Neutral weak currents in pion electroproduction on the nucleon.

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Parity violating asymmetry in inclusive scattering of longitudinally polarized electrons by unpolarized protons with $\pi^0$ or $\pi^+$ meson production, is calculated as a function of the momentum transfer squared $Q^2$ and the total energy $W$ of the $\pi N$-system. This asymmetry, which is induced by the interference of the one-photon exchange amplitude with the parity-odd part of the $Z^0$-exchange amplitude, is calculated for the $\gamma^* (Z^*) + p \rightarrow N + \pi$ processes ($\gamma^*$ is a virtual photon and $Z^*$ a virtual Z-boson) considering the $\Delta$-contribution in the $s$-channel, the standard Born contributions and vector meson ($\rho$ and $\omega$) exchanges in the $t$-channel. Taking into account the known isotopic properties of the hadron electromagnetic and neutral currents, we show that the P-odd term is the sum of two contributions. The main term is model independent and it can be calculated exactly in terms of fundamental constants. It is found to be linear in $Q^2$. The second term is a relatively small correction which is determined by the isoscalar component of the electromagnetic current. Near threshold and in the $\Delta$-region, this isoscalar part is much smaller (in absolute value) than the isovector one: its contribution to the asymmetry depend on the polarization state (longitudinal or transverse) of the virtual photon.

I. INTRODUCTION

Parity violation (PV) was discovered in 1956 in nuclear beta-decay by C.S. Wu [1], following a suggestion of T.D. Lee and C. N. Yang [2]. In 1960, Ya. Zeldovich [3] pointed out that PV should lead to parity-odd (P-odd) terms also in electron-hadron interactions. These are now considered as a manifestation of the electroweak interaction, whose properties are dictated by the Standard Model (SM). Several P-odd observables have since been studied, in two types of PV experiments, namely in atomic physics [4,5] (at very low energy and momentum transfer) and in electron scattering (at relatively high energies and non-zero momentum transfers).

At first, these experiments were aiming at testing the SM and measuring the Weinberg angle. A pioneering experiment was performed at SLAC on a deuterium target [6], followed 10 years later by experiments at Mainz on $^9\text{Be}$ [7] and Bates on $^{12}\text{C}$ [8]. Their determination of the Weinberg angle were confirmed later on, within their stated accuracy of 10%, by high energy experiments. Since $\sin^2 \theta_W$ is now known to three decimal places [$\sin^2 \theta_W = 0.23124(24)$] [9], the emphasis of e-p scattering today, is to make use of the SM to learn about the internal structure of the nucleon.

Until recently, it has been assumed that the nucleon was only made of u and d valence or sea quarks, but there are indications that the nucleon carries also hidden strangeness:

- the sigma-term (deduced from the pion-nucleon scattering length) is very different from the theoretical value calculated within the chiral perturbation theory (which is a realization of the SM at low energy), indicating that 35% of the nucleon mass might be carried out by strange quarks [10–12],
- experiments of polarized Deep-Inelastic-Scattering (DIS) of leptons show that up to 10-20% of the nucleon spin could be carried by strange quarks [13–16],
- elastic scattering of neutrinos and anti-neutrinos by protons can only be explained by taking into account strange quarks in the nucleon [17,18],

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These experiments are sensitive to various aspects of nucleon structure: for example, the sigma-term and the results of \(NN\) or \(\pi N\) experiments are sensitive to the scalar part of the hadronic current, polarized DIS and elastic scattering of neutrinos (or anti-neutrinos) by protons are sensitive to the vector-axial current. In this respect, PV in electron-nucleon scattering seems the most attractive way of measuring the \textit{strange} vector current, thanks to a clean theoretical interpretation through the SM.

The SAMPLE collaboration at MIT-Bates, has measured PV asymmetries in \(\vec{e}p\) elastic scattering at \(Q^2 = 0.1\) (GeV/c)^2 (\(Q^2 = |k^2| = -k^2\), where \(k\) is the four-momentum transferred squared) and large angle [23], which allowed them to obtain the first experimental determination of the weak magnetic form-factor of the proton. From this measurement and the knowledge of the proton and neutron electromagnetic form-factors, one could extract a strange magnetic form-factor \(G_M^s = (0.61 \pm 0.17 \pm 0.21)\) \(\mu_N\). Note that most calculations based on QCD or quark models predict negative values for \(G_M^s\) (see e.g. [24–32]).

Another experiment, done by the HAPPEX collaboration at Jefferson Lab, [33] has done a measurement at \(Q^2 = 0.48\) (GeV/c)^2 and small scattering angle \(\theta_e = 35^\circ\) where the sensitivity to the weak electric form factor \(G_E^s\) is enhanced. Here the measured asymmetry \(A = (-14.2 \pm 2.2) \cdot 10^{-6}\) is consistent with the SM prediction in the absence of \(<ss>\) components in the nucleon sea. From this asymmetry, one can deduce the following contribution to the strange form-factor:

\[
G_H^s = G_E^s + 0.39G_M^s = (0.023 \pm 0.034 \pm 0.22 \pm 0.026)\ \mu_N,
\]

compatible with zero within the error bars.

These results have stimulated a strong interest and many predictions, for both \(G_E^s\) and \(G_M^s\), have been published, whether within quark models [24], Chiral Perturbation Theories [26,27] or Lattice QCD calculations [29]. These calculations predict that while \(G_M^s\) is essentially constant as a function of \(Q^2\), \(G_E^s\) may vary rapidly. They also indicate that there might be some cancellation between \(G_E^s\) and \(G_M^s\) which are predicted of different signs. Therefore new e-p experiments are being set up in order to check these predictions: at \(Q^2 = 0.225\) (GeV/c)^2 at Mainz [34], at \(Q^2 = 0.1\) (GeV/c)^2 and forward angles by the HAPPEX collaboration in order to do a Rosenbluth separation of \(G_E^s\) and \(G_M^s\) in combination with the SAMPLE results, and finally a full separation of \(G_E^s\) and \(G_M^s\) in the momentum transfer range \(Q^2 = 0.12-1.0\) (GeV/c)^2 is foreseen by the \(G^0\) collaboration at Jefferson Lab [35].

It should be stressed that, due to their present high level of precision and their firm theoretical basis, PV experiments can also be used to answer other important physics questions, besides the existence of a possible \(<ss>\) component in the nucleon. Let’s mention for example:

- search and test for new physics beyond the SM [36–42]. Such effects could manifest themselves through, e.g. extra \(Z^\prime\)-bosons (heavier than the standard \(Z^0\)) or leptoquarks indicating that there are substructures common to both leptons and quarks. The use of the \(G^0\) experimental set-up for such studies is being discussed [43],

- study of the axial part of the hadronic weak neutral current: It has been shown that, for the special case of a \(I = 3/2, J = 3/2\) spin-isospin transition (\(\Delta\)-excitation), there is a particular sensitivity to \(G^A_{N\Delta}\) , the axial-vector transition form-factor which could be determined independently of PCAC and free of uncertainties from extrapolation of low energy theorems [44],

• measurement of the neutron charge form-factor [45–47].

The reactions \(e + p \rightarrow e + p + n^0\) and \(e + p \rightarrow e + n + \pi^+\) are of practical interest for experimentalists as they may contaminate the elastic peak. It is therefore important to determine their own asymmetries since, if they are much larger than or, even, of different sign from the elastic one, they might be a source of errors or large uncertainties.

In 3-body reactions, besides the weak PV asymmetries, there are also strong (parity-conserving) interactions, due to the so-called 5th response function [48], which are generally much larger (of the order of \(10^{-2} – 10^{-3}\) instead of \(10^{-5} – 10^{-7}\)) than PV asymmetries but which cancel in inclusive reactions or when detectors have an azimuthal symmetry.

Pion production has been studied previously [49,50] in quasi 2-body models with stable \(\Delta\) isobar. A more complete calculation including background (Born) terms with pseudoscalar \(\pi N\) coupling with the \(\Delta\) treated as a Rarita-Schwinger field with phenomenological \(\pi N\) electromagnetic transition currents can be found in [51].

