ON THE TIMESCALE FOR THE FORMATION OF PROTOSTELLAR CORES IN MAGNETIC INTERSTELLAR CLOUDS

GLENN E. CIOLEK$^1$ AND SHANTANU BASU$^2$


ABSTRACT

We revisit the problem of the formation of dense protostellar cores due to ambipolar diffusion within magnetically supported molecular clouds, and derive an analytical expression for the core formation timescale. The resulting expression is similar to the canonical expression $\tau_{\text{ff}} / \tau_{\text{ni}} \sim 10 \tau_{\text{ff}}$ (where $\tau_{\text{ff}}$ is the free-fall time and $\tau_{\text{ni}}$ is the neutral-ion collision time), except that it is multiplied by a numerical factor $C(\mu_{c0})$, where $\mu_{c0}$ is the initial central mass-to-flux ratio normalized to the critical value for gravitational collapse. $C(\mu_{c0})$ is typically $\sim 1$ in highly subcritical clouds ($\mu_{c0} \ll 1$), although certain conditions allow $C(\mu_{c0}) \gg 1$. For clouds that are not highly subcritical, $C(\mu_{c0})$ can be much less than unity, with $C(\mu_{c0}) \to 0$ for $\mu_{c0} \to 1$, significantly reducing the time required to form a supercritical core. This, along with recent observations of clouds with mass-to-flux ratios close to the critical value, may reconcile the results of ambipolar diffusion models with statistical analyses of cores and YSO’s which suggest an evolutionary timescale $\sim 1$ Myr for objects of mean density $\sim 10^4 \text{cm}^{-3}$. We compare our analytical result to the results of numerical simulations, and also discuss the effects of dust grains on the core formation timescale.

Subject headings: diffusion — dust, extinction — ISM: abundances — ISM: clouds — ISM: magnetic fields — MHD — plasmas — stars: formation

1. INTRODUCTION

Ambipolar diffusion, the drift of neutral matter with respect to magnetic field and plasma, was put forth by Mestel & Spitzer (1956) as a means by which magnetic flux can escape from interstellar clouds and thereby induce overall collapse and fragmentation. To explain the well-known inefficiency of star formation, Mouschovias (1976, 1977, 1978) proposed instead that, rather than reduce the flux of a cloud as a whole, ambipolar diffusion gravitationally redistributes matter within the central flux tubes of a magnetically supported cloud. He further suggested (Mouschovias 1979, 1982) that the formation of cores would occur on the ambipolar diffusion timescale $\tau_{\text{AD}} \sim \tau_{\text{ff}} / \tau_{\text{ni}}$, where $\tau_{\text{ff}}$ and $\tau_{\text{ni}}$ are respectively, the free-fall and neutral-ion collision times (discussion is also provided in Mouschovias 1987; McKee et al. 1993; Galli & Shu 1993; and Mouschovias & Ciolek 1999). For typical ion abundances within molecular clouds (e.g., Güellin, Langer, & Wilson 1982; Langer 1985; McKee 1989; Ciolek & Mouschovias 1994, 1998), one finds $\tau_{\text{AD}} \sim 10 \tau_{\text{ff}}$ over a wide range of densities.

Numerical simulations in axisymmetric (Fiedler & Mouschovias 1993) and disk-like (Ciolek & Mouschovias 1994 [CM94], 1995; Basu & Mouschovias 1994 [BM94], 1995a, b) model clouds confirmed the theoretical scenario described above. No magnetic flux leaked out of these model clouds, while in the interior flux tubes supercritical cores formed through ambipolar diffusion; core-envelope separation ensued, in which dense cores collapsed dynamically while embedded in massive, magnetically supported (subcritical) envelopes. For highly subcritical model clouds (i.e., with initial central mass-to-flux ratio less than $1/e$ times the critical value), cores did indeed form (i.e., the central mass-to-flux ratio reached the critical value) on a timescale $t_{\text{core}} \simeq t_{\text{ff}}$ (e.g., CM94, § 3; BM94, § 4). However, model clouds that had initial central mass-to-flux ratios $(M/\Phi_B)_{c0}$ that were less subcritical formed in a time $t_{\text{core}}$ significantly less than $\tau_{\Phi_{c0}}$ (e.g., model 5 of Basu & Mouschovias 1995b). This was also true for the recent model presented by Ciolek & Basu (2000; CB00), which successfully reproduced observations of the mass and velocity structure within the L1544 prestellar core. For this model $\mu_{c0} = 0.8$, where

$^1$New York Center for Studies on the Origins of Life (NSCORT), and Department of Physics, Applied Physics, and Astronomy, Rensselaer Polytechnic Institute, 110 8th Street, Troy, NY 12180; cioleg@rpi.edu.

$^2$Department of Physics and Astronomy, University of Western Ontario, London, Ontario N6A 3K7, Canada; basu@astro.uwo.ca.
\( \mu_{\text{c0}} = (M/\Phi_B)_{\text{c0}}/(M/\Phi_B)_{\text{crit}} \) is the initial central mass-to-flux ratio in units of the critical value for gravitational collapse, and \( t_{\text{core}} \) was found to be \( \approx 0.2 \tau_{\Phi,\text{c0}} \). This suggests that \( t_{\text{core}} \) is dependent on the value of \( \mu_{\text{c0}} \). In this paper we derive an analytical relation for \( t_{\text{core}}(\mu_{\text{c0}}) \) that is accurate for all subcritical clouds (i.e., \( \mu_{\text{c0}} \leq 1 \)).

In addition to its general theoretical interest, this investigation is particularly timely, given that a recent compilation of magnetic field strength data (Crutcher 1999) shows that many clouds have mass-to-flux ratios close to the critical value, and some recent statistical studies of ages and lifetimes of protostellar cores and young stellar objects which seem to indicate that star formation can occur on a timescale \( \sim 1 \) Myr (e.g., Lee & Myers 1999; Jijina, Myers, & Adams 1999) within objects of mean density \( \sim 10^4 \) cm\(^{-3} \); that is, it occurs in \( \sim \) few \( \tau_{\Phi} \) rather than \( \sim 10 \tau_{\Phi} \) at these densities. Our general results in this paper (as well as the specific case presented in CB00) show that these inferred lifetimes are compatible with the observed regions being slightly subcritical and evolving due to ambipolar diffusion, in addition to the possibility that they are already somewhat supercritical and evolving dynamically.

