The decay rate and the classical radiation power of an excited molecule (atom) located in the center of a dispersive and absorbing dielectric sphere taken as a simple model of a cavity are calculated adopting the Onsager model for the local field. The local-field correction factor to the external (radiation and absorption) power loss of the molecule is found to be \(|3\varepsilon(\omega)/[3\varepsilon(\omega) + 1]|^2\), with \(\varepsilon(\omega)\) being the dielectric function of the sphere. However, local-field corrections to the total decay rate (power loss) of the molecule are found to be much more complex, including those to the decay rate in the infinite cavity medium, as derived very recently by Scheel et al. [Rev. A 60, 4094 (1999)], and similar corrections to the cavity-induced decay rate. The results obtained can be cast into model-independent forms. This suggests the general results for the local-field corrections to the decay rate and to the external power loss of a molecule in an absorbing cavity valid for molecule positions away from the cavity walls.

PACS numbers: 42.50.Lc, 42.60.Da, 33.50.-j

I. INTRODUCTION

Within macroscopic electrodynamics, the decay rate \(\Gamma\) of an excited molecule (atom) at a position \(\mathbf{r}_0\) in an absorbing cavity is given by

\[
\Gamma = \frac{2\omega^2}{\hbar c^2} \mathbf{p}_{f1}^\dagger \cdot \text{Im} \tilde{G}(\mathbf{r}, \mathbf{r}_0; \omega) \cdot \mathbf{p}_{f1}, \tag{1}
\]

with \(\tilde{G}(\mathbf{r}, \mathbf{r}_0; \omega)\) being the (classical) dyadic Green function for the system, \(\mathbf{p}_{f1}\) the relevant dipole transition matrix element, and \(\omega\) the transition frequency. This result is most simply obtained using the classical theory of the molecular (radiative) decay [1] in conjunction with the correspondence principle and extending it straightforwardly to absorbing systems [2]. Within QED, however, it is derived by employing the quantized form of the macroscopic field in absorbing systems obtained rather recently [3,4] and using the Fermi golden rule [5,6] or solving Heisenberg’s equations of motion for the molecule and the field in the Markov approximation [7,8].

Splitting the Green function into the translationally invariant part \(\tilde{G}^\text{0}(\mathbf{r} - \mathbf{r}_0; \omega)\) and the scattering part \(\tilde{G}^\text{sc}(\mathbf{r}, \mathbf{r}_0; \omega)\), the decay rate can be generally expressed as

\[
\Gamma = \Gamma^\text{0} + \Gamma^\text{sc}, \tag{2}
\]

where \(\Gamma^\text{0}\) is the decay rate as would be in the infinite cavity medium and \(\Gamma^\text{sc}\) is the corresponding cavity-induced decay rate. Upon an appropriate regularization of \(G^\text{0}(\mathbf{r} - \mathbf{r}_0; \omega)\), the rate \(\Gamma^\text{0}\) is given by [6,2]

\[
\Gamma^\text{0} = \Gamma_{\text{free}} \left[\frac{3}{2} \frac{\varepsilon''(\omega)}{\varepsilon'(\omega)^2} \left(\frac{c}{\omega R_m}\right)^3 + \eta(\omega)\right], \tag{3}
\]

where

\[
\varepsilon(\omega) = \left[\eta(\omega) + i\kappa(\omega)\right]^2 \tag{4}
\]

is the dielectric function of the cavity medium, \(R_m\) an effective molecule-medium distance, and

\[
\Gamma_{\text{free}} = \frac{4\omega^3|\mathbf{p}_{f1}|^2}{3\hbar c^3} \tag{5}
\]

is the free-space spontaneous emission (SE) rate. The first contribution to \(\Gamma^\text{0}\) in Eq. (3) is identified as the nonradiative decay rate \(\Gamma^\text{0}_\text{rad}\) due to the near-field mediated transfer of the molecular energy to the surrounding medium, whereas the second one is the familiar decay rate \(\Gamma^\text{0}_\text{rad}\) due to the radiation losses of the molecule in the medium, i.e., the SE rate \(\Gamma^\text{0}_\text{SE}\) [5,9,10].

For an optically dense cavity medium, the above result has to be improved by accounting for the difference between the macroscopic field used in its derivation and the actual (local) field with which the molecule interacts. Restricting ourselves to low-density cavity media, in our previous consideration of the molecular decay [2] as well as of the spontaneous emission spectrum [11] in an absorbing planar cavity, we ignored this difference, implicitly assuming, however, that the decay rate corrected for the effect of the local field in an isotropic cavity was of the form

\[
\hat{\Gamma}_\text{loc} = \mathcal{L}[\Gamma^\text{0} + \Gamma^\text{sc}], \tag{6}
\]

where \(\mathcal{L}\) is an appropriate generalization of the local-field correction factor in the lossless case. Thus, as suggested by Barnett et al. for the decay rate \(\hat{\Gamma}^\text{0}_\text{loc}\) in the infinite cavity [5,6], one would have

\[
\mathcal{L}_{\text{Lor}}(\varepsilon) = \left|\frac{\varepsilon(\omega) + 2}{3}\right|^2 \tag{7}
\]

in the Lorentz (virtual-cavity) model [12–16] and

\[
\mathcal{L}_{\text{Oms}}(\varepsilon) = \left|\frac{3\varepsilon(\omega)}{2\varepsilon(\omega) + 1}\right|^2 \tag{8}
\]
The conjecture expressed by Eqs. (6)-(8) has recently been shown to be incorrect for both models for the local field, which we indicate in Eq. (6) by putting the tilde sign on $\Gamma$. First, Scheel et al. [21] demonstrated that, in the Lorentz model, a proper inclusion of the (quantum) noise polarization in the local field led to a more complex $\omega$-dependence of the decay rate $\Gamma_{\text{loc}}^{0}$ in an absorbing medium than that given by the simple product $L_{\text{loc}}^{0}\Gamma^{0}$ [see Eqs. (49) and (50) of Ref. [21]]. Their result is in full agreement with $\Gamma_{\text{loc}}^{0}$ in absorbing dielectrics derived by Fleischhauer using a microscopic approach [22]. In a subsequent work Scheel et al. [7] showed that the decay rate $\Gamma_{\text{loc}}^{0}$ in the Onsager model for the local field was also much more complex in the absorption case than that given by the product $L_{\text{Ons}}^{0}\Gamma^{0}$ [see Eq. (27) below].

Knowing the decay rate $\Gamma_{\text{loc}}^{0}$ in an infinite cavity (medium), it is natural to seek for the decay rate $\Gamma_{\text{loc}}$ in a finite cavity, i.e., in view of Eq. (2), for the cavity-induced rate $\Gamma_{\text{loc}}^{\text{loc}}$. Clearly, the calculation of $\Gamma_{\text{loc}}^{\text{loc}}$ is most straightforward if one adopts the Onsager model for the local field. In this model, one assumes the molecule in the center of an empty spherical cavity of radius small compared with the transition wavelength. In view of Eq. (1), the problem then reduces to the calculation of the Green function for the system with the combined symmetry of such a composite system, however, this represents a difficult task, which for most relevant cavity geometries demands approximative or numerical calculations [17–20].