In the present study, we calculate PV asymmetries in inclusive \(N(e,e')N\pi\) electroproduction, starting from threshold up to the \(\Delta\)-region in an approach similar to the one of ref. [51], but differing in the following aspects:

• the main improvement consists in including \(\omega\)- and \(\rho\)-exchange in the \(t\)-channel for \(\gamma^* (Z^*) + N \rightarrow \pi + N\) (where \(\gamma^* (Z^*)\) is a virtual photon or boson),

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we use a different parametrization for the ∆ contribution, and slightly different values of mass and width,
• crossing symmetries are treated differently (and less accurately) than in ref [51],
• we use a pseudoscalar πNN interaction in order to identify possible off-mass-shell effects.

Two remarkable results were found in the calculations of ref. [51], for which no explanation or discussion was given:
- the full (background + resonance) asymmetry A does not depend on the total energy of the hadronic system, although the separate terms show strong (and opposite) variations,
- A is, to a good approximation, a linear function of $Q^2$.

The present work agrees with these features. Moreover it gives a physical explanation to them as it shows that there is a specific parametrization of the asymmetry which allows to separate the main (isovector) contribution in a model independent way. This contribution only depends on the Fermi constant $G_F$, the fine structure constant $\alpha$ and $\sin^2 \theta_W$.

Therefore its dependence on the kinematics can be predicted exactly. Small corrections to the main contribution, due to the isoscalar part of the neutral vector and axial currents can then be calculated and their physical importance assessed.

II. P-ODD BEAM ASYMMETRY FOR $e^- + N \rightarrow e^- + N + \pi$

We shall consider here the processes $e^- + N \rightarrow e^- + N + \pi$, where $N$ is a nucleon ($p$ or $n$) and $\pi$ is a pion ($\pi^0$ or $\pi^+$). We take into account two standard mechanisms, $\gamma^-$ and $Z^-$ boson exchanges (Fig. 1), predicted by the SM. The matrix element can be written in the following form:

$$\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z,$$

$$\mathcal{M}_\gamma = -\frac{e^2}{\sqrt{2}} \ell_\mu J^{(em)}_\mu,$$

$$\mathcal{M}_Z = \frac{G_F}{2\sqrt{2}} \left( g_a^{(e)} \ell_\mu + g_v^{(e)} \ell_{\mu,5} \right) \left( J^{(nc)}_\mu + J^{(nc)}_{\mu,5} \right),$$

where $G_F$ is the Fermi constant of the weak interaction, $J^{(em)}_\mu$ is the electromagnetic current for $\gamma^- + N \rightarrow N + \pi$, $J^{(nc)}_\mu$ and $J^{(nc)}_{\mu,5}$ are the vector and vector-axial parts of the neutral weak current for $Z^- + N \rightarrow N + \pi$. The four-vector $\ell_\mu$ and $\ell_{\mu,5}$ are the vector and vector-axial parts of the neutral weak current of a point-like electron:

$$\ell_\mu = \slashed{p}(k_2)\gamma_\mu u(k_1),$$

$$\ell_{\mu,5} = \slashed{p}(k_2)\gamma_5\gamma_\mu u(k_1)$$

where $k_1$ ($k_2$) is the four-momentum of the initial (final) electron. The notation for the particle four momenta is explained in Fig. 1. Note that the formula for $\mathcal{M}_Z$, Eq. (2.1), is valid in the so-called local limit, where

$$-k^2 \ll M_Z^2 \simeq 8100 \text{ (GeV/c)}^2.$$

In the Standard Model the constants $g_a^{(e)}$ and $g_v^{(e)}$ are determined by the following expressions:

$$g_a^{(e)} = 1, \quad g_v^{(e)} = 1 - 4\sin^2 \theta_W,$$

where $\theta_W$ is the Weinberg angle. Then:

$$g_v^{(e)} \simeq 0.076, \text{ i.e. } g_v^{(e)} \ll g_a^{(e)}.$$  

From Eq. (2.1) it appears that P-odd effects in $e^- + N \rightarrow e^- + N + \pi$ result from the interference between $\mathcal{M}_\gamma$ and $\mathcal{M}_Z^{(-)}$, which is the parity-violating part of $\mathcal{M}_Z$:  

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The P-odd asymmetry in the scattering of longitudinally polarized electrons can be written as:

$$A = \frac{N_+ - N_-}{N_+ + N_-} = -\frac{G_F |k^2|}{2\sqrt{2}\pi\alpha} W^{-},$$  \hspace{0.5cm} (2.6)

with two different contributions to $W^{-}$:

$$W^{-} = g_{\alpha}^{(e)} \vec{W}_1 + g_{\nu}^{(e)} \vec{W}_2,$$  \hspace{0.5cm} (2.7)

where $W^{(em)}$ is proportional to $|M|^2$:

$$W^{(em)} = \ell_{\mu\nu} W^{(em)}_{\mu\nu},$$  \hspace{0.5cm} (2.8)

$$W^{(em)}_{\mu\nu} = \overline{\mathcal{J}}_{\mu}^{(em)} \mathcal{J}_{\nu}^{(em)*},$$  \hspace{0.5cm} (2.9)

$$\ell_{\mu\nu} = 2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} - g_{\mu\nu} k_1 \cdot k_2),$$  \hspace{0.5cm} (2.10)

and the overline in Eq. (2.9) stands for the sum over the final nucleon polarizations and the average over the polarizations of the initial nucleon in the process $\gamma^* + N \rightarrow N + \pi$. The quantities $\vec{W}_1$ and $\vec{W}_2$ in Eq. (2.7) characterize the interference of the electromagnetic hadronic current $\mathcal{J}^{(em)}$ with the vector and axial parts of the weak neutral current:

$$\vec{W}_1 = \ell_{\mu\nu} W^{(v)}_{\mu\nu},$$  \hspace{0.5cm} (2.11)

$$W^{(v)}_{\mu\nu} = \frac{1}{2} \overline{\mathcal{J}}_{\mu}^{(em)} \mathcal{J}_{\nu}^{(nc)*},$$  \hspace{0.5cm} (2.12)

$$\vec{W}_2 = \ell_{\mu\nu} W^{(a)}_{\mu\nu},$$  \hspace{0.5cm} (2.13)

$$W^{(a)}_{\mu\nu} = \frac{1}{2} \overline{\mathcal{J}}_{\mu}^{(em)} \mathcal{J}_{\nu,5}^{(nc)*},$$  \hspace{0.5cm} (2.14)

and

$$\ell^{(a)}_{\mu\nu} = 2i\epsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta},$$  \hspace{0.5cm} (2.15)

where $\epsilon_{\mu\nu\alpha\beta}$ is the usual antisymmetric tensor.

In the product of the tensors $\ell^{(a)}_{\mu\nu}$ and $W^{(a)}_{\mu\nu}$, only the antisymmetric part of $W^{(a)}_{\mu\nu}$ contributes, whereas the quantity $\vec{W}_1$, Eq. (2.11), is determined by the symmetrical part of the tensor $W^{(v)}_{\mu\nu}$.

According to Eq. (2.4), we can neglect the $\vec{W}_2$ contribution (the second P-odd contribution, which is induced by the axial part of the neutral weak current, is more model dependent and it will be the object of a detailed analysis in a subsequent paper).