## 2. Derivation of the Core Formation Timescale

The fundamental equations we use were developed for disk-like model molecular clouds (using a cylindrical polar coordinate system \( r, \phi, z \)), and were presented in § 3 of Ciolek & Mouschovias (1993; CM93) and § 2 of BM94; the results we derive here can be generalized to other geometries in a straightforward manner. Model clouds are isothermal, embedded in an external medium with constant pressure \( P_{\text{ext}} \), and contain neutral molecules and atoms (H\(_2\) with a cosmic abundance of He), trace amounts of ions (atomic and molecular), electrons, and charged and neutral dust grains. The clouds are self-gravitating and magnetically supported, with \( \mu_{\text{c0}} \leq 1 \).

As in Ciolek & Mouschovias (1996; CM96) we start by considering the motion of a neutral fluid element in a flux tube centered on the axis of symmetry \( (r = 0) \) containing mass \( M \) and magnetic flux \( \Phi_B \) and intersecting the equatorial plane at a Lagrangian radius \( r_M(t) \). In a frame moving with the neutrals,

\[
\frac{d}{dt} \left( \frac{M}{\Phi_B} \right) = \frac{M}{\Phi_B} \frac{d\Phi_B}{dt} = -2\pi \frac{M}{\Phi_B} \frac{d}{dt} \int_{r_M}^{r_M(t)} d\ell_B(r) B_z(r) v_D(t),
\]

where \( B_z \) is the magnetic field and \( v_D \equiv v_1 - v_n \) is the radial ion-neutral drift velocity; \( v_n = dr_M/dt \) is the neutral velocity and \( v_1 \) is the ion velocity. In deriving this equation we have used the magnetic induction equation \( \partial B_z/\partial t = -(1/r)(r B_z v_1)/\partial r \). The induction equation reflects the freezing of magnetic flux in the ions, which is valid for the typical density regime in which cores form (\( n_a \leq 10^3 \) cm\(^{-3} \); see § 3.1.2 of CM93). Core formation takes place in the innermost flux tubes of a cloud, and so we consider equation (1) in the limit \( r_M \to 0 \). The numerical simulations cited in § 1 found that \( B_z \) was spatially uniform in this limit, therefore we may take \( \Phi_B \sim \pi B_z r_M^2 \). Inserting this relation into equation (1), and dividing both sides by \( (M/\Phi_B)_{\text{crit}} \), yields

\[
\frac{1}{\mu_c} \frac{d\mu_c}{dt} = \frac{2v_D}{r_M} = \frac{1}{\tau_{\Phi,\text{c}}}. \tag{2}
\]

Ambipolar diffusion in the central flux tubes results in \( v_D > 0 \), which means that plasma and magnetic flux are “left behind” as neutrals drift inward toward the symmetry axis (due to self-gravity, see below), and equation (2) shows that the dimensionless central mass-to-flux ratio \( \mu_c \) will increase with an instantaneous e-folding timescale \( \tau_{\Phi,\text{c}} \equiv r_M/2v_D(r_M) \). Given the definition of the ambipolar diffusion timescale \( \tau_{AD} \equiv r/v_D \) (e.g., Spitzer 1978, § 13.3c; Mouschovias 1979), it immediately follows that \( \tau_{\Phi,\text{c}} = \tau_{AD}/2 \). The behavior of \( \tau_{\Phi,\text{c}} \) during the evolution of various cloud models is depicted in figures 2d and 4d of CM94, and figures 2c and 6c of BM94; examination of these figures indicates that \( \tau_{\Phi,\text{c}} \) normally decreases as a cloud evolves (i.e., as \( \mu_c \) increases with time).

A supercritical core exists for \( \mu_c \geq 1 \). Core formation occurs at a time \( t_{\text{core}} \) when \( \mu_c(t_{\text{core}}) = 1 \). Integrating equation (2) from \( t = 0 \) to \( t = t_{\text{core}} \) gives

\[
t_{\text{core}} = \int_{\mu_{\text{c0}}}^{1} \frac{d\mu_c'}{\mu_c'} \tau_{\Phi,\text{c}}(\mu_c'). \tag{3}
\]

Thus, we may write

\[
t_{\text{core}} = C(\mu_{\text{c0}})\tau_{\Phi,\text{c0}}, \tag{4a}
\]

where

\[
C(\mu_{\text{c0}}) = \int_{\mu_{\text{c0}}}^{1} \frac{d\mu_c'}{\mu_c'} \tau_{\Phi,\text{c}}(\mu_c'), \tag{4b}
\]

and \( \tau_{\Phi,\text{c0}} \equiv \tau_{\Phi,\text{c}}(\mu_{\text{c0}}) \). The factor \( C(\mu_{\text{c0}}) \) defined in equations (4a) and (4b) is the initial flux constant of a cloud. In a sense, it is a measure of the “distance a cloud has to go” in its central mass-to-flux ratio to attain the critical state \( \mu_c = 1 \), given a particular initial value \( \mu_{\text{c0}} \). The “closer” \( \mu_{\text{c0}} \) is to unity, the smaller we expect \( C(\mu_{\text{c0}}) \) to be. Since \( \tau_{\Phi,\text{c}}(\mu_c) \leq \tau_{\Phi,\text{c0}} \) for \( \mu_c \geq \mu_{\text{c0}} \) (as noted above), equation (4b) provides the limiting relation

\[
C(\mu_{\text{c0}}) \leq \ln \left( \frac{1}{\mu_{\text{c0}}} \right). \tag{5}
\]
Thus, \( C(\mu_c) \to 0 \) as \( \mu_c \to 1 \).

To proceed further, we need an expression for \( \tau_{i,c}(\mu_c) \) to evaluate the integral in equations (3) and (4b). This can be obtained from the force equation (per unit area) for the neutrals, which is given by equations (28a)-(28c) and (50) of CM93. In the next subsection we first consider models that account only for neutral-ion collisions and neglect the friction due to grains. The effect of neutral-grain collisions are then discussed in the following subsection.

2.1. Clouds With Only Neutral-Ion Collisions

The numerical simulations referred to in § 1 generally found that the evolution during the core formation epoch is quasistatic, with little or no acceleration of the neutral particles. Ignoring the acceleration of the neutrals, as well as thermal pressure stresses (which are also negligible at these densities and lengthscales), the evolution of the neutrals is determined by a balance between gravitational and collisional forces:

\[
0 \simeq \sigma_n g r + \frac{\sigma_n}{\tau_{ni}} v_D,
\]

where \( \sigma_n \) is the mass column density of the neutrals, \( g_r \) is the radial gravitational field, and \( \tau_{ni} \) is the neutral-ion collision (momentum exchange) time. Solving this equation, we find \( v_D = \tau_{ni}|g_r| \). Whence, again in the limit \( r_M \to 0 \),

\[
\tau_{\phi,c} = \frac{\tau_M}{2 v_D(r_M)} = \frac{1}{2} \left( \frac{r_M}{\tau_{ni}} \right)^2 c \simeq \frac{1}{2} \left( \frac{\tau_{gr}}{\tau_{ni}} \right)_c, \tag{7}
\]

where \( \tau_{gr} \equiv (r/|g_r|)^{1/2} \) is the gravitational contraction timescale (Ciolek & Königl 1998), which is \( \simeq \tau_{ni} \). (Note: in both CM94 and BM94, \( \tau_{gr} \) is referred to as the dynamical timescale \( \tau_{dyn} \).