In view of Eq. (1), the problem then reduces to the calculation of the Green function for the system through

$$W = \frac{\omega}{2} \text{Im} \mathbf{p}^* \cdot \mathbf{E}(\mathbf{r}, \mathbf{r}_0; \omega)$$

lost by the dipole in supporting its own field. Equation (1) is then obtained introducing the Green function of the system through

$$\mathbf{E}(\mathbf{r}, \mathbf{r}_0; \omega) = \frac{\omega^2}{c^2} \mathbf{\hat{G}}(\mathbf{r}, \mathbf{r}_0; \omega) \cdot \mathbf{p},$$

noting that $\mathbf{\hat{G}}(\mathbf{r}_0, \mathbf{r}; \omega)$ is the diagonal dyadic and using the correspondence principle to let $\mathbf{p} \rightarrow 2\mathbf{p}_{fi}$ [33].

The dipole field (Green function) in a cavity can always be written as

$$\mathbf{E}(\mathbf{r}, \mathbf{r}_0; \omega) = \mathbf{E}^{0}(\mathbf{r}; \omega) + \mathbf{E}^{\text{loc}}(\mathbf{r}, \mathbf{r}_0; \omega),$$

where [2]

$$\mathbf{E}^{0}(\mathbf{r}; \omega) = \frac{1}{\varepsilon} \left[ \frac{3\mathbf{R} - \mathbf{I}}{R^3} (1 - ikR) \right]$$

$$- \frac{4\pi}{3} \mathbf{I} \delta(\mathbf{R}) + k^2 \mathbf{I} - \frac{\mathbf{R} \mathbf{R}}{R} \right] \cdot \mathbf{p} e^{i k R},$$

with $\mathbf{I}$ being the unit dyadic, $\mathbf{R} = \mathbf{r} - \mathbf{r}_0$, $\hat{\mathbf{R}} = \mathbf{R}/R$, and

$$k(\omega) \equiv \sqrt{\varepsilon(\omega) \omega} = k'(\omega) + i k''(\omega),$$

Owing to its intuitive clarity and easy visualization, we prefer a classical discourse in this work. Therefore, in Sec II we recall the classical approach to the problem of the decaying molecule and rederive Eq. (3) raising, through a plausible argument, the question on the completeness of this result for absorbing media. In Sec. III we first obtain formal results for the decay rate and the power loss in the external layer of an excited molecule in the center of an absorbing multilayered spherical system. We then apply these general results to a medium with the Onsager cavity and provide an alternative derivation of $\Gamma_{\text{loc}}^{\text{loc}}$ to that given in Ref. [7]. Subsequently, by considering an absorbing dielectric sphere with and without the Onsager cavity, we determine $\Gamma_{\text{loc}}^{\text{loc}}$ and $W_{\text{ext}}^{\text{loc}}$ in terms of the corresponding quantities for the bare sphere. In Sec IV we briefly illustrate the effect of the local field in this model on $\Gamma_{\text{loc}}^{\text{loc}}$ and the total rate $\Gamma_{\text{loc}}$. The main points of this work are summarized in Sec. V and the necessary mathematical background is given in Appendices A and B.

**II. PRELIMINARIES**

In the classical approach, an excited molecule is represented by a point dipole $\mathbf{p} \exp(-i \omega t)$ at the molecular position $\mathbf{r}_0$ oscillating with the frequency of the transition $\omega$. The molecular decay rate $\Gamma$ is then related through

$$\Gamma = W/h\omega$$

Owing to its intuitive clarity and easy visualization, we prefer a classical discourse in this work. Therefore, in Sec II we recall the classical approach to the problem of the decaying molecule and rederive Eq. (3) raising, through a plausible argument, the question on the completeness of this result for absorbing media. In Sec. III we first obtain formal results for the decay rate and the power loss in the external layer of an excited molecule in the center of an absorbing multilayered spherical system. We then apply these general results to a medium with the Onsager cavity and provide an alternative derivation of $\Gamma_{\text{loc}}^{\text{loc}}$ to that given in Ref. [7]. Subsequently, by considering an absorbing dielectric sphere with and without the Onsager cavity, we determine $\Gamma_{\text{loc}}^{\text{loc}}$ and $W_{\text{ext}}^{\text{loc}}$ in terms of the corresponding quantities for the bare sphere. In Sec IV we briefly illustrate the effect of the local field in this model on $\Gamma_{\text{loc}}^{\text{loc}}$ and the total rate $\Gamma_{\text{loc}}$. The main points of this work are summarized in Sec. V and the necessary mathematical background is given in Appendices A and B.

**II. PRELIMINARIES**

In the classical approach, an excited molecule is represented by a point dipole $\mathbf{p} \exp(-i \omega t)$ at the molecular position $\mathbf{r}_0$ oscillating with the frequency of the transition $\omega$. The molecular decay rate $\Gamma$ is then related through

$$\Gamma = \frac{\omega}{2} \text{Im} \mathbf{p}^* \cdot \mathbf{E}(\mathbf{r}, \mathbf{r}_0; \omega)$$

lost by the dipole in supporting its own field. Equation (1) is then obtained introducing the Green function of the system through

$$\mathbf{E}(\mathbf{r}, \mathbf{r}_0; \omega) = \frac{\omega^2}{c^2} \mathbf{\hat{G}}(\mathbf{r}, \mathbf{r}_0; \omega) \cdot \mathbf{p},$$

noting that $\mathbf{\hat{G}}(\mathbf{r}_0, \mathbf{r}; \omega)$ is the diagonal dyadic and using the correspondence principle to let $\mathbf{p} \rightarrow 2\mathbf{p}_{fi}$ [33].

The dipole field (Green function) in a cavity can always be written as

$$\mathbf{E}(\mathbf{r}, \mathbf{r}_0; \omega) = \mathbf{E}^{0}(\mathbf{r}; \omega) + \mathbf{E}^{\text{loc}}(\mathbf{r}, \mathbf{r}_0; \omega),$$

where [2]

$$\mathbf{E}^{0}(\mathbf{r}; \omega) = \frac{1}{\varepsilon} \left[ \frac{3\mathbf{R} - \mathbf{I}}{R^3} (1 - ikR) \right]$$

$$- \frac{4\pi}{3} \mathbf{I} \delta(\mathbf{R}) + k^2 \mathbf{I} - \frac{\mathbf{R} \mathbf{R}}{R} \right] \cdot \mathbf{p} e^{i k R},$$

with $\mathbf{I}$ being the unit dyadic, $\mathbf{R} = \mathbf{r} - \mathbf{r}_0$, $\hat{\mathbf{R}} = \mathbf{R}/R$, and

$$k(\omega) \equiv \sqrt{\varepsilon(\omega) \omega} = k'(\omega) + i k''(\omega),$$
is the field of the dipole as would be in the infinite cavity (medium) and $\mathbf{E}^{\text{inc}}(\mathbf{r}, \mathbf{r}_0; \omega)$ is the component of the dipole field scattered from the cavity walls. Owing to the singular longitudinal (near-field) component of $\mathbf{E}^0(\mathbf{R}; \omega)$

$$E_0^0(\mathbf{R}; \omega) = \frac{1}{\varepsilon} \left[ \frac{3R \hat{R} - \mathbf{I}}{R^3} - \frac{4\pi}{3} \mathbf{I} \delta(\mathbf{R}) \right] \cdot \mathbf{p}. \quad (14)$$

it is the calculation of $\Gamma^0$ that represents a difficult step in determining the decay rate in absorbing systems.

