma$ is fixed, changes $r_0$ only by a factor of at most 20 per cent. More precisely, we find a slight decrease of the correlation length with increasing $\Omega_{\text{m0}}$. All the catalogues, both in optical and in X-ray bands, display a positive redshift evolution of the clustering, i.e. the estimates of $r_0$ are larger for clusters at high redshifts. This result, which confirms a previous analysis by Moscardini et al. (2000b), is due to the large increase of the bias factor with redshift; this increase overcompensates the corresponding decrease of the dark matter correlation function. In fact, considering high redshifts, galaxy clusters become rarer objects, connected to higher density fluctuations.

The relatively low values of the predicted correlation length for the EIS catalogue show that its cluster number density corresponds to objects with a small mass, with some possible contamination coming from false candidates, which would reduce the amplitude of the expected clustering.

Finally, we notice that the XMM/LSS sample has smaller $r_0$ than the ABRIXAS survey considered in our previous analysis (Moscardini et al. 2000b). Moreover the increase of clustering with redshift is less evident for the XMM/LSS sample than for the ABRIXAS catalogue. This is what we expect: when the limiting flux is decreased, a large number of small clusters enter in the catalogue, resulting in a smaller correlation function (see also Moscardini et al. 2000b).

Figure 5 shows the predicted correlation length for the EIS catalogue when the geometry of the universe is fixed and the values of $\Gamma$ and $\sigma_8$ are varied. We consider here the Einstein-de Sitter model and the open and flat models with $\Omega_{\text{m0}} = 0.3$. The small differences in the corresponding results confirm the slight dependence on the cosmology chosen. We find that, at a given $\sigma_8$, $r_0$ is a decreasing function of $\Gamma$, while, fixing $\Gamma$, $r_0$ is always an increasing function of $\sigma_8$.

A similar analysis has been performed using the characteristics of the XMM/LSS survey. Comparing the results, shown in Figure 6, with the EIS ones, we can
Figure 4. The predicted value of the correlation length (in $h^{-1}$ Mpc) for different future surveys in the optical band (EIS, left column) and in the X-ray band (XMM/LSS, left column) as a function of the present matter density content $\Omega_{0m}$. Results are presented for CDM models with normalization fitted to reproduce the cluster abundance. The different lines refer to different values of the $\Gamma$ parameter: 0.1, 0.2, 0.3, 0.4 and 0.5, from top to bottom. Dotted lines: flat cosmological models (i.e. with non-zero cosmological constant); solid ones: open models with vanishing $\Omega_{0\Lambda}$. The upper panels show the results obtained using clusters with $z \leq 0.3$, while the lower ones are for $z > 0.3$ clusters.
Figure 5. The predicted values of the correlation length (in $h^{-1}$ Mpc) for the EIS survey as a function of the value of the spectrum normalisation $\sigma_8$. Results are presented for CDM models with $(\Omega_0, \Omega_0) = (1, 0)$ (left panels), (0.3,0) (solid lines in the right panels) and (0.3,0.7) (dotted lines in the right panels). The different lines refer to different values of $\Gamma$: 0.1, 0.2, 0.3, 0.4 and 0.5, from top to down. The upper panels shows the results obtained using clusters with $z \leq 0.3$, while the lower one are for $z > 0.3$ clusters.
notice that the $\sigma_8$-dependence of $r_0$ is different: for very small normalisations, the correlation length is a decreasing function of $\sigma_8$. This effect is due to the decrease of the effective bias, which is more rapid than the growth of the dark matter correlation function.

It is interesting to note that, given a cosmological model, the predicted values of the correlation length for EIS and XMM/LSS are quite different, but the spread of $r_0$ obtained in different cosmological models is expected to be of the same order of magnitude for the two cases. Unfortunately this does not allow to understand whether future X-ray or optical surveys will better constrain cosmological parameters.

Another important issue to be discussed is the constraining ability of the clustering analysis with respect to the standard analysis based on cluster counts.

In Figure 7 we show the expected number of galaxy clusters in the XMM/LSS survey as a function of the spectrum normalisation $\sigma_8$. Results refer to the same cosmological models presented in the previous figures. We notice that these numbers are obtained directly from the model described above and do not pretend to reproduce the observed $\log N - \log S$ relation for clusters. To this aim it would be necessary to introduce an ad-hoc redshift evolution of the temperature-luminosity relation with one more parameter (see Section 3.2 and the discussion in Moscardini et al. 2000b). As already known, the results show a strong dependence on $\sigma_8$ and a relatively weak dependence on $\Gamma$. For instance, using the values of the normalisation suggested by equation 7, the change in the predicted number of clusters for $0.1 \leq \Gamma \leq 0.3$ (which includes the values suggested by
Figure 7. The expected number of galaxy clusters with \( z \leq 0.3 \) (upper panels) and \( z > 0.3 \) (lower panels) in the XMM/LSS survey as a function of the value of the spectrum normalisation \( \sigma_8 \). Results are presented for CDM models with \((\Omega_0 m, \Omega_0 \Lambda) = (1,0)\) (left panels), \((0.3,0)\) (solid lines in the right panels) and \((0.3,0.7)\) (dotted lines in the right panels). The different lines refer to different values of the \( \Gamma \) parameter: 0.1, 0.2, 0.3, 0.4 and 0.5, from bottom to top.