In this approximation, the P-odd asymmetry is solely determined by the vector part of the hadronic neutral weak current:

$$A = -\frac{G_F |k^2|}{2\sqrt{2}\pi\alpha} \vec{W}_1,$$  \hspace{0.5cm} (2.16)

In order to calculate the ratio $\vec{W}_1/W^{(em)}$, we shall use the isotopic structure of the vector neutral current, which holds in the SM when neglecting the contributions of the isoscalar quarks ($s, c, ...$):

$$\mathcal{J}^{(nc)}_{\mu} = 2\mathcal{J}^{(1)}_{\mu} - 4\sin^2 \theta_W \mathcal{J}^{(em)}_{\mu} = 2(1 - 2\sin^2 \theta_W)\mathcal{J}^{(em)}_{\mu} - 2\mathcal{J}^{(0)}_{\mu},$$  \hspace{0.5cm} (2.17)
and $\mathcal{J}_\mu^{(0)}$ and $\mathcal{J}_\mu^{(1)}$ are the isoscalar and isovector components of the electromagnetic hadronic current. Considering the specific isotopic structure of $\mathcal{J}_\mu^{(nc)}$, Eq. (2.17), the asymmetry $A$ for any process $e^+ N \to e^+ N + \pi$ can be written as:

$$A = -\frac{G_F |k|^2}{2\sqrt{2}\pi \alpha} \left[ 1 - 2 \sin^2 \theta_W + \Delta^{(s)} \right],$$  

(2.19)

where the quantity $\Delta^{(s)}$ results from the interference of the isoscalar component $\mathcal{J}_\mu^{(0)}$ of the electromagnetic current with the full electromagnetic current in $\mathcal{J}_\mu^{(em)}$, i.e.:

$$\Delta^{(s)} = \frac{W^{(0)}}{W^{(em)}}, \quad W^{(0)} = -\ell_{\mu\nu} \overline{\mathcal{J}_\mu^{(em)}} \mathcal{J}_\nu^{(0)*}. \quad (2.20)$$

One can see from Eq. (2.19) that the isovector part of the electromagnetic current induces a definite contribution to the P-odd asymmetry $A$, which is model independent and can be predicted in terms of the fundamental constants $G_F$, $\alpha$ and $\sin^2 \theta_W$. Note that this contribution depends only on the variable $k^2$. Therefore, for reactions such as $e^- + N \to e^- + \Delta$, $e^- + d \to e^- + d + \pi^0$, where the electromagnetic current is pure isovector (and therefore $\Delta^{(s)} = 0$), the asymmetry can be predicted exactly:

$$A = -\frac{G_F |k|^2}{2\sqrt{2}\pi \alpha} \left[ 1 - 2 \sin^2 \theta_W \right], \quad (2.21)$$

in agreement with ref. [52] and neglecting the small contributions from the axial hadronic current, which is not considered here (note that $\frac{G_F}{2\sqrt{2}\pi \alpha} = 1.8 \cdot 10^{-4}$). In particular, for the reaction $e^- + p \to e^- + \Delta^+$ this model-independent estimate of $A$ together with the possibility of a precise measurement of the P-odd asymmetry, open new ways to look for new physics [39] and to study effects due to the axial current.

In the next section, we will show that the quantity $\Delta^{(s)}$, in the near-threshold region for $e^- + N \to e^- + N + \pi$, as well as in the region of the $\Delta$ excitation, can be considered as a small correction to the main isovector contribution. Therefore, the uncertainty in the estimate of $\Delta^{(s)}$ will affect very little the results.

Note that there is a model independent relation between the isoscalar components of the electromagnetic currents, for the considered processes $\gamma^* + p \to n + \pi^+$ and $\gamma^* + p \to p + \pi^0$, which holds for any interaction mechanism:

$$\mathcal{J}_\mu^{(s)}(\gamma^* p \to n\pi^+) = -\sqrt{2} \mathcal{J}_\mu^{(s)}(\gamma^* p \to p\pi^0).$$

From Eq. (2.19) it appears that the inclusive asymmetry $A$ depends on the variables $E_1$ and $W$ only through the correction $\Delta^{(s)}$:

$$\Delta^{(s)} = \Delta^{(s)}(k^2, W, E_1).$$

Taking into account the longitudinal and transversal polarizations of the virtual $\gamma$ and $Z$-boson, the following representation for the correction $\Delta^{(s)}$ can be written (in case of a single channel: $e + p \to e + p + \pi^0$ or $e + p \to e + n + \pi^+$):

$$\Delta^{(s)} = \frac{\sigma_T^{(s)} + \epsilon \frac{(-k^2)}{k_0^2}}{\sigma_T + \epsilon \frac{(-k^2)}{k_0^2}} \frac{\sigma_L^{(s)}}{\sigma_L}, \quad (2.22)$$

$$\epsilon^{-1} = 1 - 2 \frac{(-k^2)}{k^2} \tan^2 \theta_e \frac{1}{2}, \quad k_0 = \frac{W^2 + k^2 - m^2}{2W},$$

where $\sigma_T(k^2, W)$ and $\sigma_L(k^2, W)$ are the total cross sections of virtual photon absorption in $\gamma^* + N \to N + \pi$:

$$\sigma_L = \int \left| \mathcal{J}_\mu^{(em)} \right|^2 d\Omega_\pi, $$
\[ \sigma_T = \int \left( \left| J_z^{(em)} \right|^2 + \left| J_y^{(em)} \right|^2 \right) \, d\Omega_\pi, \]

(2.23)

d\Omega_\pi being the element of solid angle of the produced pion (in the CMS of the process \( \gamma^* + N \rightarrow N + \pi \)). We use here a coordinate system in which the z-axis is along the three momentum of the virtual photon, and \( J_z^{(em)} \), \( J_y^{(em)} \) and \( J_z^{(em)} \) are the space components of the hadronic electromagnetic current.

The total cross sections:

\[ \sigma_T^{(s)}(k^2, W) = \int d\Omega_\pi \Re \left( J_z^{(em)} J_z^{(em)} \right), \]

(2.24)

where \( J_z^{(em)} \) are the space components of the isoscalar part of the hadronic electromagnetic current.

The interference contributions \( \sigma_T^{(s)} \) and \( \sigma_T^{(s)} \) are defined as follows:

\[ \sigma_T^{(s)}(k^2, W) = \int d\Omega_\pi \Re \left( J_z^{(em)} J_z^{(em)} + \sigma_T^{(s)} J_z^{(em)} J_z^{(em)} \right), \]

(2.25)

where \( J_z^{(em)} \) are the space components of the isoscalar part of the hadronic electromagnetic current.

The lines above the products of the components of the electromagnetic currents mean the sum over the polarizations of the final nucleons and the average over the polarizations of the initial nucleons.

The inclusive asymmetry for \( p(\bar{e}, e')N\pi \) with the contribution of two channels \( p + \pi^0 \) and \( n + \pi^+ \) in the final state, is determined by the following expressions:

\[ A = -\frac{G_F|k^2|}{2\sqrt{2}\pi\alpha} \left[ 1 - 2 \sin^2 \theta_W + \Delta_{\text{incl}}^{(s)} \right], \]

\[ \Delta_{\text{incl}}^{(s)} = \frac{\Delta^{(s)}(\gamma^*p \rightarrow n\pi^+) + R\Delta^{(s)}(\gamma^*p \rightarrow p\pi^0)}{(1 + R)}, \]

with

\[ R = \frac{\sigma_T(\gamma^*p \rightarrow n\pi^+) + \epsilon \frac{(-k^2)}{k_0^2} \sigma_L(\gamma^*p \rightarrow p\pi^0)}{\sigma_T(\gamma^*p \rightarrow p\pi^0) + \epsilon \frac{(-k^2)}{k_0^2} \sigma_L(\gamma^*p \rightarrow n\pi^+)} \]

Therefore, the P-odd inclusive asymmetry \( A \) for \( p(\bar{e}, e')N\pi, \ N\pi = (p + \pi^0) + (n + \pi^+) \) is determined by a set of four total cross sections:

\[ \sigma_T(k^2, W), \ \sigma_L(k^2, W), \ \sigma_T^{(s)}(k^2, W), \ \text{and} \ \sigma_L^{(s)}(k^2, W), \]

for each \( \gamma^* + p \rightarrow n + \pi^+ \) and \( \gamma + p \rightarrow p + \pi^0 \) processes (8 in total), as functions of two independent kinematical variables \( k^2 \) and \( W \). The polarization parameter \( \epsilon, 0 \leq \epsilon \leq 1 \), which represents the linear polarization of the virtual photon, contains the dependence on the kinematical conditions of the electrons in the initial and final states (i.e. initial energy and scattering angle).