To remove the singularity from $\mathbf{E}^0(\mathbf{R}; \omega)$ in the spirit of the macroscopic-field approach, one may average this component of the dipole field over an appropriately chosen spherical volume $V_m = (4\pi/3)R_m^3$ around the molecule [6] or regularize it by letting $\delta(\mathbf{R}) \to 1/V_m$ [2,24]. In either case, one finds that

$$E_0^0(\mathbf{R}; \omega) \bigg|_{R \to 0} = -\frac{1}{\varepsilon} \frac{4\pi}{3} \mathbf{p}. \quad (15)$$

Since for the dipole transverse field

$$E_0^0(\mathbf{R}; \omega) = E_0^0(\mathbf{R}; \omega) - E_0^0(\mathbf{R}; \omega)$$

we find

$$E_0^0(\mathbf{R}; \omega) = \frac{ik \omega^2 2}{c^2} \frac{2}{3} \mathbf{p} \quad (16)$$

as $\mathbf{R} \to 0$, this leads to the total classical dipole power loss

$$W^0 = \frac{\omega^4 |\mathbf{p}|^2}{3c^3} \left[ \frac{3}{2} \varepsilon''(\omega) + \frac{\varepsilon(\omega)}{\omega R_m} \right]^3 + \eta(\omega) \quad (17)$$

and, accordingly, to the decay rate $\Gamma^0$ given by Eq. (3).

That the above result oversimplifies the frequency dependence of the molecular decay rate in absorbing media becomes clear if one tries to calculate $\Gamma^0$ using the general relation [3]

$$\text{Im} G_{ij}(\mathbf{r}, \mathbf{r}_0; \omega) = \frac{1}{4\pi} \int d^3 s \frac{\omega^2}{c^2} \varepsilon''(\mathbf{s}, \omega) G_{ij}(\mathbf{r}, \mathbf{s}; \omega) G^*_{ij}(\mathbf{r}_0, \mathbf{s}; \omega)$$

and applying it to a homogeneous $[\varepsilon(\mathbf{r}, \omega) = \varepsilon(\omega)]$ medium to determine $\text{Im} G_{ij}^0(\mathbf{r}_0, \mathbf{r}_0; \omega)$. In view of Eqs. (9) and (10), in a less abstract language this is equivalent to using the Poynting’s theorem to obtain $W^0$. Thus, by calculating the dipole energy flow $W^0$ through a spherical surface around the dipole and the energy $W^0$ absorbed per second in the enclosed volume, we find (see Appendix A)

$$W^0 = \frac{\omega^4 |\mathbf{p}|^2}{3c^3} \left[ \frac{\varepsilon''}{\varepsilon} \right] \left[ (1 - ik R_c) e^{ik R_c} \right]^2 \left( \frac{c}{\omega R_c} \right)^3 + \eta e^{-2k'' R_c} \right], \quad (18)$$

where $R_c$ is (formally) the lower limit of the radial integration in $W^0$. Owing to their characteristic dependence on the dielectric function of the medium, we refer to two terms in this equation as the absorption ($\sim \varepsilon''$) and the radiation ($\sim \eta$) contribution to $W^0$, respectively. Since this notation may associate to $W^0_0$ and $W^0_0$ as the respective origins of these contributions, we stress that both $W^0_0$ and $W^0_0$ are needed to obtain each of them, as is clear from the derivation in Appendix A.

To obtain the dipole power loss from Eq. (18), the $R_c \to 0$ limit should eventually be taken. However, consider $R_c$ as a small ($R_c \ll \lambda$) but finite cutoff for the moment. In this case, expanding $W^0$ in powers of $\omega R_c/c$, we find [to $O((\omega R_c)^2$)]

$$W^0 = \frac{\omega^4 |\mathbf{p}|^2}{3c^3} \left\{ \frac{\varepsilon''(\omega)}{\varepsilon(\omega)} \left[ \left( \frac{c}{\omega R_c} \right)^3 + \varepsilon'(\omega) \frac{c}{\omega R_c} \right] + \frac{2}{3} \left[ \frac{\eta(\omega) \varepsilon''(\omega) + \kappa(\omega) \varepsilon'(\omega) \right] + \eta(\omega) \right\}. \quad (19)$$

In addition to a near-field term, which very much resembles the corresponding term in Eq. (17), now we have two new absorption terms. The most striking is the appearance of another $R_c$-free term which, therefore, persists even in the $R_c \to 0$ limit. This implies that a corresponding term must appear in any calculation of $W^0$, which indicates that Eq. (17) is, in this respect, incomplete.

Taking $R_c$ in Eq. (18) as a cutoff is equivalent to setting $E_0^0(\mathbf{R}; \omega) = 0$ for $R < R_c$. Since this field does not obey Maxwell’s equations, it is clearly incorrect to regard $W^0$ as the dipole power loss, which we have emphasized by using the tilde. On the other hand, extending the macroscopic field down to intermolecular distances ($R_c \to 0$) is not justified as the actual field acting on the molecule may largely differ from it. These ambiguities concerning $W^0$ are naturally resolved within an exact macroscopic-field approach in the following section, where we adopt the Onsager (real cavity) model for the local field and therefore assume the molecule in the center of an empty spherical cavity with the radius $R_c$ small compared with the transition wavelength $\lambda$. Since, in this case, the longitudinal component $E_0^0(\mathbf{R}; \omega)$ of the dipole field does not contribute to the molecular power loss, no singularity appears in the theory.

### III. LOCAL-FIELD CORRECTIONS

Consider an excited molecule (dipole) in the center of an $N$-layered spherical system, as depicted in Fig. 1. In this case, the power loss $W^{\text{inc}}$ of the molecule may be written as

$$W^{\text{inc}} = \frac{\omega}{2} \text{Im} \mathbf{p} \cdot \mathbf{E}^\text{inc}_1(\mathbf{r}; \omega)|_{r \to 0}, \quad (20)$$

where $\mathbf{E}^\text{inc}_1(\mathbf{r}; \omega) = \mathbf{E}^\text{inc}_1(\mathbf{r}, \mathbf{r}_0; \omega)|_{r \to 0}$ is the scattered part of the dipole field in the central region. The calculation
of the dipole field in this configuration is outlined in Appendix B, assuming, for simplicity, that $p = p\mathbf{z}$. Using Eq. (B6), we find from Eq. (20) that $(k_0 = \omega/c)$

$$W^{\mathrm{sc}} = W^{\mathrm{free}} \text{Re} \sqrt{\varepsilon} C_1^N, \quad W^{\mathrm{free}} = \frac{ck_0|p|^2}{3},$$

(21)

where $C_1^N$ is the corresponding reflection coefficient. In the case of the empty central region [$W^0 = W^{\mathrm{free}}$], we therefore have for the normalized total decay rate $\hat{\Gamma} = W/W^{\mathrm{free}}$ of the molecule

$$\hat{\Gamma} = 1 + \text{Re} \left[ C_1^N \right]_{\varepsilon_1 = 1}. \quad (22)$$

Identifying the central sphere with the Onsager cavity, this general result provides a direct way for inclusion of the local-field corrections to the decay rate in spherical multilayered systems.

**FIG. 1.** System considered in this paper. All layers are assumed absorbing and described by the complex dielectric function $\varepsilon_i(\omega)$.