For \( \tau_{gr} \) we may write

\[
\tau_{gr} = \left( \frac{A_{geo}}{G m_n n_i} \right)^{1/2}, \tag{8}
\]

where \( G \) is the gravitational constant, \( m_n \) is the mean mass of a neutral particle (\( = 2.33 \text{ amu} \) for an H\(_2\) gas with a cosmic helium abundance), and \( A_{geo} \) is a numerical constant that is solely dependent on the assumed geometry of a model cloud. For most geometries, \( A_{geo}^{1/2} \approx 1 \).

The neutral-ion collision time is expressed as

\[
\tau_{ni} = 1.4 \left( 1 + \frac{m_{H_2}}{m_i} \right) \frac{1}{n_i (\sigma w)_{in}}, \tag{9}
\]

where \( m_{H_2} \) and \( m_i \) are the masses of H\(_2\) molecules and ions, respectively, \( n_i \) is the ion density, and \( (\sigma w)_{in} \) is the ion-neutral collisional rate, which is \( \approx 1.7 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1} \) for H\(_2\)-Na\(^+\) and H\(_2\)-HCO\(^+\) collisions (McDaniel & Mason 1973). The factor 1.4 in equation (9) accounts for the fact that the inertial effect of helium is neglected in the neutral-ion collisional force; for further discussion on this point we refer the reader to § 2.1 of Mouschovias & Ciolek (1999).

Finally, we parameterize the central ion density through the relation \( n_{i,c} \propto n_{n,c} k \), where, in this analysis, we assume that \( k \) is a constant. Strictly speaking, \( k = k(n_{n,c}) \), with \( 0.5 \leq k \leq 1 \) for densities \( n_{n,c} \leq 10^6 \text{ cm}^{-3} \), as shown in numerical simulations (CM94) and analytical studies of the effect of ambipolar diffusion on ion abundances in contracting cores (Ciolek & Mouschovias 1998). However, as we shall see below, our neglect of the dependence of \( k \) on the density leads to only a minor error in the calculation of the integral in equation (4b). Combining this relation with equations (7), (8), and (9), and using our parameterization for \( n_{i,c}(n_{n,c}) \) discussed above, yields

\[
\frac{\tau_{\phi,c}}{\tau_{\phi,c}^0} = \left( \frac{n_{n,c}^0}{n_{n,c}} \right)^{1-k} \tag{10}
\]

Our remaining task is to determine \( n_{n,c}(\mu_c) \). In the disk model clouds, the mass density \( \rho_n(r) = m_n n_n(r) \) is related to the column density \( \sigma_n(r) \) in a magnetic flux tube through balance of thermal-pressure forces and self-gravity along the \( z \)-direction (i.e., along the direction of the magnetic field):

\[
\rho_n C^2 = P_{ext} + \frac{\pi}{2} G \sigma_n^2, \tag{11}
\]

\( C \) being the isothermal speed of sound (for a derivation, see § 3.1.1 of CM93). It then follows that

\[
n_{n,c} = \left[ \frac{\tilde{P}_{ext} + 1}{\tilde{P}_{ext} + (\sigma_n/c_n)^2} \right], \tag{12}
\]

where \( \tilde{P}_{ext} = 2 P_{ext}/\pi G \sigma_{n,c}^2 \) is the dimensionless external pressure, in units of the initial self-gravitational stress in the central flux tube. Now, in the limit \( r_M \to 0 \), \((M/\Phi_B) = \sigma_{n,c}/B_{z,c} \), and, therefore,

\[
\frac{\mu_c}{\mu_0} = \frac{(M/\Phi_B)_c}{(M/\Phi_B)_0} = \frac{\sigma_{n,c}}{\sigma_{n,c}^0} \left( \frac{B_{z,c}^0}{B_{z,c}} \right) \approx \frac{\sigma_{n,c}}{\sigma_{n,c}^0} \tag{13}
\]

In the last equality in equation (13) we have used the fact that the central magnetic field strength \( B_{z,c} \) changes very little during the core formation epoch (Fiedler & Mouschovias 1993; CM94; BM94). That is, during this stage of evolution, ambipolar diffusion
allows magnetic field lines to remain essentially “fixed in place” (Mouschovias 1978, 1979). Inserting equation (13) into (12), and then substituting into (10) and (4b), we have

$$C(\mu_0) = \int_1^{\tilde{P}_{\text{ext}}} \frac{d\mu}{y} \left( \frac{\tilde{P}_{\text{ext}} + 1}{\tilde{P}_{\text{ext}} + y^2} \right)^{1-k} , \quad (14)$$

where $y \equiv \mu c / \mu_0$. For a given value of $\tilde{P}_{\text{ext}}$, $C(\mu_0)$ can be obtained by integration of equation (14). Note that for $k = 1$, equation (14) readily yields

$$C(\mu_0) = \ln \left( \frac{1}{\mu_0} \right) , \quad (15)$$

regardless of the value of $\tilde{P}_{\text{ext}}$. Similarly, for the case $k = 1/2$, the integral (14) has the solution

$$C(\mu_0) = \left( \frac{\tilde{P}_{\text{ext}} + 1}{\tilde{P}_{\text{ext}}} \right)^{1/2} \ln \left[ \frac{\tilde{P}_{\text{ext}}^{1/2} + (\tilde{P}_{\text{ext}} + 1)^{1/2}}{\tilde{P}_{\text{ext}}^{1/2} \mu_0 + (\tilde{P}_{\text{ext}} \mu_0 + 1)^{1/2}} \right] . \quad (16)$$

The previously published model clouds cited in § 1 typically adopted a value $\tilde{P}_{\text{ext}} = 0.1$, appropriate for an isolated, gravitationally bound cloud. For such models the integral (14) may be solved by taking the limit $\tilde{P}_{\text{ext}} \rightarrow 0$, yielding

$$C(\mu_0) \approx \frac{1 - \mu_0^{2(1-k)}}{2(1-k)} \quad \text{for} \quad k < 1 \quad (17)$$