In the present calculation we shall use the following parametrization of the spin structure of the matrix element for \( \gamma^* + N \rightarrow N + \pi \), in terms of six standard contributions:

\[ \mathcal{M}(\gamma^* N \rightarrow N\pi) = \chi_1^4 \mathcal{F} \chi_1, \]

\[ \mathcal{F} = i \bar{\epsilon} \cdot \hat{K} \times \hat{q} \mathcal{F}_1 + \bar{\sigma} \cdot \bar{e} \mathcal{F}_2 + \bar{\sigma} \cdot \hat{K} \bar{\epsilon} \cdot \hat{q} \mathcal{F}_3 + \bar{\sigma} \cdot \hat{q} \bar{\epsilon} \cdot \hat{q} \mathcal{F}_4 \]

(2.26)

\[ + \bar{\epsilon} \cdot \hat{K} (\bar{\sigma} \cdot \hat{K} \mathcal{F}_5 + \bar{\sigma} \cdot \hat{q} \mathcal{F}_6), \]

where \( \chi_1 \) and \( \chi_2 \) are the two-component spinors of the initial and final nucleons, \( \bar{\epsilon} \) is the three-vector of the virtual photon polarization, \( \hat{K} \) and \( \hat{q} \) are the unit vectors along the 3-momentum of the \( \gamma^* \) and \( \pi \) in the CMS of the \( \gamma^* + N \rightarrow N + \pi \) reaction. The complex scalar amplitudes \( \mathcal{F}_i \), which are functions of three independent kinematical variables,
\( f_i = f_i(k^2, W, \cos \theta_\pi), \) can be related to the usual set of amplitudes, \( F_i, \) when the operator \( \vec{\sigma} \cdot \vec{q} \vec{\sigma} \cdot \vec{e} \times \vec{k} \) is used instead of \( i\vec{e} \cdot \hat{k} \times \vec{q}. \)

\[
\begin{align*}
  f_1 &= F_1, \\
  f_2 &= F_1 - \cos \theta_\pi F_2, \\
  f_3 &= F_2 + F_3, \\
  f_4 &= F_4.
\end{align*}
\]

The results of averaging over the polarization states of the initial nucleon and summing over the polarizations of the produced nucleon gives:

\[
|J_{\mu}^{(em)}| |J_{\nu}^{(em)}| = 2|f_2|^2 + \sin^2 \theta_\pi (|f_1|^2 + |f_3|^2 + |f_4|^2 + 2Re(f_2 f_4^* + \cos \theta_\pi f_3 f_4^*)).
\]

\[ (2.27) \]

To calculate the cross sections \( \sigma_T^{(s)} \) and \( \sigma_L^{(s)}, \) in Eqs. (2.27) the following substitutions are made:

\[
|f_i|^2 \rightarrow Re f_i f_i^{(s)},
\]

\[ 2Re f_i f_j \rightarrow Re(f_i f_j^{(s)} + f_j f_i^{(s)}), \quad i,j = 1,\ldots,6, \quad (2.28) \]

where \( f_i^{(s)} \) are the scalar amplitudes, describing the isoscalar part of the hadronic electromagnetic current \( J_\mu^{(em)}. \)

**III. MODEL FOR \( e^- + N \rightarrow e^- + N + \pi \)**

We use here the standard approach for the calculation of the electromagnetic current for the \( \gamma^* + N \rightarrow N + \pi \) processes, which describes satisfactorily well the existing photo- and electro-production data, in the region of \( W \) starting from threshold, \( W = m + m_\pi, \) up to \( W \approx 1.3 \text{ GeV} \) (the \( \Delta \) excitation region). This approach takes into account the following three contributions:

- Born terms in the \( s, t \) and \( u \) channels,
- vector meson (\( \omega \) and \( \rho \)) exchanges in the \( t \)-channel,
- \( \Delta \)-isobar excitation in the \( s \) channel.

Using the isotopic structure of the 'strong' vertices on the diagrams (Fig. 2), the scalar amplitudes for each \( \gamma^* + N \rightarrow N + \pi \) process can be written as:

\[
f_i = \sqrt{(E_1 + m)(E_2 + m)} [a_{s,i,s} + a_{a,i,u} + a_{t,i,t} + a_{\rho,i,\rho} + a_{\omega,i,\omega} + a_{\Delta,i,\Delta}], \quad (3.1)
\]

where \( f_{i,s,\ldots,i,\Delta} \) characterize the contributions of the different Feynmann diagrams to the scalar amplitudes \( f_i, \) \( i = 1-6 \).

The energies \( E_1 \) and \( E_2 \) of the initial and final nucleons are determined by the following formulae:

\[
E_1 = \frac{s + m^2 - k^2}{2s}, \quad E_2 = \frac{s + m^2 - m_\pi^2}{2s}.
\]

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The isotopic numerical coefficients $a_s...a_\Delta$ for the two processes $\gamma^* + p \to p + \pi^0$ and $\gamma^* + p \to n + \pi^+$ are shown in Table 1.

The scalar amplitudes for the isoscalar part of the electromagnetic current can be written as follows:

$$f_{i,s}^{(s)}(\gamma^* p \to p \pi^0) = - \left( f_{i,s}^{(s)} + f_{i,u}^{(s)} + f_{i,\rho} \right) \sqrt{(E_1 + m)(E_2 + m)},$$

where $f_{i,s}$ and $f_{i,u}$ are the amplitudes for the isoscalar part of the $s$- and $u$- channel Born diagrams.

One can see now that, in the framework of the considered approach, the main contributions to $\mathcal{J}_\mu^{(em)}$ have an isovector nature:

- $\Delta$-excitation in $\pi^+$ and $\pi^0$ production,
- $\pi^+$-exchange for $\pi^+$ production,
- $\omega$-exchange for $\pi^0$ production,
- contact term for $\pi^+$ production (in the case of a pseudovector $\pi NN$-interaction),
- $s + u$ Born contributions.

Therefore, the isoscalar electromagnetic current can only contain the following contributions:

- $\rho$-exchange for $\pi^0$ and $\pi^+$-production,
- the isoscalar part of the $s + u$-diagrams.

However these isoscalar contributions are small in comparison with the corresponding isovector ones. Indeed, the $\rho$-exchange term is smaller than the $\omega$ -exchange term, due to the following reasons:

- $g_{\rho\pi\gamma} \simeq \frac{1}{3} g_{\omega\pi\gamma}$: suppression at electromagnetic vertices;
- $g_{\rho NN} \simeq \frac{1}{6} g_{\omega NN}$: suppression at the strong vertex.

In the same way, the isoscalar Born contribution due to the nucleon magnetic moment, for example, is smaller than the isovector contribution:

$$\frac{|\mu_p + \mu_n|}{|\mu_p - \mu_n|} = \frac{|1.79 - 1.91|}{1.79 + 1.91} \approx 10^{-2}$$

This clearly shows that $\Delta^{(s)}$ can be considered a small correction to the model-independent prediction of Eq. (2.21).

Let us briefly discuss now the properties of the suggested model, for the $\gamma^* + p \to N + \pi$ processes.