Of obvious interest is also the radiation power $W^{\mathrm{rad}}_N$ of the molecule or, generally, the total power loss $W_N$ in the outer region of the multilayer. By comparing Eqs. (A1a) and (B5), we see that the dipole field $E_N(r; \omega)$ in this region is the same as the field produced in the infinite medium ($N$) of the dipole ($C_N^N = C_N^{N+}$):

$$p_N = \frac{\varepsilon_1}{\varepsilon_N} C_N^N p.$$

(23)

Accordingly, provided that we let $p \rightarrow p_N$, $\varepsilon \rightarrow \varepsilon_N$, and $R_c \rightarrow R_N$, we can adopt all results concerning the dipole power loss in an infinite medium derived in Appendix A. For example, with these replacements, the angular distribution of radiation $dW^{\mathrm{rad}}_N/d\Omega$ is obtained by keeping only the radiation field ($\sim 1/r^2$) contribution to Eq. (A2)

$$\frac{dW^{\mathrm{rad}}_N}{d\Omega} = \eta_N \frac{ck_0|p|^2}{8\pi} \left| \frac{\varepsilon_1}{\varepsilon_N} C_N^N \right|^2 e^{-2k_0R \sin^2 \vartheta},$$

(24)

the radiation power $W^{\mathrm{rad}}_N$ is given by the last term in Eq. (A3) and the total power loss $W_N$ by Eq. (18). Regarding the central sphere as the Onsager cavity, we see that the local-field correction factor to $W_N$ (and $W^{\mathrm{rad}}_N$) is given by $|C_N^N/\varepsilon_N|^2$ for $\varepsilon_1 = 1$ and in the limit $k_0a_1 = k_0R_c \ll 1$.

**A. Infinite medium**

To calculate local-field corrections to the decay rate in an infinite medium with the dielectric function $\varepsilon(\omega)$, we consider a two-layered system consisting of this medium with the Onsager cavity cut around the origin. Then, as was done by Scheel et al. [7], $\hat{\Gamma}^\text{loc}$ is straightforwardly obtained using Eq. (22) and expanding the reflection coefficient $C_1^N(1, \varepsilon; R_c)$ given by Eq. (B7) in powers of $\rho_1 = k_0R_c \ll 1$. It is very instructive to rederive this result by applying the Poynting’s theorem to a spherical surface around the molecule enclosing the Onsager cavity and thus determine the power loss $W^{\text{loc}}_N$ of the molecule.

We note that we have performed such a calculation in Appendix A to obtain Eq. (18). Thus, $W^{\text{loc}}_N$ is given by this equation provided that we let $p \rightarrow p_{\text{eff}}$, where [cf. Eq. (23)]

$$p_{\text{eff}} = \frac{1}{\varepsilon} C_2^N(1, \varepsilon; R_c) p,$$

and regard $R_c$ as the Onsager cavity radius.

Expanding the coefficient $C_2^N(1, \varepsilon; R_c)$ given by Eq. (B8) for small $\rho_1 = k_0R_c$, we find

$$p_{\text{eff}} = \frac{3\varepsilon}{2\varepsilon + 1} \left( 1 - \frac{10\varepsilon^2 - 9\varepsilon - 1}{10(2\varepsilon + 1)} (k_0R_c)^2 ight) p$$

$$\left[ -\frac{2\varepsilon^2(\varepsilon - 1)}{3(2\varepsilon + 1)} (k_0R_c)^3 + O[(k_0R_c)^4] \right] p.$$

(25)

Owing to the $(k_0R_c)^{-3}$ factor in the first term on the rhs of Eq. (18), in this term we must use

$$\left| (1 - i\sqrt{k_0R_c}) e^{i\sqrt{k_0R_c}} p_{\text{eff}} \right|^2 =$$

$$\frac{3\varepsilon}{2\varepsilon + 1} \left[ 1 + \frac{1}{5} \text{Re} \frac{14\varepsilon + 1}{2\varepsilon + 1} (k_0R_c)^2 ight.$$  

$$-2\text{Im} \frac{\varepsilon^2}{2\varepsilon + 1} (k_0R_c)^3 + O[(k_0R_c)^4] \right] |p|^2.$$

(26)

Since the second term on the rhs of Eq. (18) is a well-behaved function of $k_0R_c$, in this term it is sufficient to let $p_{\text{eff}} = 3\varepsilon/(2\varepsilon + 1) p$. In this way, for the normalized decay rate $\hat{\Gamma}^0_\text{loc} = W^{\text{loc}}_N/W^{\text{free}}$ we obtain [to $O(\frac{k_0R_c}{r})$]
The above result coincides with that of Scheel et al. [21]. This time, however, the origin of various contributions to \( \hat{\Gamma}_{\text{loc}}^0 \) along with their separate local-field corrections can be clearly identified. As seen from comparison with Eq. (17), while the near-field and the radiation-field terms get multiplied by \( \mathcal{L}_{\text{Ons}} \) as expected [6], it is the appearance of the additional absorption terms already signaled in Eq. (19) that represents essentially new corrections to \( \hat{\Gamma}_{\text{loc}} \) in absorbing media. Of these two terms, the most interesting is the (usually) negative \( R_c \)-free contribution. This term effectively adds to the radiation-field contribution in Eq. (19) that represents essentially new corrections when the molecule in absorbing media. Of these two terms, the most interesting is the (usually) negative \( R_c \)-free contribution.

This term effectively adds to the radiation-field contribution (\( \hat{\eta} \)) to the decay rate in absorbing media and tends to diminish the overall rate.

### B. Cavity

Having determined \( \hat{\Gamma}_{\text{loc}}^0 \), the next step is to consider the decay rate \( \hat{\Gamma}_{\text{loc}} \) in the general case when the molecule is embedded in an inhomogeneous system, i.e., in a cavity. In order to determine \( \hat{\Gamma}_{\text{loc}} \), we consider the decaying molecule in the center of a dielectric sphere of radius \( R \) and the dielectric function \( \varepsilon(\omega) \) immersed in an external medium with the dielectric function \( \varepsilon_{\text{ext}}(\omega) \). In this case, the relevant reflection coefficient to be inserted in Eq. (22) is \( C_1^3(1, \varepsilon, \varepsilon_{\text{ext}}; R_c, R) \) [Eq. (B10)] corresponding to the sphere with the Onsager cavity. Expanding this coefficient in powers of \( \rho_{11} = k_0 R_c \), we find that

\[
\begin{align*}
C_1^3(1, \varepsilon, \varepsilon_{\text{ext}}; R_c, R) &= \left( 1 - i \frac{\varepsilon}{2\varepsilon + 1} \right) - i \frac{\varepsilon(\varepsilon + 1)}{2(2\varepsilon + 1)^2} \right) (k_0 R_c)^{-1} \\
&= \frac{9\varepsilon}{2\varepsilon + 1} \frac{1}{(2\varepsilon + 1)^2} \beta_1 - \frac{1}{(2\varepsilon + 1)^2} \beta_2 - \frac{1}{(2\varepsilon + 1)^2} \beta_2 - 1 + \mathcal{O}(k_0 R_c),
\end{align*}
\]

with \( \beta_j \) given by Eq. (B14). One may recognize that

\[
\frac{2\beta_1}{\beta_1 + \beta_2} = -C_1^3(\varepsilon, \varepsilon_{\text{ext}}; R),
\]

where \( C_1^3(\varepsilon, \varepsilon_{\text{ext}}; R) \) [Eq. (B7)] is the reflection coefficient of the system without the Onsager cavity. One may also see that

\[
\text{Re} \left( \frac{9\varepsilon}{2\varepsilon + 1} \right) = \left( \frac{3\varepsilon}{2\varepsilon + 1} \right)^2 \eta - \frac{18\varepsilon''}{2\varepsilon + 1} \left( 2(\varepsilon^2 + \varepsilon') \kappa + \varepsilon'' \eta \right) \]

is equal to the \( R_c \)-free contribution to \( \hat{\Gamma}_{\text{loc}}^0 \) [Eq. (27)] and that analogous results hold for the real parts of the first two terms in Eq. (28). Therefore, from Eq. (22) we find

\[
\hat{\Gamma}_{\text{loc}} = \hat{\Gamma}_{\text{loc}}^0 + \hat{\Gamma}_{\text{loc}}^c,
\]

where

\[
\hat{\Gamma}_{\text{loc}}^c = \text{Re} \left( \frac{9\varepsilon}{2\varepsilon + 1} \right) C_1^3(\varepsilon, \varepsilon_{\text{ext}}; R)
\]

is the normalized cavity-induced decay rate with the local-field corrections.

