PROTON DECAY, BLACK HOLES, and LARGE EXTRA DIMENSIONS

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Abstract

We consider proton decay in theories that contain large extra dimensions. If virtual black hole states are allowed by the theory, as is generally the case, then proton decay can proceed via virtual black holes. The experimental limits on the proton lifetime place strong constraints on the quantum gravity scale $M_{\text{qg}}$ (the effective Planck mass). For most theories, our constraint implies a lower bound of $M_{\text{qg}} > 10^{16}$ GeV. The corresponding bound on the size of large extra dimensions is $\ell < 10^{6/n} \times 10^{-30}$ cm, where $n$ is the number of such dimensions. Regrettably, for most theories this limit rules out the possibility of observing large extra dimensions at accelerators or in millimeter scale gravity experiments. Conversely, proton decay could be dominated by virtual black holes, providing an experimental probe to study stringy quantum gravity physics.

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I. INTRODUCTION

In conventional versions of string theory (M theory), the string energy scale, the Planck mass, and the unification scale are roughly comparable and are relatively close to the standard value of the Planck mass, $M_{\text{pl}} \sim 10^{19}$ GeV. Recently, however, the possibility of a much smaller string scale and large extra dimensions – perhaps large enough to be observable in particle accelerator and gravity experiments – has sparked a great deal of interest.
One of the many constraints on such theories with large extra dimensions is the rate of proton decay. The current experimental limit on the proton lifetime depends on the particular decay mode under study [11], but for the most interesting channels the bound is approximately $\tau_p > 10^{33}$ yr [12]. In the context of theories with large extra dimensions, the existing theoretical literature includes various papers which present mechanisms to suppress “ordinary” GUT scale proton decay, i.e., decay driven by intermediate bosons that mediate baryon number non-conservation. In this case, one can invent extra symmetries to get rid of those unwanted processes and thereby suppress the proton decay rate below the experimental limit [4–7].

In this present paper, we address the idea of virtual black holes acting as the intermediate particles as they do in gravitationally induced proton decay [13–17].

This topic has some urgency: If the relevant scale (the quantum gravity scale) is as low as 1 TeV, for example, then quantum gravity effects could in principle be observed in existing (or upcoming) accelerators, or as deviations from Newtonian gravity. One of the proposed experimental signatures of quantum gravity would be virtual black holes [8,18]. Unfortunately, the same virtual black holes that could be observed would also drive proton decay at a rapid rate, larger than allowed by existing experimental bounds on the proton lifetime. As a result, gravitationally induced proton decay must be highly suppressed in any theory of quantum gravity that accommodates low energy scales and thereby allows large extra dimensions (and no known mechanism provides such a suppression). The arguments of this paper thus provide strong constraints on approaches to quantum gravity in which the theory allows for virtual black hole states at low energy scales. In particular, this bound applies to recent approaches that invoke millimeter or TeV size dimensions.

For another new approach, the Randall-Sundrum case [19,20], the Standard Model (SM) masses appear at about the TeV energy scale while the original mass parameter of the theory can remain near the (old) Planck scale $M_{\text{pl}}$. The energy scale for quantum gravity effects and virtual black holes may remain large ($\sim M_{\text{pl}}$) and hence this class of theories might evade the bounds of this paper. However, in most versions of Randall-Sundrum ideas where Kaluza-Klein (KK) states are at the TeV scale, and where black hole sizes are determined by the KK masses, our constraints will apply.
II. PROTON DECAY IN 4 DIMENSIONS

Let's first review the picture of gravitationally induced proton decay using three spatial dimensions and the traditional value of the Planck mass ($M_{\text{pl}} \sim 10^{19}$ GeV). In any quantum theory, one expects to find vacuum fluctuations associated with the fundamental excitations of the theory. In electrodynamics, for example, electron-positron pairs can form directly out of the vacuum (for a short time). Such processes can be observed indirectly by many quantum phenomena (e.g., in the Casimir effect). In general, however, one must include in the vacuum processes all possible excitations of the theory, e.g., the production of proton-antiproton pairs, or even monopole-antimonopole pairs. These processes are generally highly suppressed relative to the electron-positron amplitudes by virtue of their correspondingly large masses. In gravitation, one therefore expects not only to find virtual gravitons playing a role, but also virtual black holes. Although present uncertainties in quantum gravity theory prevent reliable calculations, even in the context of string theory, we can use a semi-classical calculation as a starting point to study such phenomena.

