ABSTRACT

We present a new model for low-luminosity X-ray sources in globular clusters, with $L_X \lesssim 10^{34}$ erg s$^{-1}$. The model we propose is that of a single neutron star accreting from cluster gas that has accumulated as a natural product of stellar evolution. An analytic luminosity function is derived under the assumption that the speed distribution of neutron stars in the central region of a cluster is described by a Maxwellian, and that the density and temperature of the gas are uniform. Predictions of the model and implications for the gas content of globular clusters are discussed.

1. INTRODUCTION

Globular clusters contain at least two varieties of X-ray sources, differentiated by their luminosities. There are a dozen bright X-ray sources observed in the Milky Way globular cluster system, with $L_X \sim 10^{36} - 10^{38}$ erg s$^{-1}$ (Deutsch, Margon, & Anderson 2000). Nearly all of these higher luminosity sources show type I X-ray bursts (Lewin, van Paradijs, & Taam 1993), and it is, therefore, clear that they are accreting neutron stars (NSs) in binary systems. In fact, 7 of the 12 bright sources have well-measured or constrained binary periods (Deutsch, Margon, & Anderson 2000). There is also a population of dim cluster X-ray sources (DCXSs), with $L_X \sim 10^{31} - 10^{34}$ erg s$^{-1}$, where the lower limit is set by the detection sensitivities. To date, $> 30$ of these dim sources have been observed, primarily with the ROSAT and Einstein satellites (Hertz & Grindlay 1983a,b; Rappaport et al. 1994; Johnston & Verbunt 1996). The nature of the DCXSs remains a mystery, largely due to the fact that sufficiently accurate positions have not been available to allow for unambiguous optical or radio identifications in the host clusters.

In a campaign to search for cataclysmic variables (CVs) in globular clusters, HST observations have discovered possible optical counterparts to several of the DCXSs in the clusters surveyed (Grindlay & Cool 1993; Cool et al. 1995). The absolute position accuracy of the HST counterparts is $\lesssim 0\farcs4$, while the ROSAT positions of the DCXSs are no better than $\sim 2\farcs1$ (Verbunt & Hasinger 1998). In addition, the DCXSs are concentrated toward the centers of their respective clusters, where the stellar density is high. High stellar density and source concentration, coupled with the disparity between HST and ROSAT position accuracies, make source confusion a serious issue. Current and pending Chandra observations are likely to resolve the problem of source identification.

As a population, the DCXSs are characterized by their low luminosities, temperatures $kT \sim 0.1 - 0.5$ keV (Johnston & Verbunt 1996) for an assumed blackbody spectrum, and the fact that they typically lie within a few core radii from the centers of their host clusters (Johnston, Verbunt, & Hasinger 1996; Verbunt & Hasinger 1998). However, this information provides only weak constraints for models of the DCXSs. The hypothesis that the DCXSs are CVs (Hertz & Grindlay 1983a) is appealing, but the X-ray luminosities of CVs in the Galactic disk typically lie in the range $L_X \sim 10^{30} - 10^{32.5}$ (see Fig. 8 in Verbunt et al. 1997), falling short of the higher luminosities observed among the DCXSs (Verbunt, van Paradijs, & Elson 1984). Other suggestions regarding the nature of the DCXSs include low-mass X-ray binaries in quiescence (Verbunt, van Paradijs, & Elson 1984), and rapidly rotating NSs that have not yet been detected as millisecond radio pulsars (see Becker & Trümper 1999 for X-ray luminosities of known millisecond radio pulsars). In this work we propose an alternate model that may explain some fraction of the DCXSs.

A NS moving through a gaseous medium will be able to accrete some of this material via the Bondi-Hoyle-Lyttleton (BHL) process (Hoyle & Lyttleton 1941; Bondi & Hoyle 1944; Bondi 1952), converting a fraction of the gravitational energy of the accreted gas into X-ray radiation. This physical picture was first proposed by Ostriker, Rees, & Silk (1970), and later extended by Treves & Colpi (1991) and Blaes & Madau (1993), for NSs in the Galactic disk, where the NS is assumed to accrete from the interstellar medium or from the gas in a giant molecular cloud. We propose the application of this basic idea to globular clusters to explain the DCXSs. This model has the rather minimal requirements that (i) there be a population of single NSs able to accrete from the intracluster gas, and (ii) the ambient gas density and temperature lie in the right range to yield the requisite accretion luminosities.