**A. Born contribution: s–channel**

Using a pseudoscalar $\pi NN$-interaction, we can write the relativistic invariant expression for the matrix element of the $\gamma + p \to n + \pi^+$ reaction in the following form:

$$\mathcal{M}_B = eg (\mathcal{M}_s + \mathcal{M}_u + \mathcal{M}_t),$$

$$\mathcal{M}_s = \overline{u}(p_2) \gamma_5 \frac{\not{p}_2 + \not{q} + m}{s - m^2} \left( F_{2p} \epsilon + \frac{2m}{F_{2p}} \frac{\sigma_{\mu\nu} e_{\mu} k_{\nu}}{2m} \right) u(p_1),$$

$$\mathcal{M}_u = \overline{u}(p_2) \left( F_{1n} \epsilon + \frac{2n}{F_{2n}} \frac{\sigma_{\mu\nu} e_{\mu} k_{\nu}}{2m} \right) \frac{\not{p}_2 + \not{q} - m}{u - m^2} \gamma_5 u(p_1),$$

$$\mathcal{M}_t = \overline{u}(p_2) \left( F_{1t} \epsilon + \frac{2t}{F_{2t}} \frac{\sigma_{\mu\nu} e_{\mu} k_{\nu}}{2m} \right) \frac{\not{p}_2 + \not{q} - m}{u - m^2} \gamma_5 u(p_1),$$
\[ \mathcal{M}_t = \frac{(2e \cdot q - e \cdot k)}{t - m_n^2} \pi(p_2) \gamma_5 u(p_1), \]

where \( s, t, \) and \( u \) are the standard Mandelstam variables:

\[ s = (p_2 + q)^2, \quad t = (p_1 - p_2)^2, \quad u = (p_2 - k)^2, \]

\( k, q, p_1 \) and \( p_2 \) are the four-momenta of \( \gamma^*, \pi, \) initial and final nucleons, \( e \) is the four-vector of the virtual photon polarization, \( g \) is the \( \pi NN \) coupling constant (for a pseudoscalar interaction), \( F_{1p}(k^2) \) and \( F_{2p}(k^2) \) \( (F_{1n}(k^2) \) and \( F_{2n}(k^2)) \) are the Dirac and Pauli electromagnetic form factors of the proton (neutron). The electromagnetic form factor of the nucleon is usually parametrized in form of a \( k^2 \)-dependence of the electric \( (G_E) \) and magnetic \( (G_M) \) nucleonic form factors:

\[ F_{1N}(k^2) = \frac{G_E(k^2) - \tau G_M(k^2)}{1 - \tau}, \]

\[ F_{2N}(k^2) = \frac{-G_E(k^2) + G_M(k^2)}{1 - \tau}, \quad \tau = \frac{k^2}{2m^2}. \]

A simple dipole dependence of \( G_{Ep}, G_{Mp}, \) and \( G_{Mn} \):

\[ G_{Ep}(k^2) = G_{Mp}(k^2)/\mu_p = G_{Mn}(k^2)/\mu_n = \frac{1}{\left[ 1 - \frac{k^2}{0.71(\text{GeV}/c)^2} \right]^2}, \]

with \( \mu_p = 2.79, \mu_n = -1.91, \) has been considered a good parametrization of the existing experimental data, in a wide region of space-like momentum transfer while \( G_{En}(k^2) = 0. \) However a very recent direct measurement [53] of the ratio \( G_{Ep}/G_{Mp} \) shows some deviation of \( G_{Ep} \) from a dipole behavior, in the region \( 0 \leq -k^2 \leq 3.5 \) (GeV/c)^2. This high precision experiment is based on the measurement of the polarization of the final protons in \( e^+p \rightarrow e^+ p' \), in the elastic scattering of longitudinally polarized electrons [54].

This effect should be taken into account in future calculations, as well as the fact that \( G_{En} \) deviates from zero, at least in the region \( k^2 \leq 1 \) (GeV/c)^2. The last direct measurement of \( G_{En} \), in \( e^+d \rightarrow e^+ X \) [55] confirms some previous estimates of the neutron form factor based on the Saclay \( ed \) elastic scattering data [56].

In the Vector Dominance Model (VDM) approach, the pion electromagnetic form factor \( F_{\pi}(k^2) \) is described by:

\[ F_{\pi}(k^2) = \left( 1 - \frac{k^2}{m_\rho^2} \right)^{-1}, \]

where \( m_\rho \) is the \( \rho \)-meson mass.

Note that the electromagnetic current for the reaction \( \gamma^* + p \rightarrow p + \pi^0 \), corresponding to the sum of the Born diagrams in the \( s \) and \( u \)-channels, is conserved for any form factors \( F_{1p} \) and \( F_{2p} \) in the whole kinematical region. This is not the case for the reaction \( \gamma^* + p \rightarrow n + \pi^+ \) (2.26), as one can show that the divergence of the corresponding electromagnetic current (in the Born approximation, for the sum of the \( s, t, \) and \( u \)-contributions) is proportional to the following combination of the electromagnetic form factors:

\[ k \cdot J^{(B)}(\gamma^*p \rightarrow n\pi^+) = eg \sqrt{2} (F_{1p} - F_{1n} - F_{2p} - F_{2n}) \pi(p_2) \gamma_5 u(p_1). \]

The simplest way to conserve the hadronic electromagnetic current, is to extend to all values of \( k^2 \) the following relation:

\[ F_{1p}(k^2) = F_{1n}(k^2) + F_{\pi}(k^2), \quad (3.2) \]

which is in general valid only for \( k^2 = 0. \) However existing data on pion and nucleon form factors are in contradiction with relation (3.2). A possible way to avoid this difficulty is to renormalize the matrix element \( \mathcal{M}_B(\gamma^*p \rightarrow n\pi^+) \) in the following way:

\[ \mathcal{M}_B \rightarrow \mathcal{M}_B' = \mathcal{M}_B + eg \frac{e \cdot k}{k^2} \pi(p_2) \gamma_5 u(p_1) \left( -F_{1p} + F_{1n} + F_{\pi} \right), \quad (3.3) \]

The electromagnetic current, corresponding to the new Born matrix element \( \mathcal{M}_B' \), is conserved for any form factor. Note that such a procedure changes only \( \sigma_L \), without any effect on the transversal cross-sections \( \sigma_T(k^2, W) \) and \( \sigma_T^{(s)}(k^2, W) \). In our considerations we shall use the procedure (3.3), while relation (3.2) was taken in ref. [57].

The scalar amplitudes \( f_i \), corresponding to different diagrams of the Born mechanism, are given in the Appendix.
The matrix element $\mathcal{M}_V$, corresponding to vector meson exchange in the $t-$channel can be written in the following form:

$$\mathcal{M}_V = \frac{e g_{V\pi\gamma^*}(k^2)}{t - m_V^2} \epsilon_{\mu\nu\alpha\beta} e_\mu k^\nu J_\alpha^{(V)} q_\beta,$$

$$J_\alpha^{(V)} = \frac{\pi(p_2)}{2m} \left[ \gamma_\alpha F_1^V(t) - \frac{F_2^V(t)}{2m} \sigma_{\alpha\beta}(p_1 - p_2)_\beta \right] u(p_1)$$

where $g_{V\pi\gamma^*}(k^2)$ is the electromagnetic form factor for the $V\pi\gamma^*$-vertex, $m_V$ is the vector meson mass, $F_1^V(t)$ and $F_2^V(t)$ are the "strong" form factors for the $V^*NN$ vertex (with a virtual V-meson). In principle the "static" values of these form factors (i.e. for $t = 0$), are related to the $\omega NN$ and $\rho NN$ coupling constants:

$$F_1^V(0) = g_{VNN}, \quad F_2^V(0)/F_1^V(0) = \kappa_V.$$

An estimate for the $\omega NN$ coupling constants, based on the Bonn potential [58], gives:

$$\frac{g_{\omega NN}^2}{4\pi} = 20, \quad \kappa_\omega = 0$$

The $\rho NN$ coupling constants can be estimated from pion photoproduction data [57]:

$$\frac{g_{\rho NN}^2}{4\pi} = 0.55, \quad \kappa_\rho = 3.7$$

Note that the constants $\kappa_V$ can be identified in VDM, with the values of the isoscalar and isovector anomalous magnetic moment of the nucleon. The VDM allows to write the following parametrization for the $k^2$ dependence of the electromagnetic form factor for the $\gamma^* + V \rightarrow \pi$ vertex:

$$g_{V\pi\gamma^*}(k^2) = \frac{g_{V\pi\gamma^*}(0)}{\Gamma - k^2/m_V^2},$$

where $m_V$ is the mass of the $\rho$ or $\omega$ vector meson.