The standard arguments for virtual black holes lead to the idea that spacetime must be filled with tiny Planck mass black holes with a density of roughly one per Planck volume [21,22]. These microscopic virtual black holes then live roughly for one Planck time. This picture of the spacetime vacuum is often called the spacetime foam (a generalization of this argument for higher dimensions is sketched in the Appendix). Since neither black holes nor the Standard Model itself conserve baryon number, these virtual black holes contribute to the rate of proton decay through their gravitational interaction. In this setting, a proton is considered to be a hollow sphere of radius $R \sim m_p^{-1} \sim 10^{-13}$ cm that contains three (valence) quarks. Suppose that two of these quarks fall into the same black hole at the same time (the quarks must be pointlike compared to the black hole scale for this argument to hold). Since the black hole will evaporate predominantly into the lightest particles consistent with conservation of charge and angular momentum, this process effectively converts the quarks into other particles; only rarely will the same quarks come out that originally went into the black hole. The output particles will often be antiquarks and leptons, and hence baryon number conservation is generally violated. In other words, quantum gravity introduces an effective
interaction leading to many final states, including processes of the form

\[ q + q \rightarrow \bar{q} + \ell, \]

where the final state can include any number additional particles (gravitons, gluons, photons, neutrinos, etc.) and where the resulting antiquark will generally hadronize (e.g., to \( \pi^0 \)). These interactions can be regarded as four-Fermi interactions whose coupling strength is determined by the Planck mass. Such processes are mediated by black holes and can violate conservation of baryon number. Notice, however, that these processes cannot be mediated by gravitons alone because such interactions conserve both electric charge and baryon number.

The probability of two quarks being within one Planck length \((\ell_{pl} \sim 10^{-33} \text{ cm})\) of each other inside a proton is about \((m_P/M_{pl})^3 \sim 10^{-57}\). This value represents the probability per proton crossing time \(\tau \sim m_P^{-1} \sim 10^{-31} \text{ yr}\), if we assume that the particles move at the speed of light. In order for an interaction to take place (such as equation [1]), a virtual black holes must be present at the same time that the two quarks are sufficiently near each other. Including this effect reduces the overall interaction probability by an additional factor of \(m_P/M_{pl}\). Converting these results into a time scale for proton decay [13–17], we find an estimated proton lifetime of

\[ \tau_P \sim m_P^{-1} \left(\frac{M_{pl}}{m_P}\right)^4 \sim 10^{45} \text{ yr}. \]

For comparison, if the proton is unstable through some process operating at the (nonsupersymmetric) unification (GUT) scale \(M_X\) [11], the corresponding time scale for proton decay becomes

\[ \tau_P \sim 10^{30} \text{ yr} \left(\frac{M_X}{10^{15} \text{ GeV}}\right)^4. \]

Thus, the proton lifetime expected from virtual black hole processes (equation [2]) is the same as that for GUT scale processes (equation [3]) in the limit that the unification scale \(M_X\) approaches the Planck scale \(M_{pl}\) and the coupling becomes of order unity. For completeness, we note that in supersymmetric theories the unification scale can be somewhat higher than in grand unified theories without SUSY; in this case, the proton lifetime can be as long as \(\tau_P = 10^{33} - 10^{34} \text{ yr}\), consistent with current experimental limits.
Most versions of string theory contain black hole states with masses comparable to the Planck mass (the quantum gravity or string scale). These black holes play the role of the $X$-boson in proton decay, independent of any specific argument about virtual black holes or spacetime foam. Furthermore, we have no reason not to believe the Hawking formula for the entropy of stringy black holes or their temperature; as a result, the density of states formula implicitly used here should be correct in string theory. What remains controversial, however, is whether the black holes genuinely lose information. Most particle physicists say they do not, whereas most relativists say they do. At present, we simply do not know. Notice that if only $B-L$ is conserved in string theory, then protons will decay with the rate derived here.

One still might be concerned about suppression of the proton decay rate due to violation of global conservation laws or information loss. Because of the t’Hooft anomaly, however, baryon number is not conserved even in the SM, although electric charge and possibly $B-L$ are. The decay described by equation (1), e.g., conserves these latter quantum numbers. Channels such as this decay, that could conserve global quantum numbers and perhaps even circumvent information loss issues, can dominate [10]. As a result, there is no compelling reason to expect large suppressions of the decay channel.