The above result for \( \hat{\Gamma}_{\text{loc}}^c \) can be transformed into a form similar to Eq. (27) for \( \hat{\Gamma}_{\text{loc}}^0 \). Letting \( \sqrt{\varepsilon} \to \sqrt{\varepsilon} C_1^3(\varepsilon, \varepsilon_{\text{ext}}; R) \) in Eq. (29), we see that \( \hat{\Gamma}_{\text{loc}}^c \) is obtained from the \( R_c \)-free contribution to \( \hat{\Gamma}_{\text{loc}}^0 \) upon replacements \( \eta \to \hat{\Delta}^c \) and \( \kappa \to 2\hat{\Delta}^c \), where

\[
\hat{\Delta}^c = \frac{1}{2} \text{Im} \left( \sqrt{\varepsilon} C_1^3(\varepsilon, \varepsilon_{\text{ext}}; R) \right)
\]

is the normalized cavity-induced decay rate [Eq. (21)] and

\[
\Delta^c = \frac{1}{2} \text{Im} \left( \sqrt{\varepsilon} C_1^3(\varepsilon, \varepsilon_{\text{ext}}; R) \right)
\]

is the normalized classical cavity-induced level shift [34] of the molecule in the bare sphere. Accordingly, from Eq. (27) we finally have

\[
\hat{\Gamma}_{\text{loc}} = \left( \frac{3\varepsilon}{2\varepsilon + 1} \right)^2 \left\{ \hat{\Gamma}_{\text{loc}}^c - \varepsilon'' \varepsilon_{\text{ext}}(\varepsilon, \varepsilon_{\text{ext}}; R) \right\}
\]

\[
\hat{\Gamma}_{\text{loc}}^c = \left( \frac{3\varepsilon}{2\varepsilon + 1} \right)^2 \left\{ \hat{\Gamma}_{\text{loc}}^c - \varepsilon'' \varepsilon_{\text{ext}}(\varepsilon, \varepsilon_{\text{ext}}; R) \right\}
\]

We note that this is not an unexpected result once we have learned the correct form of the decay rate \( \hat{\Gamma}_{\text{loc}}^0 \) in the infinite medium. In its derivation it is implicitly assumed that the radius of the sphere, i.e., the molecule-mirror distance, is much larger than the transition wavelength \( k_0 R \gg 1 \). Under these circumstances, the molecule-mirror interaction goes through the radiation-field component of the scattered field and therefore only this field component determines the cavity-induced rate. Thus, the rate \( \mathcal{L}_{\text{Ons}} \) as would be obtained by letting \( p \to 3\varepsilon/(2\varepsilon + 1)p \) in Eq. (21), is corrected for an absorption contribution in the same way as is the radiation-field contribution to the rate \( \hat{\Gamma}_{\text{loc}}^0 \) in the infinite cavity.

### C. External region

To find the local-field corrections to the molecular losses in the external region, we consider the field \( \mathbf{E}_{\text{ext}}^0(r; \omega) \) outside the sphere with the Onsager cavity. As already noted, this field is equal to the field in the
the dipole \[\text{cf. Eq. (23)}\]

\[p_{\text{ext}}^{\text{loc}} = \frac{1}{\varepsilon_{\text{ext}}} C_3^2(1, \varepsilon, \varepsilon_{\text{ext}}; R_c, R) p, \quad (35)\]

with the coefficient \(C_3^2(1, \varepsilon, \varepsilon_{\text{ext}}; R_c, R)\) given by Eq. (B12). For small \(\rho_{11} \approx k_0 R_c\), we find that \([\text{to } O[(k_0 R_c)^2]]\)

\[C_3^2(1, \varepsilon, \varepsilon_{\text{ext}}; R_c, R) = -\frac{i \varepsilon_{\text{ext}}}{\sqrt{k_0 R} \beta_1 + 1} \varepsilon^2 \frac{2}{\beta_1 + \beta_2}, \quad (36)\]

with \(\beta_j\) given by Eq. (B14). Now

\[\frac{i \varepsilon_{\text{ext}}}{\sqrt{k_0 R} \beta_1 + 1} \varepsilon^2 \frac{2}{\beta_1 + \beta_2} = -C_2^2(\varepsilon, \varepsilon_{\text{ext}}; R), \quad (37)\]

where \(C_2^2(\varepsilon, \varepsilon_{\text{ext}}; R)\) is the external-field coefficient of the system without the Onsager cavity [cf. Eq. (B8)]. To the leading term in \(k_0 R_c\), we therefore have

\[p_{\text{ext}}^{\text{loc}} = \frac{3 \varepsilon}{2 \varepsilon + 1} \varepsilon_{\text{ext}} C_2^2(\varepsilon, \varepsilon_{\text{ext}}; R) p = \frac{3 \varepsilon}{2 \varepsilon + 1} p_{\text{ext}}, \quad (38)\]

where \(p_{\text{ext}}^{\text{loc}}\) is the corresponding effective dipole moment for the bare sphere [cf. Eq. (23)]. Accordingly, for the external field we have

\[E_{\text{ext}}^{\text{loc}}(r; \omega) = \frac{3 \varepsilon(\omega)}{2 \varepsilon(\omega) + 1} E_{\text{ext}}(r; \omega), \quad (39)\]

with \(E_{\text{ext}}(r; \omega)\) being the external field for the bare sphere. Consequently, this implies \(L_{\text{Ons}}\) as the proper local-field correction factor to the external power loss

\[W_{\text{loc}} = \left| \frac{3 \varepsilon(\omega)}{2 \varepsilon(\omega) + 1} \right|^2 W_{\text{ext}}, \quad (40)\]

Note that here \(W_{\text{ext}}\) is generally given by Eq. (18), with \(k = k_{\text{ext}}\) and \(R_c = R\).

We end this section with a remark on a (plausible) generalization of the obtained results. Using Eqs. (10) and (B6), the normalized cavity-induced decay rate [Eq. (32)] and frequency shift [Eq. (33)] can be written in terms of the Green function for the bare sphere as

\[\hat{\Gamma}_{\text{sc}}^c = \frac{3}{2 k_0} \hat{p} \cdot \text{Im} \mathbf{G}_{\text{sc}}(r_0, r_0; \omega) \cdot \hat{p}, \quad (41)\]

\[\hat{\Delta}_{\text{sc}}^c = -\frac{3}{4 k_0} \hat{p} \cdot \text{Re} \mathbf{G}_{\text{sc}}(r_0, r_0; \omega) \cdot \hat{p}, \quad (41)\]

where \(\hat{p}\) gives the direction of the transition. With this inserted in Eqs. (34), any reference to the specific system considered in the derivation of this equation is formally lost. Therefore, at first sight, there is no reason why Eqs. (34) and (41) should not be taken as the general result for the molecular decay rate in an absorbing cavity valid for all positions of the molecule away from the cavity walls and for all transition dipole orientations. Similarly, in view of Eq. (10), it appears that Eq. (39) indicates a rather general relationship between the Green function elements for a system with and without the Onsager cavity:

\[\tilde{G}_{\text{loc}}(r, r_0; \omega) = \frac{3 \varepsilon(\omega)}{2 \varepsilon(\omega) + 1} \mathbf{G}_{\text{sc}}(r, r_0; \omega), \quad (42)\]

where \(r\) is in the external layer and \(r_0\) in the cavity. If this relation holds, \(L_{\text{Ons}}(\varepsilon)\) as the local-field correction factor for the molecular external power loss in the general case would then be its immediate consequence. Of course, as already stressed, these conjectures cannot be proved without the calculation of the exact Green function for the system including the Onsager cavity at the source position.