(for $k = 1/2$, this expression is also easily obtained from eq. [16] in the same limit), and for $k = 1$, $C(\mu_0)$ is given by (15) above. Figure 1 shows $C(\mu_0)$ as a function of $\mu_0$, for models with $\tilde{P}_{\text{ext}} = 0.1$ and $k = 0.5, 0.6, 0.75, \text{and} \ 1.0$, respectively. The solid lines are the solutions given by equations (15) and (17), which assume $\tilde{P}_{\text{ext}} = 0$. The curves with crosses, for the models with $k = 0.5, 0.6$, and 0.75, represent the solution for $\tilde{P}_{\text{ext}} = 0.1$, obtained from equation (14) or direct numerical integration. Examination of this figure reveals that the analytical approximation (17) for these small $\tilde{P}_{\text{ext}}$ models is in excellent agreement with the exact integration.

Also shown in Figure 1 are the results for $C(\mu_0) = t_{\text{core}} / \tau_{\Phi,c,0}$, as found in some of our earlier published core formation and collapse numerical simulations: model A of CM94 ($\tilde{P}_{\text{ext}} = 0.1, \mu_0 = 0.26, k \approx 0.65$ — see Fig. 2c of CM94), model 2 of BM94 ($\tilde{P}_{\text{ext}} = 0.1, \mu_0 = 0.324, k = 0.5$), model 5 of BM95b ($\tilde{P}_{\text{ext}} = 0.1, \mu_0 = 0.5, k = 0.5$), and the L1544 model of CB00 ($\tilde{P}_{\text{ext}} = 0.1, \mu_0 = 0.8, k \approx 0.7$). Comparison of the values found in the numerical simulations with those derived from our expressions for $C(\mu_0)$ (eqs. [14]-[17]) also show excellent agreement.

Notice that, for highly subcritical clouds ($\mu_0 \ll 1$), $C(\mu_0) \sim 1$ for most realistic values of $k$. Hence, for these clouds, $t_{\text{core}} \approx \tau_{\Phi,c,0}$, as originally argued by Mouschovias (1979, 1982). Models with $k \approx 1$ and $\mu_0 \ll 1$ are an exception: for these models $\tau_{\Phi,c,0}$ does not decrease with increasing density, since the degree of ionization $x_{i,c} = n_{i,c} / n_{\text{hi,c}} \propto n_{\text{hi,c}}^{k-1}$ in the central flux tubes remains essentially constant as the cloud evolves. Under these conditions, $\tau_{\Phi,c,0}$ is a true e-folding timescale.

However, $C(\mu_0) \ll 1$ as $\mu_0$ approaches unity, for all values of $k$. This region is of particular interest, given the recent OH Zeeman observations of magnetic field strengths in dense molecular cloud gas by Crutcher (1999). The estimated mass-to-flux ratios are close enough to the critical value that a clear observational distinction between subcritical and supercritical cores cannot be made. However, when the likely effects of random orientations of cloud axes and magnetic field projections on the plane of the sky are accounted for, Crutcher’s inferred values are consistent with $0.5 \lesssim \mu_c \lesssim 1.5$ (e.g., see Table 1 of Shu et al. 1999). Although his sample undoubtedly contains some cores that have already become supercritical (and therefore are in the early stages of collapse), these values may also be indicative of $\mu_0$ for typical star-forming molecular clouds. In this case, clouds may often be subcritical (and magnetically supported), but only moderately so. 3 From equations (15) - (17) it follows that, for $(1 - \mu_0) \ll 1$, $C(\mu_0) \approx 1 - \mu_0$. Hence, $C(\mu_0) \ll 1$ will be expected to occur for clouds with $\mu_0$ values in the range suggested by the Crutcher (1999) data. This means that clouds with these values of $\mu_0$ will evolve and form cores by ambipolar diffusion on a timescale much less than $\tau_{\Phi,c,0}$.

Let us evaluate the magnitude of this effect numerically. From equations (7), (8), and (9) we find

$$\tau_{\Phi,c,0} = 7.0 \times 10^6 A_{\text{geo}} \left( \frac{x_{i,c,0}}{10^{-7}} \right) \text{yr} , \quad (18)$$

where we have normalized $x_{i,c,0}$ to a value typical for cosmic-ray ionized gas at density $n \approx 10^4 \text{cm}^{-3}$. Since, according to our results displayed in Figure

3That clouds may have mass-to-flux ratios near the critical value is consistent with the well-known linewidth-size relation for molecular clouds (e.g., Larson 1981; Myers 1983). For instance, Mouschovias & Psaltis (1995) analyzed the data for the linewidths in 14 different clouds and showed that the ubiquitous relation $\Delta v \propto R^{1/2}$ (where $\Delta v$ is the linewidth and $R$ the radial extent of a cloud) could arise from turbulent or nonthermal linewidths resulting from hydromagnetic waves in clouds with magnetic field strengths comparable to the critical value for collapse.
1, $\mathcal{C}(\mu_0) \lesssim 0.5$ for clouds with $\mu_0 \gtrsim 0.5$, we conclude from equations (4a) and (18) that $t_{\text{core}} \lesssim 4$ Myr for moderately subcritical clouds. This effect had already been seen in the L1544 model of CB00, which had $\mu_0 = 0.8$, resulting in $\mathcal{C}(\mu_0) = 0.2$, and $t_{\text{core}} = 1.3$ Myr (see their § 3). Thus, recent statistical analyses suggesting that star formation occurs in some regions on timescales $\sim 1$ Myr may be explained by ambipolar diffusion models and observations of subcritical clouds with central mass-to-flux ratios slightly below the critical value for collapse. We also note that the much longer lifetimes of a few $\times 10^7$ yr for giant molecular clouds as a whole can be accounted for by their lower mean density, yielding a higher ionization fraction $x_1$, particularly when ionization due to background ultraviolet starlight is accounted for (e.g., Ciolek & Mouschovias 1995). The higher values of $x_1$ can greatly extend the lifetime of lower density regions even if the mass-to-flux ratio is close to critical.