In equation (1), the virtual black holes mediating the interaction appear to act like local quantum objects and thus one might be concerned that the interaction could be gauged away, much like what is done to remove unwanted interactions involving $X$-bosons. Unlike local quantum particles, however, the virtual black holes in quantum gravity processes are solitonic objects – they are less likely to be gauged away because they are extended. In order to suppress proton decay mediated by virtual black holes, one would need to make [5,6] baryon number (or an equivalent matter parity) into a local charge via an exact gauged discrete symmetry that is fully respected by the true vacuum of the theory. Even then, proton decay is generally only suppressed up to some order in the effective operators. Such a suppression requires very special arrangements of chiral fermions or other aspects of the theory. For example, as pointed out by Kakushadze [6], a $\mathbb{Z}_3$ “Generalized Baryon Parity” cannot accommodate right-handed neutrinos without adding additional matter because of anomaly cancellation constraints. As we argue below, because theories with large extra dimensions allow rapid proton decay via virtual black holes, any viable such theory must have baryon number absolutely or almost conserved in the presence of black holes.
Although it is not yet absolutely proven that baryon number cannot be effectively conserved in the presence of black holes, we find it unlikely for two reasons: 

[A] Astrophysical black holes seem to manifestly violate such a conservation law. Imagine compressing a star containing \( N_B \sim 10^{57} \) baryons into a black hole and watching it radiate away. Because the Hawking temperature is low for most of its evaporation time, the black hole radiates primarily into photons, gravitons, and neutrinos; the temperature becomes hot enough to radiate quarks, protons, or other baryonic particles only after the mass shrinks by 19 orders of magnitude. Thus, for baryon number to be conserved, the theory would have to contain extremely unusual objects with small mass and huge baryon number – this case may not be explicitly excluded but it is nonetheless extremely unlikely. (Notice that the black hole may not actually disappear, just as an electrically charged black hole may not disappear – it remains in a BPS-like configuration; the implications of this possibility remain unclear).

[B] The observed baryon asymmetry in the universe argues strongly against absolute conservation of baryon number. Unless one posits special initial conditions at the Big Bang, the cosmos had to generate a baryon excess through some process that violates conservation of baryon number. In string theory, a large number of BPS states are known to be extreme black holes [23], and presumably many other massive string states are black holes as well. All of these states can mediate decays as in equation (1). Because so few known experimental tests can probe the string nature of quantum gravity theories, we should turn our argument around: Since string theory is likely to allow proton decay via virtual black holes and since the string scale may be as low as the GUT scale, proton decay modes (such as those explored in this paper) may be a very powerful diagnostic of string theories. Furthermore, the gravitationally induced channels of proton decay should be recognizable in experiments from their observed branching ratios. This issue should be studied in greater detail in future work.

III. PROTON DECAY WITH LARGE EXTRA DIMENSIONS

We now consider the process of gravitationally induced proton decay in theories with large extra dimensions. For proton decay driven by non-gravitational means – for intermediate particles other than virtual black holes – it is possible to enforce symmetries on the theory to prevent proton decay.
at overly fast rates [4–7]. In the case of gravity, however, no such suppression seems to be allowed. Working in worlds with large extra dimensions, Emparan et al. [10] have argued that black hole evaporation occurs mostly on the brane; this result thus strengthens our approach since final states with particles lighter than a proton are not suppressed.

In a theory with large extra dimensions, two effects modify the picture of proton decay outlined above:

[A] The most important modification is that the Planck mass changes. Specifically, the energy scale of virtual black hole processes changes from $M_{\text{pl}} \approx 10^{19}$ GeV to a lower value which we denote here as $M_{\text{qg}}$. Because the new quantum gravity scale $M_{\text{qg}}$ is generally lower than both $M_{\text{pl}}$ and the GUT scale, this effect acts to reduce the proton lifetime. In particular, the Schwarzschild radius for the virtual black holes is given by $R_S \sim M_{\text{qg}}^{-1}$ [8], which determines the cross section and is much larger than before.

[B] If the number of extra large dimensions is $n > 0$, then the geometry of both the proton and the virtual black holes change. In this context, we let $d \leq n$ denote the number of extra dimensions that the quarks can propagate through. In most theories with large extra dimensions, quarks and other SM particles are confined to the usual 4-dimensional world and cannot freely propagate in the extra dimensions; for most cases, we thus have $d = 0$. In the general case with $d > 0$, the quarks that make up the proton have more dimensions in which to propagate. With more dimensions, the quarks would be less likely to encounter each other and hence this effect increases the proton lifetime. On the other hand, the black holes must be $(4 + n)$ dimensional objects and will necessarily live in the additional dimensions. The black hole interaction cross sections remain of order $R_S^2 \sim M_{\text{qg}}^{-2}$, however, even when interacting with SM particles confined to the usual 4-dimensional spacetime.