The $g_{V\pi\gamma^*}(0)$ coupling constant can be fixed by the width of the radiative decay $V \rightarrow \pi \gamma$, through the following formula:

$$\Gamma(V \rightarrow \pi \gamma) = \frac{\alpha}{24} |g_{V\pi\gamma^*}(0)|^2 \left( 1 - \frac{m_\pi^2}{2m_V^2} \right)^3.$$
Therefore any violation of this relation is an indication of the presence of an isotensor component of the electromagnetic current, which is absent, however, at the quark level. So, a precise experiment with the simultaneous determination of the two coupling constants for $\rho^0 \to \pi^0 \gamma$ and $\rho^\pm \to \pi^\pm \gamma$ would be very important. It would not only constitute a test of the isotopic properties of the hadronic electromagnetic current, but also have application in the calculation of the meson exchange current contributions (MEC) to the deuteron electromagnetic form factors. Such contributions are considered to be very important at high momentum transfer.

Note in this respect that the $t$-dependence of the form factors $F_1^\ast(t)$ and $F_2^\ast(t)$ is also important, for the correct calculation of MEC, in case of elastic electron-deuteron scattering. Moreover, the relative sign of the $\rho$ with the normalization condition:

$$
\text{Tr}\rho = 2s_\Delta + 1 = 4.
$$

In this formalism the $\Delta N\pi$-vertex can be parametrized as follows:

$$
\mathcal{M}_{\Delta N\pi} = g_{\Delta N\pi} \chi^\dagger \tilde{q} \cdot \hat{k},
$$

where $\chi$ is the 2-component spinor of the nucleon in the decay $\Delta \to N + \pi$, $\tilde{q}$ is the unit vector along the pion three momentum, in the $\Delta$ rest frame, and the constant $g_{\Delta N\pi}$ characterizes the width of the strong decay $\Delta \to N + \pi$.

Taking into account the conservation of the total angular momentum and of the P-parity in the electromagnetic decay $\Delta \to N + \gamma$ with production of M1 photons, the following expression can be written for the matrix element:

$$
\mathcal{M}_{\Delta N\gamma} = e g_{\Delta N\gamma} \chi^\dagger \tilde{e} \cdot \hat{k},
$$

where $g_{\Delta N\gamma}$ is the constant of the magnetic dipole radiation (or the magnetic moment for the transition $\Delta \to N + \gamma$), $\tilde{e}$ and $\hat{k}$ are the photon polarization three-vector and unit momentum vector along the three-momentum of $\gamma$, respectively.

In the general case, the transition $\gamma^* + N \to \Delta$ must be described by three different form factors, corresponding to the absorption of $M1, E2t$ (transversal) and $E2l$ (longitudinal) virtual photons. But the existing experimental data about pion photo and electro-production of pions on the nucleons (in the $\Delta$ resonance region) indicate that the $M1$ term is dominant [60], therefore in our analysis we will consider only this form factor.

Using expressions (3.6) and (3.7) for both the $\Delta N\pi$ and $\Delta N\gamma$ vertices, one can write the matrix element of the $\Delta$–contribution in the $s$–channel (Fig. 2e) as follows:

$$
\mathcal{M}_{\Delta N\gamma} = \frac{eG(k^2)|q|}{M_\Delta^2 - s - i\Gamma_\Delta M_\Delta} \sqrt{(E_1 + m)(E_2 + m)} \chi_2^\dagger (2\delta_{ab} - i\epsilon_{abc}\sigma_c) \chi_1 \tilde{q}_a (\tilde{e} \times \hat{k})_b,
$$

C. $\Delta$-excitation

This contribution can be analyzed in a relativistic framework [51], considering a virtual $\Delta$ as a Rarita-Schwinger field with spin 3/2 but in this approach it is difficult to treat off-shell effects. First of all, this means that $\Delta$–exchange may contain contributions from a state with spin 1/2 as well as antibaryonic terms with negative P-parity and $s=1/2$ and 3/2. Therefore the description of the $\Delta$–isobar, with $J^P = 3/2^+$, especially in the $s$–channel is not straightforward. To avoid these complications, we choose here a direct parametrization of the $\Delta$ contribution. Note that the CMS for $\gamma^* + p \to \Delta^+ \to N + \pi$ is the optimal frame, because the three-momentum of the $\Delta$ is zero, so that the $\Delta$ can be described by a two-component spinor, with a vector index, $\tilde{\chi}$, which satisfies the following auxiliary condition:

$$
\tilde{\sigma} \cdot \tilde{\chi} = 0,
$$
typical for a pure spin 3/2 state. Using this condition, it is possible to find the following expression for the $\Delta$-density matrix:

$$
\rho_{ab} = \chi_a \chi_b^\dagger = \frac{2}{3} (\delta_{ab} - \frac{i}{2} \epsilon_{abc}\sigma_c),
$$

with the normalization condition: $\text{Tr}\rho_{aa} = 2s_\Delta + 1 = 4$.

In this formalism the $\Delta N\pi$-vertex can be parametrized as follows:

$$
\mathcal{M}_{\Delta N\pi} = g_{\Delta N\pi} \chi^\dagger \tilde{q} \cdot \hat{k},
$$

where $\chi$ is the 2-component spinor of the nucleon in the decay $\Delta \to N + \pi$, $\tilde{q}$ is the unit vector along the pion three momentum, in the $\Delta$ rest frame, and the constant $g_{\Delta N\pi}$ characterizes the width of the strong decay $\Delta \to N + \pi$.

Taking into account the conservation of the total angular momentum and of the P-parity in the electromagnetic decay $\Delta \to N + \gamma$ with production of M1 photons, the following expression can be written for the matrix element:

$$
\mathcal{M}_{\Delta N\gamma} = e g_{\Delta N\gamma} \chi^\dagger \tilde{e} \cdot \hat{k},
$$

where $g_{\Delta N\gamma}$ is the constant of the magnetic dipole radiation (or the magnetic moment for the transition $\Delta \to N + \gamma$), $\tilde{e}$ and $\hat{k}$ are the photon polarization three-vector and unit momentum vector along the three-momentum of $\gamma$, respectively.

In the general case, the transition $\gamma^* + N \to \Delta$ must be described by three different form factors, corresponding to the absorption of $M1, E2t$ (transversal) and $E2l$ (longitudinal) virtual photons. But the existing experimental data about pion photo and electro-production of pions on the nucleons (in the $\Delta$ resonance region) indicate that the $M1$ term is dominant [60], therefore in our analysis we will consider only this form factor.

Using expressions (3.6) and (3.7) for both the $\Delta N\pi$ and $\Delta N\gamma$ vertices, one can write the matrix element of the $\Delta$–contribution in the $s$–channel (Fig. 2e) as follows:

$$
\mathcal{M}_{\Delta N\gamma} = \frac{eG(k^2)|q|}{M_\Delta^2 - s - i\Gamma_\Delta M_\Delta} \sqrt{(E_1 + m)(E_2 + m)} \chi_2^\dagger (2\delta_{ab} - i\epsilon_{abc}\sigma_c) \chi_1 \tilde{q}_a (\tilde{e} \times \hat{k})_b,
$$

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where $M_\Delta$ and $\Gamma_\Delta$ are the mass and width of $\Delta$ and $G(k^2)$ is proportional to the magnetic form factor of the $\gamma^* + N \rightarrow \Delta$ transition.

The following $\Delta$ contributions to the scalar amplitudes, $f^\Delta_i$ can be derived:

$$f^\Delta_1 = 2\Pi(s, k^2)$$
$$f^\Delta_2 = \cos \theta_\pi \Pi(s, k^2),$$
$$f^\Delta_3 = -\Pi(s, k^2),$$
$$f^\Delta_4 = f^\Delta_5 = f^\Delta_6 = 0,$$

where we use the notation:

$$\Pi(s, k^2) = \frac{G(k^2)|q^\Delta|}{M^2_\Delta - s - i\Gamma_\Delta M_\Delta}.$$  

The role of the factor $q^\Delta$ is to correctly describe the threshold behavior of the $M1$ amplitude for $\gamma + p \rightarrow N + \pi$, according to the P-wave nature of the produced pion. We shall use the following formula for the $k^2$-dependence of the transition electromagnetic form factors:

$$G(k^2) = \frac{G(0)}{\left(1 - \frac{k^2}{0.71 \text{ (GeV/c)}^2}\right)^2 \left(1 - \frac{k^2}{m^2_\pi}\right)}.$$  

The factor $(1 - \frac{k^2}{m^2_\pi})^{-1}$, with $m_\pi = 6 \text{ GeV}^2$, is included in order to take into account a steeper decreasing of $G(k^2)$ in comparison with the dipole behavior of the elastic electromagnetic form factors of the nucleons [60].