**IV. DISCUSSION**

To illustrate the effect of the local field on the decay rate in absorbing cavities, we exploit the above simple model and consider the decaying molecule in the center of an absorbing dielectric sphere surrounded by air (\(\varepsilon_{\text{ext}} = 1\)). The dielectric function of the sphere is modeled as

\[\varepsilon(\omega) = \varepsilon_b + \frac{\Omega^2}{\omega_b^2 - \omega^2 - i \omega \gamma}, \quad (43)\]

where \(\varepsilon_b\) is the background (high-frequency) dielectric constant, \(\omega_0\) and \(\gamma\) are, respectively, the center frequency and the width of the absorption resonance and its strength is controlled by \(\Omega^2\). In this analysis, a relatively large background dielectric constant \(\varepsilon_b = 5\) is chosen to strengthen the cavity effect of the sphere.

**FIG. 2.** Cavity-induced decay rate with (upper curve) and without (lower curve) local-field corrections. The radius of the sphere is \(R = \lambda_0 / \pi\) and the material parameters are \(\varepsilon_b = 5, \Omega = 0.5 \omega_0\) and \(\gamma = 0.1 \omega_0\).
enhancement cavity with respect to the SE rate at $\omega = \omega_0$. For these parameters, the sphere therefore acts as an enhancement cavity with respect to the SE rate at $\omega = \omega_0$ and, since the molecule decays only radiatively, the decay rate is equal to the SE rate. Away from the resonance, the system is lossless and, owing to the cavity effect of the sphere, the two (SE) rates exhibit the familiar oscillations with $\omega$ with the amplitudes of oscillations scaled by the usual local-field correction factor $\mathcal{L}_{\text{Ons}}(\varepsilon_b) \approx 1.85$ [cf. Eq. (34)]. A somewhat different effect of the local field is observed in the region of the resonance $\omega \approx \omega_0$. For the medium parameters chosen, the second (absorption) term on the rhs of Eq. (34) is small compared with the first one. Moreover, according to our calculations, $\hat{\Gamma}^{\text{loc}}$ cannot be distinguished on this scale from the decay rate $\hat{\Gamma}^{\infty}$ over a wide range of the parameters $\Omega$ and $\gamma$. The cavity-induced rate in this frequency region is therefore (again) predominantly determined by the product of $\mathcal{L}_{\text{Ons}}$ and $\hat{\Gamma}^{\infty}$. Each of these quantities exhibits a characteristic asymmetric dispersion around $\omega_0$. In addition, the sphere is (nearly) in resonance with the medium. As a consequence, instead of a peak at $\omega = \omega_0$ appearing in the nonabsorbing ($\Omega = 0$) case, owing to the resonant absorption, the rates $\hat{\Gamma}^{\text{loc}}$ and $\hat{\Gamma}^{\infty}$ exhibit asymmetric double-peak structures, which very much resemble the "cavity-polariton" part of the SE spectrum in the bad-cavity case [11].

Figure 3 represents the total decay rates $\hat{\Gamma}^{\text{loc}}$ [Eq. (30)] in conjunction with Eqs. (27) and (31)] and $\hat{\Gamma}$ [Eq. (2) in conjunction with Eqs. (3) and (32)] with and without the local-field corrections, respectively, along with the decay rate $\hat{\Gamma}^{\text{loc}} = \mathcal{L}_{\text{Ons}}\hat{\Gamma}$ [Eqs. (6) and (8)] with the local-field correction factor assumed in our previous work [2,11]. In plotting these curves, we have taken that the molecule-medium distance $R_m$ figuring in $\hat{\Gamma}^0$ [Eq. (3)] is equal to the Onsager cavity radius, $R_m = R_c$. Away from the resonance, $\hat{\Gamma}^{\text{loc}}$ and $\hat{\Gamma}$ merely reproduce the corresponding curves of Fig. 2, with values enhanced by the off-resonance infinite-cavity decay rates $\hat{\Gamma}^{\text{loc}}_0 = \mathcal{L}_{\text{Ons}}(\varepsilon_b)\sqrt{\varepsilon_b} \approx 4.1$ and $\hat{\Gamma}^0 = \sqrt{\varepsilon_b} \approx 2.2$, respectively. As before, the situation around $\omega_0$ is different. For this $R_c$ and the medium parameters chosen, the absorption contributions to $\hat{\Gamma}^{\text{loc}}_0$ [Eq. (27)] and $\hat{\Gamma}^{\infty}_0$ [Eq. (34)] are (still) small compared with the corresponding radiation contributions. Accordingly,

$$\hat{\Gamma}^{\text{loc}} \approx \mathcal{L}_{\text{Ons}}(\eta) + \hat{\Gamma}^{\infty}$$

holds. For $R_m \approx R_c$, the same conclusion applies to the decay rate $\hat{\Gamma}$ as well, so that

$$\hat{\Gamma} \approx \eta + \hat{\Gamma}^{\infty}.$$  

The different shapes of the curves in Figs. 3 when compared with those in Fig. 2 in this region are therefore caused by the (anomalous) dispersion of the superimposed $\mathcal{L}_{\text{Ons}}\eta$ and $\eta$, respectively. This also explains why $\hat{\Gamma}^{\text{loc}}$ (upper solid line) practically cannot be distinguished on this scale from the decay rate $\hat{\Gamma}^{\text{loc}} = \mathcal{L}_{\text{Ons}}\hat{\Gamma}$ (dotted line), although the corresponding absorption contributions to these rates are different [cf. Eqs. (27) and (3)]. We note that Eqs. (44) and (45) hold for a range of the Onsager cavity radii and/or medium parameters. In these cases, our previous conjecture concerning the local-field corrections to the decay rate, as expressed by Eq. (6), remains (approximately) valid.

![FIG. 3. Total decay rate with (upper solid line) and without (lower solid line) local-field corrections for the system described in Fig. 2. The dotted line represents the macroscopic decay rate with the usual local-field correction factor. The Onsager cavity radius is $R_c = 0.1\lambda$ and $R_m = R_c$.](image)

![FIG. 4. Same as in Fig. 3, but for the Onsager cavity radius $R_c = 0.03\lambda$.](image)
The difference between \( \hat{\Gamma}_{loc} \) (upper solid line) and \( \hat{\Gamma} \) (lower solid line) this time arises not only because of the overall extra factor \( \mathcal{L}_{\text{Ons}} \), but also because of the 3/2 times larger nonradiative contribution to \( \hat{\Gamma} \) [cf. Eqs. (27) and (3)]. This 3/2-factor in the corresponding nonradiative rates also leads to a significantly larger decay rate \( \hat{\Gamma}_{loc} \) (dotted line) than the true rate \( \hat{\Gamma}_{loc} \) (upper solid line). We note, however, that the comparison between \( \hat{\Gamma}_{loc} \), \( \hat{\Gamma} \), and \( \hat{\Gamma}_{loc} \) is given here only for illustrative purposes as their difference is a consequence of our arbitrary input: \( R_m = R_c \). Indeed, in addition to demanding, on physical grounds, the same order of magnitude for \( R_m \) and \( R_c \), any other relation between these parameters may also be assumed.