In the limit $P_{\text{ext}} \gg 1$, corresponding to model clouds situated in a high-pressure external medium, appropriate for cloud fragments or sheets located in a dense or massive cluster or cloud complex, or in a particularly hot medium or intense radiation field (perhaps suitable for proplyds), the integral in equation (14) is found in the limit $P_{\text{ext}} \rightarrow \infty$ to be the same as that given by equation (15). For this case, clouds with $\mu_0 < 1/e$ have $t_{\text{core}} > \tau_{\Phi,0}$ since $\tau_{\Phi,0}$ is again an $e$-folding timescale. This occurs in this limit because the mass density $\rho_n$ given by equation (11) is insensitive to the amount of mass within a flux tube (given by the column density $\sigma_n$). This is seen from equation (12), which implies that the fractional variation of the density $\delta n_{c,n,c}$ is related to the fractional variation in column density $\delta n_{c,n,c}$ by

$$\frac{\delta n_{c,n,c}}{n_{c,n,c}} = 2\frac{(\sigma_{n,c}/\sigma_{n,0}) (\delta \sigma_{n,c}/\sigma_{n,0})}{(\sigma_{n,c}/\sigma_{n,0})^2 + P_{\text{ext}}} \approx 2\frac{(\mu_0/\mu_0) (\delta \mu_0/\mu_0)}{(\mu_0/\mu_0)^2 + P_{\text{ext}}},$$

where the last approximate equality is due to equation (13). For $P_{\text{ext}} \rightarrow 0$, it follows from (19) that $\delta n_{c,n,c} / n_{c,n,c} \rightarrow 2\delta \sigma_{n,c}/\sigma_{n,0} \approx 2\delta \mu_0/\mu_0$, i.e., the central density increases twice as fast as the column density and mass-to-flux ratio. In the opposite extreme, $P_{\text{ext}} \rightarrow \infty$, $\delta n_{c,n,c} / n_{c,n,c} \rightarrow 2(\sigma_{n,c}/\sigma_{n,0}) (\delta \sigma_{n,c}/\sigma_{n,0}) / P_{\text{ext}} \approx 2(\mu_0/\mu_0) (\delta \mu_0/\mu_0) / P_{\text{ext}} \rightarrow 0$. For these models, $n_{c,n,c}$ is determined primarily by vertical confinement (compression) of the disk by the external bounding pressure $P_{\text{ext}}$, and, as a result, there is little change in the density as the column density and mass-to-flux ratio increases in the central flux tubes. Since the density does not increase much in this case, neither does the degree of ionization and $\tau_{\Phi,0}$ is effectively constant as the cloud evolves; this follows from equation (10), which yields $\delta \tau_{\Phi,0}/\tau_{\Phi,0} = -(1 - k)\delta \sigma_{n,c}/n_{n,c}$. (As noted earlier, $k = 1$ also yields $\delta \tau_{\Phi,0} = 0$.) Figure 2 displays $\mathcal{C}(\mu_0)$ for two model clouds with $P_{\text{ext}} = 20$; one has $k = 0.5$ and the other $k = 1$. The high-pressure analytical result $\mathcal{C}(\mu_0) = \ln(1/\mu_0)$, is an approximation for these models. In summary, we have demonstrated that ambipolar diffusion driven evolution actually occurs less rapidly in regions of high external pressure, since the density, which is then determined to a great extent by the bounding pressure, increases less rapidly as the central mass-to-flux ratio increases.

Finally, before ending this subsection we make an important physical distinction. As described above, our new analysis shows that, for clouds with $\mu_0 \sim 1$, $\mathcal{C}(\mu_0) = t_{\text{core}} / \tau_{\Phi,0} \ll 1$. Despite the fact that a core forms at a faster rate under these conditions (when compared to the uncorrected initial flux-loss timescale), it should not be considered to be dynamical. Recall that our results are obtained using a quasi-static approximation for the equation of motion of the neutrals during the core formation epoch, consistent with behavior seen in our detailed numerical simulations. Therefore, although the timescale for core formation in this situation is much less than that which would have been predicted by the “classical” expression for a core forming due to ambipolar diffusion (i.e., $t_{\text{core}} \simeq \tau_{\Phi,0}$), it should not be taken to mean that the mass in the inner flux tubes is undergoing dynamical or accelerated collapse during this phase of evolution. As described in detail in the numerical simulations cited in § 1, magnetically-diluted collapse occurs within a supercritical core only after it forms, leading to star formation in an additional few $\tau_{\text{ff}}$.

2.2. Inclusion of Grain Collisional Friction

We now consider the effect of neutral-grain collisions. Here we restrict our analysis to standard models with $P_{\text{ext}} \ll 1$. Detailed discussion of the effects of grains (charged and neutral) on the evolution of self-consistent model clouds is provided in CM93, CM94, and CM95. For typical grain sizes in dense molecular clouds (radii $a \gtrsim 10^{-6}$ cm) the bulk of the grains have a charge $-e$, where $e$ is the electronic charge. Moreover, as shown in CM93-95, rapid inelastic capture of ions and electrons on grains transfers momentum between charged and neutral grains, which can also result in effective coupling of the neutral grains to the magnetic field.

We again make use of the fact that the evolution due to ambipolar diffusion during the core formation...
epoch is well-approximated as being quasistatic. The motion of the neutrals is therefore given by

\[ 0 \simeq \sigma_n g_r + \frac{\sigma_n}{\tau_{ni}} v_D + \frac{\sigma_n}{\tau_{ng}} \Delta_g v_D. \]  

(20)

The latter term on the RHS of equation (20) is the force (per unit area) from neutral-grain collisions. The quantity \( \tau_{ng} \) is the neutral-grain collision time and \( \Delta_g \equiv (v_g - v_n)/v_D \), where \( v_g \) is the grain velocity and \( v_n \) is the neutral velocity. \( \Delta_g \) expresses the coupling of charged dust grains to the magnetic field: grains that are attached to magnetic field lines will have \( v_g = v_n \), and, therefore, \( \Delta_g = 1 \); in the opposite limit of unattached grains (due to collisions with neutrals), \( v_g = v_n \) and \( \Delta_g = 0 \).