The normalization constant $G(0)$ can be found according to the following procedure. Using Eq. (3.8), let us calculate first the differential cross section for $\pi^0$-photoproduction:

$$\frac{d\sigma}{d\Omega}(\gamma p \rightarrow p \pi^0) = \frac{\alpha}{32\pi} \frac{q^\Delta_1 (E_1\Delta + m)(E_2\Delta + m)}{M^4_\Delta \Gamma^2_\Delta} G^2(0)(5 - 3\cos^2 \theta_\pi),$$  

at $s = M^2_\Delta$, where the $\Delta$-excitation in the $s$-channel is the main mechanism. So, our parametrization of the $\Delta$-contribution describes correctly the angular dependence $(5 - 3\cos^2 \theta_\pi)$, typical for the magnetic excitation of a $\frac{3}{2}^+$ state in $\gamma + p \rightarrow \Delta^+ \rightarrow N + \pi$. Therefore, the total cross section can be written as:

$$\sigma_t(\gamma p \rightarrow p \pi^0) = \frac{\alpha}{2} \frac{q^\Delta_1 (E_1\Delta + m)(E_2\Delta + m)}{M^4_\Delta \Gamma^2_\Delta} G^2(0),$$

where:

$$E_1\Delta = \frac{M^2_\Delta + m^2}{2M_\Delta}, \quad E_2\Delta = \frac{M^2_\Delta + m^2 - m^2_\pi}{2M_\Delta},$$

$$k^\Delta = \frac{M^2_\Delta - m^2}{2M_\Delta}, \quad q^\Delta = \sqrt{E^2_2\Delta - m^2}.$$  

We can approximate with a good accuracy $\sigma_t(\gamma p \rightarrow p \pi^0)$ by a single $\Delta$-resonance contribution. For a numerical estimate of $G(0)$ we use $\sigma_t \simeq 250 \cdot 10^{-30} \text{ cm}^2$.

Note again that this procedure cannot determine the sign of $G(0)$. However for the $\gamma + p \rightarrow n + \pi^+$ reaction, there is a strong interference between the pion diagram and the $\Delta$-contribution. The comparison of the calculations using different signs with the experimental $\theta_\pi$-dependence in the resonance region allows to fix the corresponding relative sign. Two remarks should be done about this procedure:
there is no ambiguity concerning off-mass shell effects for the $\Delta$-contribution, at least in the $s$–channel,

this special contribution is gauge invariant.

We neglect in our consideration the $\Delta$-exchange in the $u$–channel. The main reason to include this contribution is to have the crossing symmetry of the model. This is in principle an important property of the photoproduction amplitude, in particular in connection with the dispersion relation approach. However in the framework of phenomenological approaches, this symmetry is strongly violated. For example, the $s$–channel $\Delta$–contribution induces an amplitude which is mostly complex (with a typical Breit-Wigner behavior), whereas the $u$-channel contribution results in a real amplitude. The inclusion of different form factors for the $s$– and $u$–channel violates the crossing symmetry, which is important for the Born contributions. This appears clearly for the reaction $\gamma + p \to p + \pi^0$, because here the crossing symmetry is correlated with the gauge invariance of the electromagnetic interaction, and the violation of the crossing symmetry has for direct consequence the violation of the current conservation. On the contrary, for the $\Delta$-contribution, this important correlation is absent.

IV. NUMERICAL PREDICTIONS AND DISCUSSION

Having determined all the parameters of the model, it is possible in principle to calculate all observables for the processes $e^- + N \to e^- + N + \pi$ (on proton and neutron targets) in the kinematical region from threshold to the $\Delta$-resonance region ($W \leq 1300$ MeV), for any value of the pion production angle, $\theta^\pi$, (in the CMS of the $\pi N$-system,) and of the four momentum transfer $k^2$.

In order to test the present model, we used existing experimental data on the angular dependence of the differential cross sections for both the $\gamma + p \to p + \pi^0$ and $\gamma + p \to n + \pi^+$ reactions. This comparison allowed to fix empirically the relative sign of the different contributions: Born, $\Delta$-excitation (in the $s$–channel) and vector meson exchange (in the $t$–channel). The relative sign of all three diagrams for the Born approximation in the case of the process $\gamma + p \to n + \pi^+$ are fixed by gauge invariance, but it is necessary to find the relative signs between the Born amplitudes, on one side, and the $\Delta$-isobar and vector meson exchange contributions, on another side. The $\gamma + p \to n + \pi^+$ reaction is more sensitive to the signs of the $\Delta$-contribution and $\rho^+$-exchange. Then the data about the differential cross sections for $\gamma + p \to p + \pi^0$ allow a further check and give a constrain for the $\omega$–exchange amplitude.

Note that the sign of the $\rho$–exchange contribution relative to the Born contribution (in both the $\gamma + p \to p + \pi^0$ and $\gamma + p \to n + \pi^+$ reactions) has to be the same as the relative sign of meson exchange currents (due to the $\rho \pi \gamma^*$ meson-exchange mechanism in the calculation of the electromagnetic form factors of the deuteron) with respect to the amplitude in the impulse approximation for elastic $ed$–scattering. This represents an important link between very different physical problems.

In order to obtain a good description of the experimental data for $\gamma + p \to p + \pi^0$ and $\gamma + p \to n + \pi^+$ we introduced small corrections to the different contributions. For the reaction $\gamma + p \to p + \pi^0$ a form factor was added to the Born contribution. The $u$–channel nucleon contribution for $\gamma + p \to n + \pi^+$ can be neglected without violating gauge invariance, because its magnetic content satisfies alone the current conservation condition. As a matter of fact this contribution has a diverging behavior at large angle, which is typically corrected by introducing an ad-hoc form factor. We choose to replace this contribution with a somewhat simplified phenomenological ($S$-wave like) contribution: $a \left(1 - \frac{t}{1.2}\right)^{1.2} \frac{1.2 \text{ GeV}}{W}$, where $a$ is a parameter which is adjusted in order to reproduce at best the $\pi^0$ photoproduction data.

We did not attempt to reproduce with a good accuracy the threshold behavior of the $\gamma + p \to p + \pi^0$ and $\gamma + p \to n + \pi^+$ amplitudes. A precise description of this behavior, in particular for the process $\gamma + p \to p + \pi^0$, can be obtained, for example, in the framework of the Chiral Perturbative Theory approach [61]. For inclusive calculations, a qualitative description of the data in the threshold region is sufficient.