6 CONCLUSIONS

In this paper we discussed the model constraints that it is possible to infer from the analysis of the observational data on the clustering of galaxy clusters. A set of 3 optical (coming from the APM and EDCC catalogues) and 4 X-ray catalogues (RASS1 Bright Sample, BCS, XBACs, REFLEX) has been considered. The theoretical predictions for the different cosmological models have been obtained by using a model which accounts for the clustering of observable objects in our past lightcone and for the redshift evolution of both the underlying dark matter covariance function and the cluster bias factor. A linear treatment of redshift-space dis-
tortions has been also included. In the case of X-ray selected clusters we followed the approach of Moresco et al. (2000b), which makes use of theoretical and empirical relations between mass, temperature and X-ray luminosity to convert the limiting flux of catalogues into a corresponding minimum mass for the dark matter haloes hosting the clusters. In the optical band, we used the mean distance (i.e. the number density) of the observed clusters to fix the minimum mass required by the model.

Our theoretical predictions have been compared with the observed correlation lengths by means of a maximum-likelihood analysis. We considered cosmological models belonging to the cold dark matter class, defined by four parameters: the closure density in dark matter and vacuum energy ($\Omega_m$ and $\Omega_\Lambda$, respectively); the power-spectrum shape parameter $\Gamma$ and normalisation $\sigma_8$.

Our main results can be summarized as follows:

- The constraints coming from X-ray and optical clusters are consistent but the former appear to be tighter.
- If we fix the power-spectrum shape parameter $\Gamma = 0.2$, in the range suggested by different observational datasets, and the power-spectrum normalisation to reproduce the cluster abundance, we obtain strong constraints on the value of the matter density parameter, independently of the presence of a cosmological constant: $0.064 \Omega_m + 0.069 \Omega_\Lambda$ for flat and open models, respectively.
- If we allow the shape parameter to vary, we find that the clustering properties of clusters are only weakly dependent on $\Omega_m$ and $\Omega_\Lambda$. On the contrary, the results appear to be much more strongly sensitive to $\Gamma$. In fact, smaller shape parameters correspond to higher correlation lengths.
- Considering models with the normalisation coming from the cluster abundance, we find that the centre of the region allowed by the maximum-likelihood analysis of the optical data is described by the relation $\Gamma \sim 0.382 - 0.797 \Omega_m + 1.029 \Omega_\Lambda$ and $\sigma_8 \sim 0.73 \Omega_m + 0.933 \Omega_\Lambda$ for flat and open models, respectively. Considering the catalogues in the X-ray band, we find $\Gamma \sim 0.487 - 1.342 \Omega_m + 1.687 \Omega_\Lambda$ and $\sigma_8 \sim 0.22$ for the flat models and $\Gamma \sim 0.394 - 0.934 \Omega_m + 1.084 \Omega_\Lambda$ and $\sigma_8 \sim 0.44 \Omega_\Lambda$ for the open models.
- Using X-ray selected data only, we find that for the Einstein-de Sitter model the value of $\Gamma$ has to be quite close to 0.1 with $0.45 \lesssim \sigma_8 \lesssim 1.1$; for open and flat models with $\Omega_m = 0.3$ the $2\sigma$ region has $0.14 \lesssim \Gamma \lesssim 0.22$ and $0.6 \lesssim \sigma_8 \lesssim 1.3$.

We also used our model to make predictions on the clustering properties of galaxy clusters expected in future surveys. We considered an optical catalogue with characteristics similar to the EIS project and a deep X-ray catalogue with the characteristics of the XMM/LSS survey. From this analysis we can conclude that:

- Clusters at high redshifts are expected to have a larger correlation function than at low redshifts.
- Again, predictions are almost insensitive to the presence of a cosmological constant while they are strongly dependent on the shape parameter.
- The predicted clustering for the EIS catalogue is relatively small for all cosmological models suggesting that its cluster number density corresponds to objects with small mass, including some possible false candidates.
- The correlation length for X-ray selected clusters is confirmed to depend on the limiting flux of the survey: the deeper the catalogue, the smaller $r_0$. Moreover, the redshift evolution of clustering is less evident in deeper catalogues.

In conclusion, our results show that the existing data on the clustering properties of clusters can be successfully used to put constraints on the cosmological parameters. The future availability of deeper surveys can increase the power of this approach, which can be considered complementary to the traditional study of cluster abundances.

Acknowledgments.

This work has been partially supported by Italian MURST, CNR and ASI, and by the TMR european network “The Formation and Evolution of Galaxies” under contract n. ERBFMRX-CT96-086. LM thank the Max-Planck-Institut für Astrophysik for its hospitality during the visit when this work was completed. We want to warmly thank Sabrina De Grandi for stimulating comments and helpful suggestions. We are grateful to C. Collins for having provided the clustering results of the REFLEX catalogue before publication and to Hervé Aussel, Marco Scodeggio and Bepi Tormen for clarifying discussions. We thank the anonymous referee for useful suggestions which helped to improve the presentation of our results.

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