One major point of uncertainty regarding BHL accretion by the Galactic disk population of single NSs is their distribution of space velocities (see Hansen & Phinney 1997). If the NS is moving supersonically through a gas, with speed $v$, then the BHL accretion luminosity is proportional to $v^3$, implying that an uncertainty in the speed distribution is considerably amplified when computing the luminosity function of the accreting sources. The extension of the BHL accretion model to NSs in globular clusters does not suffer from this indeterminacy. For the more centrally concentrated globular clusters (e.g., 47 Tuc, M15, NGC 6397) the central relaxation time is typically $\lesssim 8$ Gyr, so that at the current epoch the stars in the cluster core region should be nearly in thermal equilibrium (see Waters, Joshi, & Rasio 2000) and the speed distribution for objects of a certain mass should be roughly Maxwellian for speeds less than the cluster escape speed. With a well-defined speed distribution, it is straightforward to compute the shape of the lumi-
nosity function for the isolated, accreting cluster NSs (IACNs). In this regard, a model qualitatively similar to our own (using a Maxwellian speed distribution), but applied to the Galactic center, was proposed by Zane, Turolla, & Treves (1996) to explain the diffuse X-ray emission from the Galactic center.

In §2 we compute the accretion luminosity for a NS moving through a gaseous medium. We discuss the range of intracluster gas density and temperature required to explain the DCXSs. In §3 the luminosity function for the IACNs is derived under the assumptions of a Maxwellian distribution in NS speeds, and a constant gas density and temperature over the region of interest. Caveats concerning our model are discussed in §4. Finally, in §5 we summarize some of the fundamental points of the IACN model.

2. ACCRETION FROM THE INTRACLUSTER GAS

A NS of mass \( M_{\text{ns}} \), moving with relative speed \( v \) through a gas of ambient density \( \rho \) and sound speed \( c_s \), accretes at the BHL rate (see Foglizzo & Ruffert 1997),

\[
M \simeq 4\pi (GM_{\text{ns}})^2 \rho \left(\frac{v^2 + c_s^2}{2}\right)^{3/2}.
\]

(1)

If we define \( V \equiv \left(\frac{v^2 + c_s^2}{2}\right)^{1/2} \) and \( \rho \equiv m_p n \), where \( m_p \) is the proton mass and \( n \) is the hydrogen number density, then for a NS of mass \( M_{\text{ns}} = 1.4 M_\odot \) and radius \( R_{\text{ns}} = 10 \) km, the corresponding X-ray luminosity is given by

\[
L_X = \epsilon GM_{\text{ns}} MR_{\text{ns}}^{-1} \simeq 10^{32} \epsilon n \left(\frac{V}{10 \text{ km s}^{-1}}\right)^{-3} \text{ erg s}^{-1},
\]

(2)

where \( \epsilon \simeq 1 \) is the efficiency for converting gravitational energy into X-ray radiation, and the density is in units of \( \text{cm}^{-3} \).

In order for the IACN model to successfully describe the DCXSs, the predicted X-ray luminosity must naturally span the range of the observed luminosities. This requires that the resultant speed \( V \) not be too large, and that the gas density not be too small. For those globular clusters where it is possible to measure the radial velocity dispersion, \( \sigma \), of the central population of stars, one finds \( \sigma \sim 5 - 20 \) km s\(^{-1}\) (Dubath, Meylan, & Mayor 1997). Generally, it is the light from giants, with mass \( \sim 0.8 M_\odot \), that dominates the sample used in the velocity dispersion measurements. Energy equipartition implies that the one-dimensional velocity dispersions \( \sigma_{m1} \) and \( \sigma_{m2} \), for objects of mass \( m_1 \) and \( m_2 \), are related by \( m_1 \sigma_{m1}^2 = m_2 \sigma_{m2}^2 \) (Binney & Tremaine 1987). If the dynamical temperature of the NS population is the same as that of the turn-off stars with measured velocity dispersions, it follows that the one-dimensional velocity dispersion of NSs is \( \sigma_{\text{ns}} \lesssim 15 \) km s\(^{-1}\), so that the speed itself should not present a problem for the IACN model of the DCXSs. The density of the gas and its thermal speed are more uncertain, and these uncertainties are tied to the larger problem of the gas content in globular clusters, which we now discuss.