Equation (20) yields

\[ v_D(r_M) = \left( \frac{\sigma_n g_r |\tau_{ni}|}{1 + (\tau_{ni}/\tau_{ng}) \Delta_g} \right) r_M. \]  

(21)

As in the preceding section, in the limit \( r_M \to 0 \) it follows from equations (2) and (21) that

\[ \tau_{\phi,c} = \frac{1}{2} \left( \frac{\tau_{gr}^2}{\tau_{ni}} \right)_c + \frac{1}{2} \left( \frac{\tau_{gr}^2}{\tau_{ng}} \Delta_g \right)_c. \]  

(22)

In the derivation of \( C(\mu_c) \), the evaluation of the first term on the RHS of equation (22) — the contribution to the flux-loss timescale due to the ions alone (which would occur for \( \Delta_g = 0 \) or \( \tau_{ni}/\tau_{ng} \ll 1 \) — follows exactly as presented in § 2.1 above. We focus now on the second term on the RHS of equation (22). To determine its effect on \( C(\mu_c) \) involves evaluating the integral

\[ \int_1^\infty \frac{d\mu_c}{\mu_c^2} \Delta_g(r_M, \mu_c) = \left( \frac{\tau_{gr}^2}{\tau_{ng}} \right)_c \int_1^\infty \frac{d\mu_c}{\mu_c^2} \left( \frac{\tau_{gr,c}}{\tau_{gr,c}} \right)^2 \Delta_g(r_M, \mu_c). \]  

(23)

Simplification comes about since

\[ \left( \frac{\tau_{gr,c}}{\tau_{gr,c}} \right)^2 \tau_{ng,c} = \frac{n_{nc,0}}{n_{nc,0} x_{g,c,0}} = \frac{x_{g,c}}{x_{g,c,0}}; \]  

(24)

where \( n_g \) is the density of grains and \( x_g = n_g/n_n \) is the relative abundance of grains. In the reduction of equation (24), we have used equation (8) and the fact that the neutral-grain collision time

\[ \tau_{ng} = \frac{m_n + m_g}{m_g n_g (\sigma w)_{gn}}; \]  

(25)

\( m_g (\gg m_n) \) is the grain mass, and \( (\sigma w)_{gn} \) is the grain-neutral collision rate, equal to \( \pi a^2 (8 k_B T / \pi m_n)^{1/2} \), where \( a \) is the radius of the grains, \( k_B \) is the Boltzmann constant, and \( T \) is the gas temperature.

Further progress is made by using the analysis of CM96, who studied the effect of ambipolar diffusion on dust-to-gas ratio during the formation of cores. As CM96 described at length, ambipolar diffusion can reduce the abundance of grains in a core when the inwardly drifting neutrals “leave behind” the magnetic field lines and the grains that are attached to them. In particular, they showed that during the core formation epoch, \( \Delta_g, c \frac{d\mu_c}{\mu_c} \neq 0 \) (corresponding to some degree of attachment of grains to the magnetic field), the relative abundance of grains decreases as the central mass-to-flux ratio increases. 4 Using this relation, along with equation (24), the integral (23) becomes

\[ \int_1^\infty \frac{d\mu_c}{\mu_c^2} \Delta_g(r_M, \mu_c) = \left( \frac{\tau_{gr}^2}{\tau_{ng}} \right)_c \left( 1 - \frac{x_{g,c}}{x_{g,c,0}} \right) \]  

(26)

where \( x_{g,c} \) is the abundance of grains in the central flux tubes at time \( t_{core} \), \( \mu_c = 1 \). CM96 (eq. [11]) also showed that

\[ \frac{x_{g,c}}{x_{g,c,0}} = \frac{\mu_c}{1 + A_g \mu_c}, \]  

(27)

where \( A_g \) is a dimensionless quantity whose inverse is related to the strength of the attachment of charged grains to the magnetic field. Specifically, equations (7), (8), and (9a)-(b) of CM96 yield

\[ \Delta_{g,c,0} = \frac{1}{1 + A_g^2 \mu_c^4}; \]  

(28)

and therefore, for \( A_g \to 0 \), \( \Delta_{g,c,0} \to 1 \) (grains are frozen to magnetic field lines), while, conversely for \( A_g \to \infty \), \( \Delta_{g,c,0} \to 0 \) (grains are completely unattached). Numerically,

\[ A_g = 2.32 \times 10^{-2} \left( \frac{B_{z,c,0}}{35 \mu G} \right) \left( \frac{T}{10^6 \text{ cm}} \right)^2 \times \left( \frac{m_n}{2.33 \text{ amu}} \right)^{1/2} \left( \frac{10 \text{ K}}{T} \right)^{1/2}. \]  

(29)

(CM96, eq. [10]). For most situations of interest \( (10^{-6} \text{ cm} \lesssim a \lesssim 10^{-5} \text{ cm}, B_{z,c,0} \sim 10 \mu G, \text{ and } T \simeq 10 \text{ K}), A_g \ll 1 \).

We now have the necessary information to derive \( C(\mu_c) \) in models clouds including the effect of grain

\[ 4 \text{This particular result is unique to our ambipolar diffusion models. It also affects the ion chemistry in contracting cores, as discussed in CM94 and Ciebciak & Mouschovias (1998).} \]
friction. Combining equations (4b), (22), and (26)-(28), along with the results for the ion contributions described in § 2.1, we have, in the limit \( \dot{P}_{\text{ext}} \ll 1 \),

\[
C(\mu_{c0}) = \frac{1 - \mu_{c0}^2(1 - k)}{2(1 - k)} + \left( \frac{\tau_{ni}}{\tau_{ng,c0}} \right)_{c0} \left[ 1 - \mu_{c0} \left( \frac{1 + A_g^2 A_{\chi g}^2 \mu_{c0}^2}{1 + A_g^2 A_{\chi g}^2} \right)^{1/4} \right] \\
+ \left( \frac{\tau_{ni}}{\tau_{ng,c0}} \right)_{c0} \left( \frac{1}{1 + A_g^2 A_{\chi g}^2 \mu_{c0}^2} \right) \left( \frac{1}{1 + A_g^2 A_{\chi g}^2} \right)^{1/4}
\]

(30a)

if \( k < 1 \),

\[
\ln \left( \frac{1}{\mu_{c0}} \right) + \left( \frac{\tau_{ni}}{\tau_{ng,c0}} \right)_{c0} \left[ 1 - \mu_{c0} \left( \frac{1 + A_g^2 A_{\chi g}^2 \mu_{c0}^2}{1 + A_g^2 A_{\chi g}^2} \right)^{1/4} \right] \\
1 + \left( \frac{\tau_{ni}}{\tau_{ng,c0}} \right)_{c0} \left( \frac{1}{1 + A_g^2 A_{\chi g}^2 \mu_{c0}^2} \right) \left( \frac{1}{1 + A_g^2 A_{\chi g}^2} \right)^{1/4}
\]

(30b)

if \( k = 1 \).