Of course, owing to the strong increase of the \( R_c \)-dependent terms, for even smaller \( R_c \) the decay rate \( \Gamma_0 \) largely exceeds the cavity-induced rate \( \Gamma_{loc} \), so that \( \Gamma_{loc} \approx \Gamma_0 \) holds. For a detailed discussion of the decay rate in this regime, we therefore refer the reader to Ref. [7].

V. SUMMARY

In this work we have calculated the decay rate and the classical external power loss of an excited molecule located in the center of an absorbing and dispersive dielectric sphere by adopting the Onsager (real cavity) model for the local field. We have found that the external fields of a dipole classically representing the molecule calculated with and without the Onsager cavity scale with \( 3\epsilon(\omega)/[3\epsilon(\omega) + 1] \). This immediately gives \( 3\epsilon(\omega)/[3\epsilon(\omega) + 1] \) as the correct local-field correction factor for the (radiation and absorption) power loss of the molecule outside the sphere in the absorbing case. Whereas this result could have been guessed on the basis of a straightforward analytical continuation of the corresponding result in the lossless case, the local-field corrections to the total decay rate (power loss) of the molecule are found to be much more complex. The total decay rate is found to consists of the decay rate for the infinite sphere, as very recently obtained for an absorbing medium in Ref. [7], and of the cavity-induced rate, for which we have obtained a similar expression. When expressed in terms of the Green function for the sphere, these results become formally system-independent. This suggests the general results for the local-field corrections to the decay rate and to the external power loss of a molecule in an absorbing cavity and located away from the cavity walls.

APPENDIX A: DIPOLE POWER LOSS

To calculate \( W^0 \) using the Poynting’s theorem, it is convenient to rewrite the dipole electromagnetic field in spherical coordinates. With \( \mathbf{I} = \hat{\mathbf{r}} \times (\hat{\mathbf{θ}} + i \hat{\phi}) \) in Eq. (12), letting the origin at the dipole site and assuming, for simplicity, that \( \mathbf{p} = p \hat{\mathbf{z}} \), we have for \( r \neq 0 \) (\( k_0 = \omega/c \))

\[
\mathbf{E}^0(\mathbf{r}; \omega) = ikk_0^2p \left\{ \frac{2h_1^{(1)}(kr)}{kr} \cos \varphi \mathbf{r} + \left[ \frac{h_1^{(1)}(kr)}{kr} - \frac{h_0^{(1)}(kr)}{kr} \right] \sin \varphi \hat{\mathbf{θ}} \right\},
\]

\[
\mathbf{B}^0(\mathbf{r}; \omega) = k^2k_0ph_1^{(1)}(kr) \sin \varphi \hat{\mathbf{φ}},
\]

where

\[
h_0^{(1)}(z) = -\frac{i e^{iz}}{z}, \quad h_1^{(1)}(z) = -\frac{e^{iz}}{z}(1 + i),
\]

are the spherical Hankel functions of the first kind. The radial component of the Poynting’s vector

\[
\mathbf{P}(\mathbf{r}; \omega) = \frac{c}{8\pi} \text{Re} \mathbf{E}^0(\mathbf{r}; \omega) \times [\mathbf{B}^0(\mathbf{r}; \omega)]^* \]

is then easily found to be

\[
\hat{\mathbf{r}} \cdot \mathbf{P}(\mathbf{r}; \omega) = \frac{c}{8\pi} \text{Im} \frac{|k_0^3| \mathbf{p}|^2}{\sqrt{\epsilon} \sqrt{\epsilon^*}} \left\{ -\frac{h_1^{(1)}(kr)}{kr} + h_0^{(1)}(kr) \left[ \frac{h_1^{(1)}(kr)}{kr} \right]^* \sin^2 \vartheta \right\}.
\]

This determines the angular distribution of the energy flow \( dW_f/d\Omega = r^2 \hat{\mathbf{r}} \cdot \mathbf{P}(\mathbf{r}; \omega) \) through a spherical surface of radius \( r \) around the dipole. Upon the integration over the angles, we therefore have

\[
W_f(\mathbf{r}) = \frac{\omega |\mathbf{p}|^2}{3} \frac{\epsilon'' |(1 - ikr)e^{ikr}|^2}{|\mathbf{r}|^3} + \eta \frac{\omega k_0^3 |\mathbf{p}|^2}{3} r^2 e^{-2k'r}.
\]  

(A3)

The dipole energy absorbed per second within the volume \( V_r \) of the sphere is given by

\[
W_a(\mathbf{r}) = \frac{\omega \epsilon''}{8\pi} \int_{V_r} d^3 \mathbf{r} |\mathbf{E}^0(\mathbf{r}; \omega)|^2.
\]  

(A4)

With

\[
|\mathbf{E}^0(\mathbf{r}; \omega)|^2 = \frac{|k^3| \mathbf{p}|^2}{|\mathbf{r}|^2} \left\{ 4 \frac{h_1^{(1)}(kr)}{kr} \right\}^2 \cos^2 \vartheta + \left[ \frac{h_1^{(1)}(kr)}{kr} - \frac{h_0^{(1)}(kr)}{kr} \right]^2 \sin^2 \vartheta
\]
and performing the angular integration, we have

\[
W_0^f(r) = \frac{\omega |\mathbf{p}|^2}{3} \left| \frac{\varepsilon''}{\varepsilon} \right|^2 \int_{R_c} r dr \left\{ \frac{3}{r^2} |(1 - ikr) e^{ikr}|^2 - 2 \frac{k'^2 - k'^2 - k''^2}{r^2} |k|^2 e^{-2k''r} + |k|^4 e^{-2k''r} \right\}. \tag{A5}
\]

Partially integrating the first term and noticing that the remaining integrals involving the dipole near field cancel, we obtain

\[
W_0^f(r) = \frac{\omega |\mathbf{p}|^2}{3} \left| \frac{\varepsilon''}{\varepsilon} \right|^2 \left[ \frac{3}{r^2} |(1 - ikr) e^{ikr}|^2 \right]_{R_c} + \eta \frac{\omega |\mathbf{p}|^2}{3} \left( e^{-2k''r_R} - e^{-2k''r} \right). \tag{A6}
\]

This, together with Eq. (A2), gives Eq. (18) for the total dipole power loss \(W_0^f(r) + W_0^a(r)\).