A star with the cluster turn-off mass (\( \sim 0.8 M_\odot \)) sheds most of its \( \sim 0.2 M_\odot \) envelope as it ascends the AGB, with a wind outflow speed of \( \sim 10 - 20 \) km s\(^{-1}\) (Knapp & Morris 1985). Therefore, if the central escape speed of a globular cluster is \( \gtrsim 20 \) km s\(^{-1}\), mass should accumulate in the central regions of the cluster. For a cluster that contains a mass \( M_\odot \) of stars within a few core radii from its center, the expected total rate of mass-loss from stars is \( M_{\text{wind}} \sim 10^6 M_\odot / 10^8 M_\odot \) yr\(^{-1}\) (see Knapp, Rose, & Kerr 1973; Tayler & Wood 1975). As a globular cluster passes through the midplane of the Galaxy, it is expected that any accumulated gas will be swept out by ram pressure (Tayler & Wood 1975). In the \( \sim 10^8 \) years between midplane crossings, a globular cluster with \( M_\odot = 10^5 M_\odot \) could thereby accumulate of order \( 100 M_\odot \) of gas. This estimate is reduced considerably (by more than a factor of 10 in some cases) in more sophisticated treatments of the gas-flow problem in globular clusters which include heating and cooling processes (Scott & Rose 1975; Vandenberg & Faulkner 1977; Knapp et al. 1996).

Direct observational searches for cluster gas in the form of molecular, neutral, and ionized hydrogen have yielded only non-detections (Smith et al. 1990; Smith, Woodsworth, & Hesser 1995; Knapp et al. 1996), implying upper limits to the total gas content in the range \( 0.1 - 10 \) km s\(^{-1}\). In a search for ionized hydrogen in six globular clusters, Knapp et al. (1996) found \( M_{\text{HII}} \lesssim 0.1 M_\odot \) within about one core radius for the clusters observed, implying upper limits \( n_{\text{HII}} \sim 50 - 100 \) cm\(^{-3}\). A density of \( 100 \) cm\(^{-3}\), however, gives a BHL accretion luminosity of \( 10^{34} \) erg s\(^{-1}\), which is certainly high enough to explain the brightest of the DCXSs.

A simplistic argument can be made to determine a lower limit to the density of the intracluster gas. Suppose that the wind from each star in the central region of a globular cluster is allowed to flow freely, unimpeded by the gravity of the cluster and interactions with the wind from other stars. Furthermore, assume that the spatial distribution of stars is uniform and confined within a sphere of radius \( r_s \). In this approximation, it can be shown that for \( r < r_s \),

\[
n > \frac{3M_{\text{wind}}}{8\pi r_s^2 v_{\text{wind}}^2 \rho_{\text{fp}}} \sim 1 \left( \frac{M_\odot}{10^5 M_\odot} \right) \left( \frac{v_{\text{wind}}}{20 \text{ km s}^{-1}} \right)^{-1} \left( \frac{r_s}{0.5 \text{ pc}} \right)^{-2} \text{ cm}^{-3}.
\]

(3)

where the scale \( r_s = 0.5 \text{ pc} \) has been chosen to represent the characteristic radius of a dense globular cluster core. When the cluster gravity and the interaction between stellar winds is taken into account, we suspect that the gas density can be substantially larger than this value, although a hydrodynamic investigation is certainly merited.

At least two lines of argument suggest that the temperature of the intracluster gas should be \( \gtrsim 10^4 \) K, with a sound speed \( c_s \sim 10(10^4 \text{ K})^{1/2} \text{ km s}^{-1} \). Only one or two hot, post-AGB stars are required to provide sufficient ultraviolet flux to leave the gas near the cluster center in a warm (\( T \sim 10^4 \) K), photoionized state (see Osterbrock 1989; Knapp et al. 1996). However, a simple calculation shows that only one order in one ten clusters should contain a hot, post-AGB star, owing to the short lifetime (\( \lesssim 10^5 \) yr) of the phase. We return to this point in §4 and discuss the consequences of a predominantly neutral intracluster gas for the IACN model. In addition to discrete ionizing energy sources within the cluster, complete thermalization of the outflow speeds of \( \sim 10 - 20 \) km s\(^{-1}\), via collisions between stellar winds, would also translate to a temperature of \( \sim 10^4 \) K.