Including the above two modifications in estimating the proton decay rate through virtual black hole processes, we find the proton lifetime

$$\tau_P \sim m_P^{-1} \left( \frac{M_{\text{qg}}}{m_P} \right)^{4+d}.$$  \hspace{1cm} (4)

The current experimental bound on the proton lifetime [12] can be written in the form

$$\tau_P > 10^{33} \text{ yr} \equiv m_P^{-1} \left( \frac{\Lambda}{m_P} \right)^4,$$  \hspace{1cm} (5)
where we have defined an energy scale \( \Lambda \equiv (m_P^5 10^{33} \text{yr})^{1/4} \approx 1.4 \times 10^{16} \text{ GeV} \). Combining the general expression (4) with the experimental bound (5), we thus obtain a bound on the scale \( M_{\text{qg}} \) of quantum gravity:

\[
M_{\text{qg}} > (m_P^d \Lambda^4)^{1/(4+d)} = 10^{64/(4+d)} \text{ GeV},
\]

where we have used \( m_P \sim 1 \text{ GeV} \) and \( \Lambda \sim 10^{16} \text{ GeV} \) to evaluate the bound in the second equality. This result greatly constrains the possibility of having a low quantum gravity scale that could be observed in present-day or future accelerators. For the most likely case \( d = 0 \), so that quarks are confined to propagate in our 4-dimensional brane, the quantum gravity scale must be comparable to the (usual) GUT scale, i.e., \( M_{\text{qg}} \geq 10^{16} \text{ GeV} \). For \( d = 1 - 2 \), the quantum gravity scale remains quite high. The weakest constraint arises if \( d = 7 \), which corresponds to the (unlikely) case in which all of the possible extra dimensions are large and the valence quarks within the proton are allowed to propagate freely through all dimensions; in this case, the limit on the quantum gravity scale is \( M_{\text{qg}} > 700 \text{ TeV} \). This scale remains interesting in terms of modifying the hierarchy problems associated with a high quantum gravity scale \( (M_{\text{pl}} \sim 10^{19} \text{ GeV}) \), but unfortunately it remains safely out of experimental reach.

We can also find corresponding bounds on the size scales of the extra dimensions. This size scale \( \ell \) is determined by

\[
\ell^n = M_{\text{pl}}^2 M_{\text{qg}}^{-(2+n)},
\]

where \( n \) is the number of extra large dimensions (see Ref. [1–10]). For the most likely case with \( d = 0 \), equation (6) implies the bound

\[
\ell < (M_{\text{pl}}/\Lambda)^{2/n} \Lambda^{-1}.
\]

As a result, the “large” extra dimensions in such a theory would actually be rather small, \( \ell < 10^{6/n} \times 10^{-30} \text{ cm} \). These size scales would be impossible to observe in modified gravity experiments.

Other rare decays mediated by virtual black holes, such as \( \mu \rightarrow e\gamma \) or neutrino disappearance, may also provide limits on large extra dimensions and low quantum gravity scales. We leave the study of these effects to a future analysis.
IV. SUMMARY

This paper argues that gravitationally induced proton decay – virtual black hole processes that violate baryon number conservation – enforces strong constraints on theories of quantum gravity with large extra dimensions. In particular, our analysis suggests that the observed absence of proton decay via virtual black holes puts a lower limit on the quantum gravity scale $M_{qg}$ and a corresponding upper limit on the size $\ell$ of large extra dimensions. In the weakest (and unlikely) case in which quarks propagate in $n = d = 7$ large extra dimensions, the limit is $M_{qg} > 700$ GeV. This bound rapidly increases to $M_{qg} > 10^{16}$ GeV for any number $n$ of large extra dimensions, if quarks move only in 3 spatial dimensions ($d = 0$) as is generally required for theories to retain the usual SM physics [1–10,24]. The corresponding bound on the size scale of the extra dimensions is $\ell < 10^{6/n} \times 10^{-30}$ cm.

Because the required interactions with black holes are very general, this limit is robust and will not be affected by the domination of specific decay channels. It could be modified if quark sizes (perhaps set by the string scale) are larger than the Planck size, but this does not occur in most approaches. Our limit may not apply if the generally accepted picture of spacetime foam – every Planck volume of spacetime typically contains a virtual black hole for a Planck time – is not valid, or if virtual black hole states are charged under some conserved discrete gauge symmetry (as discussed earlier).