**APPENDIX B: DIPOLE FIELD IN MULTILAYERED SPHERICAL MEDIA**

Generalizing the result for an infinite medium [Eq. (A1b)], the magnetic field of a dipole oscillating along the z-axis in the center of an \(N\)-layered spherical system [Fig. 1] can be written in the form

\[
\mathbf{B}(r;\omega) = \varepsilon_1 k_0^3 |f(r)| \sin \vartheta \hat{\phi}. \tag{B1}
\]

Through

\[
\mathbf{E}(r;\omega) = \frac{i}{k_0 \varepsilon(r)} \nabla \cdot \mathbf{B}(r;\omega),
\]

the electric field is therefore given by

\[
\mathbf{E}(r;\omega) = i k_0^2 \frac{\varepsilon_1}{\varepsilon(r)} \left\{ \frac{2f(r)}{r} \cos \vartheta \hat{r} - \frac{rf(r)''}{r} \sin \vartheta \hat{\vartheta} \right\}, \tag{B2}
\]

with the prime denoting the derivative of the function in the brackets. The general form of the function \(f(r)\) in the \(l\)th layer is

\[
f_l(r) = h_1^{(1)}(kr) \delta_{l1} + C_{l1}^N h_1^{(1)}(kr) + C_{l2}^N h_1^{(2)}(kr), \tag{B3}
\]

where \(h_1^{(i)}(z)\) are spherical Hankel functions. The last two terms here are the solutions of the homogeneous Maxwell’s equations [35] and give the scattered field \(\mathbf{E}^{sc}(r;\omega)\) in the system. The regularity of this field at the origin demands that \(C_{l+}^N = C_{l-}^N = C_{l+}^N/2\), while the outgoing wave condition at infinity demands that \(C_{l-}^N = 0\). The rest of the coefficients \(C_{l \pm}^N\) are determined from the boundary conditions

\[
f(r_l^−) = f(r_l^+) \quad \text{and} \quad \frac{rf(r_l^+)}{\varepsilon(r_l^+)} = \frac{rf(r_l^−)}{\varepsilon(r_l^−)}
\]
at the layer interfaces. The scattered field in the central layer is therefore generally given by

\[
\mathbf{E}_l^{sc}(r;\omega) = ik_0 k_0^3 \varepsilon_1 \left\{ C_{l1}^N \left[ \frac{2h_1^{(1)}(kr)}{k_0} \cos \vartheta \hat{r} - \frac{kr h_1^{(1)}(kr)'}{k_0} \sin \vartheta \hat{\vartheta} \right] + C_{l2}^N \left[ \frac{2h_1^{(2)}(kr)}{k_0} \cos \vartheta \hat{r} - \frac{kr h_1^{(2)}(kr)'}{k_0} \sin \vartheta \hat{\vartheta} \right] \right\}, \tag{B4}
\]

where \(j_1(z)\) is the spherical Bessel function. In the other layers it is given by \((\mathbf{E}_l = \mathbf{E}_l^{sc})\)

\[
\mathbf{E}_l(r;\omega) = \frac{i k_0 k_0^3 \varepsilon_1}{3} \left\{ C_{l1}^N \left[ \frac{2h_1^{(1)}(kr)}{k_0} \cos \vartheta \hat{r} - \frac{kr h_1^{(1)}(kr)'}{k_0} \sin \vartheta \hat{\vartheta} \right] + C_{l2}^N \left[ \frac{2h_1^{(2)}(kr)}{k_0} \cos \vartheta \hat{r} - \frac{kr h_1^{(2)}(kr)'}{k_0} \sin \vartheta \hat{\vartheta} \right] \right\}, \tag{B5}
\]

with \(C_{l-}^N = 0\). Specially, since \(j_1(z)/z \to 1/3\) and \([j_1(z)]'/z \to 2/3\) for small \(z\), the scattered field in the center of the system is given by

\[
\mathbf{E}_1^{sc}(r;\omega)|_{r=0} = ik_0 k_0^3 \varepsilon_1 \left\{ C_{11}^N \left[ \frac{2h_1^{(1)}(kr)}{k_0} \cos \vartheta \hat{r} - \frac{kr h_1^{(1)}(kr)'}{k_0} \sin \vartheta \hat{\vartheta} \right] + C_{12}^N \left[ \frac{2h_1^{(2)}(kr)}{k_0} \cos \vartheta \hat{r} - \frac{kr h_1^{(2)}(kr)'}{k_0} \sin \vartheta \hat{\vartheta} \right] \right\} \quad \left( B_6 \right)
\]

For \(N = 2\), we find \((\rho_i = k_i r_i)\)

\[
C_{11}^2(\varepsilon_1, \varepsilon_2; r_1) = \frac{1}{D} \left[ \varepsilon_2 h_1^{(1)}(\rho_2) |p_1 h_1^{(1)}(\rho_1)|' - \varepsilon_1 h_1^{(1)}(\rho_1) |p_2 h_1^{(1)}(\rho_2)|' \right], \quad \left( B_7 \right)
\]

and

\[
C_{2+}^2(\varepsilon_1, \varepsilon_2; r_1) = \frac{i \varepsilon_2}{\rho_1 D}; \quad \left( B_8 \right)
\]

where

\[
D = \varepsilon_1 j_1(\rho_1) |p_2 h_1^{(1)}(\rho_2)|' - \varepsilon_2 h_1^{(1)}(\rho_2) |p_1 j_1(\rho_1)|'. \quad \left( B_9 \right)
\]

For \(N = 3\), we have \((\rho_{ij} = k_i r_j)\)

\[
C_{11}^3(\varepsilon_1, \varepsilon_2, \varepsilon_3; r_1, r_2) = \frac{1}{j_1(\rho_{11})} \left[ \frac{\beta_2 h_1^{(1)}(\rho_{21}) - \beta_1 h_1^{(1)}(\rho_{12})}{\alpha_1 \beta_2 - \alpha_2 \beta_1} - h_1^{(1)}(\rho_{11}) \right], \quad \left( B_{10} \right)
\]

\[
C_{2+}^3(\varepsilon_1, \varepsilon_2, \varepsilon_3; r_1, r_2) = \frac{\beta_2}{\alpha_1 \beta_2 - \alpha_2 \beta_1}, \quad \left( B_{11a} \right)
\]

\[
C_{2-}^3(\varepsilon_1, \varepsilon_2, \varepsilon_3; r_1, r_2) = \frac{-\beta_1}{\alpha_1 \beta_2 - \alpha_2 \beta_1}. \quad \left( B_{11b} \right)
\]
\[
C_{3+}^3(\varepsilon_1, \varepsilon_2, \varepsilon_3; r_1, r_2) = \frac{i\varepsilon_3}{\rho_{22} \alpha_1 \beta_2 - \alpha_2 \beta_1}, \tag{B12}
\]

where

\[
\alpha_j = -\frac{i\rho_{11}}{\varepsilon_2} \varepsilon_1 \varepsilon_3 [\varepsilon_1 j_1(\rho_{11})|\rho_{21} h_1^{(j)}(\rho_{21})[^j] - \varepsilon_2 h_1^{(j)}(\rho_{21})|\rho_{11} j_1(\rho_{11})[^j]], \tag{B13}
\]

and

\[
\beta_j = \varepsilon_3 h_1^{(1)}(\rho_{32})|\rho_{22} h_1^{(j)}(\rho_{22})[^j] - \varepsilon_2 h_1^{(j)}(\rho_{22})|\rho_{32} h_1^{(1)}(\rho_{32})[^j]. \tag{B14}
\]

[7] S. Scheel, L. Knöll, and D.-G. Welsch, Phys. Rev. A 60, 4994 (1999); ibid. 61, 069901(E) (2000). There is an obvious misprint in Eq. (53) of this reference: In the second term on the rhs of this equation \(16e_R(\omega_\lambda)\) should stay instead of \(12e_R(\omega_\lambda)\).
[34] Using the reaction-force type of arguments [see, e.g., Ref. [27]], the classical cavity-induced frequency shift \(\Delta^{sc} = \omega_{\lambda}^{sc}/\gamma_0\) of the molecule normalized to the (free-space) electron radiation damping rate \(\gamma_0\) can be given as

\[
\Delta^{sc} \approx -(3/4k_0^3|p|^2)\text{Re} \cdot \mathbf{E}^{sc}(r_0, r_0; \omega).
\]

With \(\mathbf{E}^{sc}(r_0, r_0; \omega)\) from Eq. (B6), this gives Eq. (33) for \(\Delta^{sc}\), while using Eq. (10) one obtains the general result given in Eq. (41).