We proceed under the assumption that there are no processes whereby the gas is continuously heated so that it maintains a very high temperature (\( \gtrsim 10^5 \) K), or is removed so efficiently as to render moot the discussion of the intracluster gas and the IACN model. A number of energetic gas removal mechanisms have been suggested (for a list, see Smith, Woodsworth, & Hesser 1995). Of these mechanisms, the most powerful is perhaps the sweeping/heating of the intracluster gas by relativistic winds or low-frequency radiation from millisecond pulsars (Spergel 1991). Millisecond pulsars are known to be abundant.
in some globular clusters (see Camilo et al. 2000), but there is no direct evidence yet that the pulsar wind sweeping mechanism is actually operative.

3. THE LUMINOSITY FUNCTION

The derivation of the luminosity function (LF) is simplified considerably if the assumption is made that the gas density, \( n \), the sound speed, \( c_s \), and the one-dimensional velocity dispersion of NSs, \( \sigma_{\text{ns}} \), are constant over the region of interest. These assumptions should be reasonable within the core region of a globular cluster. Given the NS speed distribution, \( p(v) \), the X-ray LF is obtained by computing \( p(L) = \frac{dL}{dL} p(v) \), where we have dropped the subscript on \( L \). We assume that the speed distribution in the central region of a globular cluster is well-represented by a Maxwellian:

\[
p(v)dv = \frac{2}{\pi^{\frac{3}{2}}} \frac{v^2}{\sigma_{\text{ns}}} \exp\left(-\frac{v^2}{\sigma_{\text{ns}}^2}\right) dv.
\]

This is certainly an idealization, since a more realistic speed distribution would vanish beyond the cluster escape speed. However, we would like to stress that the LF derived here is for illustrative purposes only. There are a number of important processes which may lead to a LF that is quite different from the one given below (see §4).

For convenience, we define the following dimensionless quantities:

\[
\ell \equiv L/L_{\text{max}}; \mathcal{M} \equiv v/c_s; \mu \equiv \frac{\sigma_{\text{ns}}}{c_s},
\]

where \( L_{\text{max}} \equiv 4\pi\varepsilon G^2 M_{\odot} R_{\text{BH}}^2/c_s^2 \) is the maximum accretion luminosity (eq. [2], with \( v = 0 \)). In terms of the Mach number \( \mathcal{M}(\ell) = (\ell^{-1/3} - 1)^{1/2} \), the LF is

\[
p(\ell)d\ell = \mu^{-3} \frac{2}{9\pi} (1 + \mathcal{M}^2)^{3/2} \mathcal{M} \exp\left(-\frac{1}{2} \mathcal{M}^2 / \mu^2\right) d\ell.
\]

In Figure 1 we plot the LF and the corresponding cumulative distribution for three plausible values of the characteristic Mach number, \( \mu \). For the values of \( \mu \) shown in Fig. 1, the LF peaks at \( \ell \gtrsim 10^{-2} \) and then drops off rather sharply for decreasing \( \ell \). Furthermore, note that the LF is not well-fit by a power-law over more than one decade in luminosity. Therefore, if we suppose that the bright end of the LF is being observationally selected, then this model shows that a rising observed LF for decreasing luminosity does not necessarily mean that we are seeing the "tip of the iceberg," as inferred by extrapolating a power-law LF.

4. DISCUSSION

In deriving the LF (eq. [6]) using the standard BHL formula for the accretion rate (eq. [11]), we have made some rather restrictive assumptions. The simple BHL formalism assumes that the flow is hydrodynamic (i.e., collisional), that the gas density and temperature are uniform well beyond the accretion radius of the NS, and that the interaction between the flow and the magnetic field of the NS can be neglected. Each of these simplifying assumptions is carried over into the derivation of the LF. A more realistic treatment of the accretion process is beyond the scope of this paper, but we can discuss certain phenomenological consequences of lifting some of the aforementioned restrictions.