Examination reveals that, in the limits \( (\tau_{ni}/\tau_{ng,c0}) \to 0 \) (dynamically unimportant dust grains), or \( A_g \to \infty \) (grains are not well-coupled to field lines and move with the neutrals, thereby not retarding contraction), both equations (30a) and (30b) revert to the expressions found in § 2.1 due to the effects of ions alone, namely, equations (17) and (15). Furthermore, for most circumstances, because \( 10^{-6} \text{ cm} \lesssim a \lesssim 10^{-5} \text{ cm} \), resulting in \( A_g \ll 1 \) (see eq. [29]), clouds with \( 1 - \mu_{c0} \ll 1 \) have \( C(\mu_{c0}) \approx 1 - \mu_{c0} \), just as we had found earlier.

Our derivation can be compared to numerical simulations of model clouds that account for the dynamical effects of grains. In model B of CM94 (see their §§ 3.1 and 3.3), \( \mu_{c0} = 0.26 \), \( B_{z,c0} = 35 \mu \text{G} \), \( T = 10 \text{ K} \), \( x_{i,c0} = 2.0 \times 10^{-7} \), \( x_{g,c0} = 7.1 \times 10^{-11} \), and the radius of the grains \( a = 3.75 \times 10^{-6} \text{ cm} \). Inserting these values into the expressions for \( \tau_{ni,c0} \) (eq. [9]), \( \tau_{ng,c0} \) (eq. [25]), and \( A_g \) (eq. [29]), and taking \( k = 0.64 \) (see Fig. 4c of CM94), equation (30a) gives \( C(\mu_{c0}) = 0.86 \). In actuality, CM94 model B had \( C(\mu_{c0}) = t_{\text{core}}/\tau_{\phi,c0} = 0.92 \). Hence, our analytical expression for \( C(\mu_{c0}) \) is again in very good agreement with the results of detailed evolutionary calculations.

The core formation timescale \( t_{\text{core}} = C(\mu_{c0}) \tau_{\phi,c0} \).

Numerically, when the collisional effects of grains are accounted for, \( \tau_{\phi,c0} \) is again given by equation (18), but multiplied by a factor

\[
1 + \left( \frac{\tau_{ni}}{\tau_{ng,c0}} \right)_{c0} \frac{1}{1 + A_g^2 A_{\chi g}^2 \mu_{c0}^2} \times \left( \frac{T}{10 \text{ K}} \right)^{1/2} \left( \frac{3 \text{ g cm}^{-3}}{\rho_s} \right) \left( \frac{10^{-6} \text{ cm}}{a} \right) \times \left( \frac{10^{-7} \text{ cm}}{x_{i,c0}} \right) \left( \frac{x_{g}}{0.01} \right) = 1 + 2.6 \times 10^{-4} \times \frac{T}{10 \text{ K}} \times \left( \frac{3 \text{ g cm}^{-3}}{\rho_s} \right) \times \left( \frac{10^{-6} \text{ cm}}{a} \right) \times \left( \frac{10^{-7} \text{ cm}}{x_{i,c0}} \right) \times \left( \frac{x_{g}}{0.01} \right),
\]

where equations (9) and (25) were used to evaluate \( \tau_{ni,c0} \) and \( \tau_{ng,c0} \), and we also made the approximation \( A_g^2 \mu_{c0} \ll 1 \). In equation (31) we have used \( \rho_s = 4\pi \rho_s a^3/3 \), where \( \rho_s \) is the density of the solid grain material, and also the relation \( x_{g,c0} = (m_{n}/m_{g}) \chi_g \), where \( \chi_g \) is the dust-to-gas mass ratio normalized to the canonical value for the interstellar medium. From equations (30a) and (30b) it follows that clouds with \( \mu_{c0} > 0.5 \) still have \( C(\mu_{c0}) \ll 1 \), and therefore, \( t_{\text{core}} \ll \tau_{\phi,c0} \). For the values of \( \tau_{\phi,c0} \) given by equations (18) and (31), depending on the properties of grains in those clouds, values of \( t_{\text{core}} \) in the range 1 to 5 Myr are still likely.

3. RELEVANCE TO OBSERVATIONS

Our results reveal that dense cores may be regarded as an ensemble of objects which are closely clustered on either side of the subcritical-supercritical boundary, as implied by recent observations (Crutcher 1999). The subcritical objects are still evolving due to ambipolar diffusion, and the supercritical objects have crossed the critical barrier and may show extended infall (e.g., the case of L1544; CB00). This picture seems consistent with the observation that only some cores show extended infall, and that even L1544 requires significant magnetic forces to keep the infall speeds to subsonic levels (CB00), given that it has little turbulent support. However, the mean lifetime for all the cores, from statistical studies, can still be \( \lesssim \) few Myr, due to the relatively short duration of the subcritical phase at the observed densities.

Statistical studies of dense cores, usually defined as regions in which the mean density \( n \gtrsim 10^4 \text{ cm}^{-3} \), since they excite the NH\(_3\) \((J,K) = (1,1)\) line, reveal a lifetime \( \sim 1 \text{ Myr} \) for starless cores (Lee & Myers 1998; Jijina et al. 1999). There are considerable uncertainties (at least of order one) in this determination, since the lifetimes are based upon a bootstrapping from inferred pre-main-sequence (PMS) lifetimes; the relative statistics of cores with embedded infrared sources to PMS sources yields a lifetime for the embedded phase, and the relative statistics of cores with and without embedded sources yields a lifetime for the starless phase. This is done under the assumption that the formation times of the cores are uncorrelated and uniformly spread in time up to the present. There is also a question as to whether the samples are complete, as many of them are biased toward cores that already contain stars, and regions that have already undergone considerable star formation activity.

A cautionary note regarding lifetimes comes from the submillimeter continuum studies of Ward-Thompson et al. (1994, 1999). In all starless cores detected in the submillimeter regime, the inferred
central density is in the range $10^5$ cm$^{-3}$ to $10^6$ cm$^{-3}$, meaning that the cores have a much shorter gravitational contraction timescale than cores whose inferred central density is only $10^3$ cm$^{-3}$. In particular, at a central density $n_{\text{c},0} = 3 \times 10^5$ cm$^{-3}$, the spherical free-fall time $\tau_{\text{ff},c} = (3\pi/32Gn_{\text{c},0})^{1/2} = 6 \times 10^4$ yr, compared to $3 \times 10^5$ yr at a density $10^4$ cm$^{-3}$. This means that if the submillimeter cores have a lifetime $\sim 1$ Myr, then they live for $\sim 20\tau_{\text{ff}}$, making ambipolar diffusion very necessary at this stage (see Ward-Thompson et al. 1999, who even conclude that some ambipolar diffusion models evolve too rapidly)

The resolution to this apparent discrepancy can only come if it is demonstrated that the submillimeter cores represent a small subset of the ammonia cores, and high resolution studies establish that most of the ammonia cores actually have central densities close to $10^4$ cm$^{-3}$. A complete sample of starless cores (with high angular resolution) is thus very necessary before drawing definitive conclusions about lifetimes.