In general, the resulting limit on the quantum gravity scale is so high that it removes most motivation for large extra dimensions. Even with this new constraint, however, the quantum gravity scale $M_{qg}$ could still be somewhat lower than before ($M_{qg} \sim 10^{16}$ GeV $< M_{pl} \sim 10^{19}$ GeV), which could help alleviate hierarchy problems. Unfortunately, our argument rules out observable effects at colliders and in millimeter-scale gravity experiments. Nonetheless, the signatures of virtual black hole processes might be observable in proton decay experiments, which may eventually provide a powerful experimental probe of quantum gravity and string theories.

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APPENDIX:
VIRTUAL BLACK HOLES AND SPACETIME FOAM

In this Appendix, we present a version of the standard argument for virtual black holes filling the vacuum and thereby producing a spacetime foam. In this case, however, we generalize the calculation for higher dimensions. We consider gravity to propagate in $4+n$ dimensions, so that $n$ is the number of large extra dimensions. Notice that $n$ depends on the scale. On sufficiently large spatial scales $n \to 0$ and we must recover old (4-dimensional) Einstein gravity; in this context we are interested in small spatial scales where black holes must be $(4+n)$-dimensional. Gravity is controlled by the action [21,22,25]

$$I[g] = \frac{-1}{16\pi G_n} \int_M R (-g)^{1/2} d^{4+n}x - \frac{1}{8\pi G_n} \int_{\partial M} K (-h)^{1/2} d^{3+n}x + C[h], \quad (9)$$

where $G_n$ is the gravitational constant in $(4+n)$ dimensions and $R$ is the Ricci scalar for the metric $g_{ab}$, which is defined on the spacetime $M$. The spacetime boundary $\partial M$ has the induced metric $h_{ab}$. The quantity $K$ is the trace of the second fundamental form on the boundary $\partial M$ and $C[h]$ is a functional of $h$ defined so that the action of Minkowski space vanishes. Extremization of this action for fixed metric on the boundary leads to the Einstein equations for $g_{ab}$ in $M$.

The path integral for gravity is

$$Z \sim \int \mathcal{D}[g] \ e^{iI[g]}, \quad (10)$$

where the integral is taken over all metrics $g$. Our goal is to investigate how black holes contribute to any amplitude in quantum gravity. We first assume that this integral can be approximated by the usual Euclidean continuation. The action for a single Schwarzschild black hole of mass $m$ is then given by

$$I_1 \sim \frac{m^{2+n}}{M_{Pl}^{2+n}}, \quad (11)$$
where $M_{pl*}$ is the Planck mass in $(4 + n)$ dimensions ($M_{pl*} \approx M_{qg}$). This equation should also contain additional geometrical factors, but these are of order unity and convention dependent (depending on how the mass $m$ is defined). Ignoring interactions between the black holes, we find the action for a collection of $N$ black holes to be

$$I_N \sim \frac{N m^{2+n}}{M_{pl*}^{2+n}}. \quad (12)$$

In the path integral, the black holes are indistinguishable; each is independent of the others and can be positioned anywhere in space. Since $N$ is undetermined, we can evaluate $Z$ in a box of volume $V_n$ (in $3 + n$ spatial dimensions) to obtain the result

$$Z \sim \int_0^\infty dm \sum_{N=0}^\infty \exp[-4\pi N m^{2+n}/M_{pl*}^{2+n}] \frac{1}{N!} \left( \frac{V_n}{\ell_{pl*}^{3+n}} \right)^N. \quad (13)$$

The factor of $V_n$ comes from accounting for the black holes being anywhere in the box, and the factor of $1/N!$ arises from their indistinguishability.

The combination of these results thus defines a probability distribution for having $N$ black holes with mass $m$. Elementary calculations yield the corresponding expectation values for the number density $n_{bh}$ of black holes and for the black hole mass $m_{bh}$, i.e.,

$$\langle n_{bh} \rangle \sim \frac{V_n}{\ell_{pl*}^{3+n}} \quad \text{and} \quad \langle m_{bh} \rangle \sim M_{pl*}, \quad (14)$$

where $\ell_{pl*}$ is the Planck length in the $(4+n)$-dimensional spacetime. As before in the case of 4 dimensions, we find that spacetime must be filled with tiny Planck mass black holes with a density of roughly one per Planck volume. These microscopic virtual black holes live roughly for one Planck time. In this generalized case, however, the black holes are $(4+n)$-dimensional objects and the Planck mass, the Planck volume, and the Planck time are now given by the new (lower) energy scale $M_{pl*}$ (where $M_{pl*} \approx M_{qg}$). This picture of the spacetime vacuum is thus a generalization of the spacetime foam for the case of $(4 + n)$ dimensions.
REFERENCES