Roughly speaking, the gas will accrete hydrodynamically if the mean free path of the atomic or molecular constituents of the gas is shorter than the BHL accretion radius (Begelman 1977; Alcock & Illarionov 1980), \( R_{\text{acc}} = 2GM_{\odot}V^2 \sim 3 \times 10^{14} (M_{\odot}/M_{\odot}) (V/10 \text{ km s}^{-1})^2 \text{ cm} \), where \( V \) is the resultant speed in eq. (2). For neutral hydrogen, the mean free path is \( \sim 10^{16} \text{ cm} \). So, if the ambient gas is predominantly neutral, then a density \( \gtrsim 100 \text{ cm}^{-3} \) is required for hydrodynamic accretion. On the other hand, the mean free path of a proton in an ionized gas is orders of magnitude shorter than the atomic mean free path for \( T \sim 10^6 \text{ K} \) (Alcock & Illarionov 1980). Therefore, for globular clusters where the gas is ionized, the BHL accretion process might not be relevant. For simplicity, we restrict our discussion to those globular clusters where it is likely that the gas near the cluster center is photoionized, 47 Tuc being a case in point (see O’Connell et al. 1997).

The accretion flow onto a rotating, magnetized NS will differ substantially from the case where the NS is nonmagnetic. At the very least, it is expected that the flow will be inhibited in the former case, either due to a relativistic pulsar wind or something akin to the “propeller” mechanism (Illarionov & Sunyaev 1975). It is likely that the detailed consideration of the influence of the magnetic field on the infalling plasma, coupled with distributions in field strength and rotation frequency, will yield a broader LF than the one shown in Fig. 1.

There are essentially two factors which determine how much gas is present in a globular cluster at the current epoch: (i) the mass and central escape speed of the cluster, and (ii) the orbit of the cluster and its current position and velocity. Massive globular clusters with a large central escape speed will tend to accumulate more gas between crossings through the Galactic disk. Also, a cluster will likely contain more gas at the current epoch if it is near the top of, or on the descent from, a moderately high-altitude Galactic orbit.

If the gas density is highly inhomogeneous, e.g., strongly enhanced around a discrete number of stars with large mass-loss rates, the LF derived in §3 would not be valid. On the
other hand, a more clumpy gas distribution may help to explain the presence of DCXSs at more than a few core radii from the center of their host cluster (see Johnston, Verbunt, & Hasinger 1996), where the density of an otherwise smooth background of gas should be markedly reduced below its central value (Vandenbroucke & Faulkner 1977). In this case, only when a NS passes within $\lesssim 0.1$ pc of a mass-losing AGB star might the density be sufficiently large to yield an accretion luminosity $\lesssim 10^{30}$ erg s$^{-1}$.

Several of the DCXSs show evidence for significant X-ray variability over timescales of $\sim 1$ yr (Verbunt & Hasinger 1998; Verbunt & Johnston 2000). Within the IACN model, such time variability may arise due to spatial nonuniformity in the density or temperature of the ambient gas over length scales $\lesssim 2$ AU (for a NS speed of 10 km s$^{-1}$). Hydrodynamic instabilities, possibly induced by the NS magnetic field, may lead to variability on much shorter timescales. Large uncertainties associated with each of these mechanisms prevent us from making any clear predictions for the time variability of the X-ray luminosity of the IACNs. However, we note that numerical simulations of BHL accretion show a strong tendency toward nonsteady behavior (see Benensohn, Lamb, & Taam 1997). In contrast, there is no obvious reason why an isolated rotation-powered pulsar should show any significant variability in the mean X-ray intensity (averaged over possible X-ray pulsations), which may detract from the millisecond pulsar hypothesis if many of the DCXSs are shown to be single and variable.

5. SUMMARY AND CONCLUSIONS

The model we have presented for the DCXSs is rather generic, since the requirements of the model should not be difficult to satisfy, at least in some globular clusters. If it turns out that the IACN model explains a significant fraction of the DCXSs, then much can be learned about the population of single NSs in globular clusters, the properties of the intracluster gas, and the BHL accretion process. On the other hand, if very few of the DCXSs are explained by this model (perhaps because most of the DCXSs are shown to be millisecond pulsars or binaries), then strong constraints can be placed on the gas content of globular clusters and/or the number of single NSs able to accrete efficiently from the intracluster gas.