Another caution against preliminary conclusions about the timescale for protostellar evolution comes from a recent study by Kontinen et al. (2000), who measured the abundances of various molecular species in two dense cores: the starless, prestellar core Cr A C located in Coronae Australis, and the Cha MMS1 core in the Chamaleon I dark cloud, which contains a class 0 source. Comparing the results of their observational study with time dependent chemical evolution models, they argue that Cr A C exhibits ‘later time chemistry’ than that which is seen in the Cha MMS1 core. Kontinen et al. suggest, as one possibility, that the reason the Cr A C core appears to be chemically or chronologically older than the more physically evolved Cha MMS1 core is due to the Coro-

nae Australis cloud having had a longer ambipolar diffusion timescale than in Chamaleon I. Of course, based on our discussion in this paper, differences in the ambipolar diffusion and core formation timescales between these two parent clouds may reflect differences in the values of $\mu_{\text{c}}$ (and, thus, $C(\mu_{\text{c},0})$), as well as disparities in $x_{\text{c},0}$ and the initial central dust abundance $x_{\text{g},0}$ (which could result from variations in the local cosmic-ray ionization rate $\zeta_{\text{CR}}$ and/or dust grain properties — see CM98 for a discussion on how the fractional abundances of charged particles depend on these particular quantities), which would fix the value of $\tau_{\phi,c}$ (see eqs. [18] and [31]) in each cloud.

Finally, it is of interest to note that the recent Zeeman measurements cited above seem to indicate that the mass-to-flux ratio in some molecular clouds is comparable to the critical value for collapse. That it is so close to the critical value likely indicates the important role of magnetic fields in the formation, support, and evolution of molecular clouds. Unfortunately, since a rigorous and comprehensive theory for the formation of molecular clouds from out of the diffuse interstellar medium is still lacking (clearly an astrophysical problem of some importance, yet outside the purview of our present study), 5 more definitive statements about the specific nature of $\mu_{\text{c}}$, and $\mu_{\text{c},0}$, may take on values $\sim 1$ for observed clouds cannot be made at this time. However, our lack of knowledge as to why $\mu_{\text{c},0}$ may take on certain values does not affect our analysis in this paper (nor our previously published models and parameter studies, cited in § 1), as we have formulated our result in such a way that it can be applied to all subcritical ($\mu_{\text{c},0} \leq 1$) clouds or cloud fragments.

4. SUMMARY

In this paper we have reexamined the formation of protostellar cores by ambipolar diffusion in magnetically supported interstellar molecular clouds. Starting from basic principles, and utilizing the results of detailed numerical simulations, we have derived an analytical relation for the timescale for the formation of supercritical cores $t_{\text{core}}$ within initially subcritical clouds as a function of $\mu_{\text{c},0}$, the initial central mass-to-flux ratio in units of the critical value for gravitational collapse. The resulting expression is given simply by $t_{\text{core}} = C(\mu_{\text{c},0})\tau_{\phi,c}$, where $\tau_{\phi,c}$ is the initial central flux-loss timescale, and the dimensionless factor $C(\mu_{\text{c},0})$ is the initial flux constant of a cloud. Our analytical expressions are found to be in excellent agreement with the results of detailed numerical simulations of core formation by ambipolar diffusion in disk-like molecular clouds, including a recent model presented by Ciolek & Basu (2000) that accurately reproduces the density and velocity structure within the L1544 protostellar core. Our analysis has also included the dynamical effects of interstellar dust grains, which is also found to agree well with our earlier numerical models.

We have derived simple, yet accurate expressions for $C(\mu_{\text{c},0})$, valid for all $\mu_{\text{c},0} \leq 1$. A strict upper limit $C(\mu_{\text{c},0}) \leq \ln(1/\mu_{\text{c},0})$ is obtained. Clouds with $\mu_{\text{c},0} < 1$ generally have $C(\mu_{\text{c},0}) \sim 1$, and therefore $t_{\text{core}} \approx \tau_{\phi,c}$, in agreement with the early analysis by Mouschovias (1979). However, clouds with $\mu_{\text{c},0} \gtrsim 0.5$ have $C(\mu_{\text{c},0}) \ll 1$, thereby substantially reducing $t_{\text{core}}$ with respect to $\tau_{\phi,c}$. Clouds with these values of $\mu_{\text{c},0}$, which seem to be indicated in certain regions by recent OH Zeeman observations, have numerical values of $t_{\text{core}} \lesssim \text{few Myr}$. Hence, the evidence for

5Interestingly, ambipolar diffusion can provide a means by which $\mu_{\text{c},0}$ can be deduced for molecular clouds. As discussed in CM96, reduction of the dust-to-gas ratio by ambipolar diffusion (see § 2.2, and eq. [27] above) can be used to infer $\mu_{\text{c},0}$ from comparative observations of the relative abundances of grains in the cores and envelopes of molecular clouds.
comparably short timescales for protostellar collapse and evolution (e.g., Jijina et al. 1999; Lee & Myers 1999) is not in disagreement with a picture in which observed dense cores are either somewhat subcritical (and evolving towards the critical value due to ambipolar diffusion) or are already somewhat supercritical and have started accelerated or dynamical collapse.

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Fig. 1.— Initial flux constant $C(\mu c_0) = t_{\text{core}}/\tau_{\text{E,core}}$ for several model clouds with $k = 0.5$, 0.6, 0.75, and 1.0, respectively. Crosses represent the exact solution of equation (14) for $P_{\text{ext}} = 0.1$ and $k = 0.5$ (eq. [16]), 0.6, and 0.75. Solid lines are the solutions given by equations (15) or (17) for $P_{\text{ext}} = 0$. Also shown are the values found from previously published numerical simulations: model A of CM94 (located by the center of $\otimes$), model 2 of BM94 (⊕), model 5 of BM95b (•), and the L1544 model cloud of CB00 (⋄). For most models with $\mu c_0 \ll 1$, $C(\mu c_0) \sim 1$. Above $\mu c_0 \gtrsim 0.5$, however, $C(\mu c_0)$ is significantly less than unity. For $1 - \mu c_0 \ll 1$, $C(\mu c_0) \approx 1 - \mu c_0$. 
Fig. 2.— Same as in Figure 1, except in the limit of high pressure, $\tilde{P}_{\text{ext}} = 20$, for models with $k = 1.0$ and 0.5, given by equations, (15) and (16), respectively.