We list a number of key points regarding the BHL accretion scenario and the IACN population in globular clusters.

1. Globular clusters with large central escape speeds which are presently high above the Galactic plane should contain the largest proportion of IACNs (see §4). The globular cluster 47 Tuc is a prime candidate in this regard. The central escape speed and height above the Galactic disk for 47 Tuc are $\sim 60$ km s$^{-1}$ and $\sim 3.2$ kpc, respectively. We also note that there is weak observational evidence for the presence of intracluster gas in 47 Tuc, based upon diffuse UV emission from the central region of the cluster (O’Connell et al. 1997), as well as diffuse soft X-ray emission from a possible bow shock resulting from the interaction between the cluster gas and the low-density Galactic halo gas (Krockenberger & Grindlay 1995). In addition, we note that the dispersion measures for 20 of the millisecond pulsars in 47 Tuc exhibit a range of $\pm 0.2$ cm$^{-3}$ pc around a central value of $25.5$ cm$^{-3}$ pc (Freire et al. 2000). Given that these pulsars are distributed within the inner $\sim 1$ pc of the cluster, and if we assume that the pulsars are spread along the line of sight by approximately this same amount, this suggests that the electron density within the central region of the cluster could be $\sim 0.2$ cm$^{-3}$.

2. If the accreted gas is thermalized at the surface of the NS, we would expect a blackbody spectrum, with temperature $kT \sim 50 (L/10^{32} \text{ erg s}^{-1})^{1/4} f^{-1/4}$ eV, where $f$ is the fraction of the NS surface onto which material is accreted. For a 50 eV black body, the apparent $B$ magnitude is $\sim 32$ at a distance of 2 kpc. Therefore, if the accreting NSs radiate as blackbodies, and if a negligible amount of the soft X-ray and UV radiation is reprocessed into optical light, then these objects should not have detectable optical counterparts.

3. Theoretical studies indicate that the spectra of the IACNs should be nearly blackbody (see Alme & Wilson 1973; Zane, Turolla, & Treves 2000). However, based on how poorly understood are the X-ray spectra from better studied, more luminous accreting NSs, we would not be surprised if the X-ray spectra of actual BHL accreting NSs turned out to differ markedly from Planckian.

4. We would like to stress that if the IACN model we propose accounts for a very small fraction of the DCXSs, then rather stringent constraints may be placed on the properties of the intracluster gas and/or the number of NSs able to accrete. For instance, if we assume a gas temperature of $10^{4}$ K, then the maximum accretion luminosity of a nonmagnetic NS is $L_{X} \sim 10^{32} n$ erg s$^{-1}$ (eq. [2]). If no IACNs are seen above the detection threshold, $L_{X} \gtrsim 10^{30}$ erg s$^{-1}$, then the gas density must be $n \lesssim 10^{-2}$ cm$^{-3}$, potentially making this the most sensitive available test for the presence of intracluster gas.

5. If accretion onto magnetized, rapidly rotating, NSs is strongly inhibited, then an absence of IACNs could also imply that recycled pulsars dominate the population of single NSs. This is certainly not an untenable hypothesis, since it is likely that a NS will undergo at least one mass and angular momentum accretion episode in a binary system over the lifetime of the cluster (Rasio, Pfahl, & Rapaport 2000; Pfahl, Rapaport, & Rasio 2000, in preparation).

6. A strong density enhancement accompanies the large mass-loss rates and slow wind speeds from AGB stars. This suggests that some DCXSs may be spatially correlated with AGB stars if the IACN model is successful. Therefore, it may be worthwhile to calculate a projected 2D correlation function for DCXSs and AGB stars for those globular clusters that have been observed with both Chandra and HST.

7. Finally, as an aside, we note that the $M^{3}$ dependence of the BHL accretion luminosity (see eqs. [1] and [2]) implies that a black hole of mass $\sim 10 M_{\odot}$ could be as much as $10^{3}$ times more luminous that an accreting NS for the same ambient gas density and temperature (see Grindlay 1978). There are theoretical and observational reasons to believe that there are very few such objects in globular clusters (see Sigurdsson & Hernquist 1993); however, those that remain might be easily detectable near the cluster centers even if they are not in binary systems.

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