The quality of our model is shown in Fig. 3, where we present the comparison of our predictions with the experimental data on the differential cross sections for the $\gamma + p \to p + \pi^0$ and $\gamma + p \to n + \pi^+$ reactions, in the kinematical region where our model can be considered a reasonable approach. Indeed the unpolarized differential cross sections are well described. We did not apply the model to polarization observables. In particular different $T$–odd observables, such as, for example, the target asymmetry or the polarization of the final nucleons, are very sensitive to the relative phases of the different contributions. A good description requires a very precise treatment of the unitarity condition as well as of $T$-invariance of the hadron electromagnetic interaction, which are not so important for the differential or total cross section.
Therefore, after having determined the relative signs of the different contributions, our model can be generalized to pion electroproduction. Our aim is the calculation of the inclusive P-odd asymmetry $A$, in $p(\vec{e}, e')X$, for the sum of two possible channels, $X = p + \pi^0$ and $X = n + \pi^+$. One can see, from Eq. (2.25), that such asymmetry is determined by the following ratios of inclusive cross sections:

$$
R_L^{(s)} = \frac{\sigma_L^{(s)}(k^2, W)}{\sigma_T(k^2, W)}, \quad R_T^{(s)} = \frac{\sigma_T^{(s)}(k^2, W)}{\sigma_T(k^2, W)}, \quad R_{LT} = \frac{\sigma_L(k^2, W)}{\sigma_T(k^2, W)},
$$

for both channels, $\gamma^* + p \rightarrow p + \pi^0$ and $\gamma^* + p \rightarrow n + \pi^+$, and

$$
R_{pn} = \frac{\sigma_T(\gamma^* p \rightarrow p\pi^0)}{\sigma_T(\gamma^* p \rightarrow n\pi^+)},
$$

which characterizes the relative role of the two channels. The 2-dimensional plots of these ratios as functions of $k^2$ and $W$ are shown in Fig. 4 and 5, for the reactions $\gamma^* + p \rightarrow p + \pi^0$ and $\gamma^* + p \rightarrow n + \pi^+$, respectively.

For $\pi^0$- electroproduction, both $R_L^{(s)}$ and $R_T^{(s)}$ are small corrections to $A$. In the considered kinematical region, they are positive and tend to decrease in the region of the $\Delta$ resonance, due to the dominance of the isovector resonance contribution. The behavior of all these ratios in the threshold region can be improved, as we discussed above.

In the case of the $\gamma^* + p \rightarrow n + \pi^+$ reaction, the corresponding corrections are also small, especially $R_L^{(s)}$. Note that $R_T^{(s)}$ is negative in the whole region of $k^2$ and $W$.

Combining these results it is possible to calculate the resulting asymmetry $A$ for the sum of both channels, again in a 2-dimensional representation (Fig. 6). The dependence on the detailed electron kinematics for $p(\vec{e}, e')X$ (energies of the initial and final electron and electron scattering angle) is contained in the single parameter $\epsilon$, for which we used three different values: $\epsilon = 0$, $1/2$ and $1$. In order to extract the strong $k^2$ dependence of $A$, the "reduced" asymmetry $A_0 = -A/|k^2|$ is shown.

In this picture one can see that the behavior of $A$ versus $k^2$ and $W$, in the region $1.08 \leq W \leq 1.26$ GeV and in a wide region of momentum transfer $k^2$, is smooth everywhere and negative (note the $-1/|k^2|$ factor in the formula). Such a behavior results from the isovector nature of the electroproduction processes which we have considered.

The role of the different contributions is illustrated in Figs. 7, 8 and 9. In Fig. 7 (Fig. 8) the ratio of the cross sections $R_L^{(s)}$ and $R_T^{(s)}$ is reported as a function of $W$, for two fixed values of $|k^2|$, (a) $|k^2| = 0.5$ and (b) $1.0$ (GeV/c)$^2$, for the reaction $\gamma^* + p \rightarrow p + \pi^0$ ($\gamma^* + p \rightarrow n + \pi^+$). The $\Delta$ contribution (dashed-dotted line) vanishes, while the Born terms (dotted line) give the largest contribution at forward angles. The contribution given by the vector meson ($\rho$ and $\omega$) exchange diagrams is not so essential here.

The different contributions to the total asymmetry $A$ are shown in Fig. 9. Fig. 9 is a projection of Fig. 6 showing the resulting reduced asymmetries $A_0 = -A/|k^2|$ as a function of $W$ at a fixed value of the virtual photon polarization $\epsilon = 0.5$ and for two values of the momentum transfer (a) $|k^2| = 0.5$ (GeV/c)$^2$; (b) $1.0$ (GeV/c)$^2$. The $\Delta$-contribution only is constant as a function of $W$ due to its isovector dominance, the vector-meson exchange gives a rather small contribution at low $W$ (below $1.2$ GeV) and it is negligible above. The full calculation gives values of $A$ varying smoothly from $-7 \cdot 10^{-5}$ at $W = 1.1$ GeV (close to the elastic region) to $-8 \cdot 10^{-5}$ at $W = 1.25$ GeV, in the region of the $\Delta$ at $|k^2| = 1$ (GeV/c)$^2$.

One purpose of the present paper is to give an estimate of a possible contamination of the elastic peak by $\pi$-production and to assess its effect on the measured asymmetry. Although a detailed comparison of elastic and inelastic channels can only be made for specific geometries at the same incident energies and scattering angles (or equivalently, same $k^2$ and $\epsilon$), a rough estimate can be made using inclusive cross-sections.

Weak asymmetries have been calculated in the Standard Weinberg-Salam model assuming no strangeness in the nucleon. The results depend on the electromagnetic form factors for protons and neutrons, and give therefore different predictions depending on which values are taken. Calculations of ref. [65] give $A/|k^2| = -(1.4 \pm 1.5 \cdot 10^{-5})$, at $|k^2|=0.1$ (GeV/c)$^2$ up to $-(2.2 - 2.8 \cdot 10^{-5})$, at $|k^2|=0.3$ (GeV/c)$^2$ for $E_e = 2$ GeV.

The SAMPLE Collaboration [23] has measured $A = -4.92 \pm 0.61 \pm 0.73 \cdot 10^{-6}$ at $|k^2|=0.1$ (GeV/c)$^2$ and backward angle. Weak asymmetry calculations have been done for the $G^0$ [35], PVA4-Mainz [34] and HAPPEX [33] experimental conditions, predicting values ranging from $-0.3 \cdot 10^{-6}$ at $|k^2|=0.1$ (GeV/c)$^2$ to $-2.0 \cdot 10^{-5}$ at $|k^2|=0.5$ (GeV/c)$^2$ [66].

As one can see all calculations agree to predict negative asymmetries except one [52] where small positive values have been obtained in an extension of the SM when taking right-handed doublets of $t$ and $d$ quarks into account (in this early paper, $\sin^2(\theta_W)$ is taken equal to $1/3$). Note that one can also find in this paper a simple explanation of the negative sign of the asymmetry in terms of $Z^0$ coupling to quarks and leptons.
Now comparing elastic scattering and inclusive $\pi$-production (Fig. 9), we see that they are both negative and of the same order of magnitude. Moreover $A$ is smaller in the region $W=1.1$ GeV (close to elastic scattering) and larger in the $\Delta$ region. Therefore we can conclude that a small admixture of $\pi$-production events in the region of the elastic peak, is not going to produce a large uncertainty in the elastic PV asymmetry. Specific cases can be computed using Fig. 6 or from the corresponding numerical values available from the authors.

V. CONCLUSIONS

We have derived the dependence of the P-odd asymmetry for inclusive scattering of longitudinally polarized electrons by unpolarized protons with production of neutral and positive pions $p(e',e'')X$, with $X = p + \pi^0$ or $X = n + \pi^+$. Using the known isotopic properties of the electromagnetic current for the $\gamma^* + p \rightarrow p + \pi^0$ and $\gamma^* + p \rightarrow n + \pi^+$ processes and the vector part of the hadronic weak neutral current for the $Z^* + p \rightarrow p + \pi^0$ and $Z^* + n \rightarrow n + \pi^+$ processes, we derived an original expression for the inclusive asymmetry $A$. Without approximations, it is possible to disentangle the main contribution to $A$, which depends only on $k^2$. The calculation of $A$ is then reduced to the analysis of specific isoscalar contributions to the electromagnetic currents. Such contributions appear, then, as corrections to the main term, which can be calculated exactly.

We have calculated the amplitudes for $\gamma^* + p \rightarrow N + \pi$, taking into account three standard contributions: Born+vector meson exchange+$\Delta$-excitation. All the necessary parameters: interaction constants and different electromagnetic form factors are taken from other sources. Small adjustments in this basic approach were done in order to obtain a good description of the differential cross section data for the $\gamma + p \rightarrow p + \pi^0$ and $\gamma + p \rightarrow n + \pi^+$ reactions. The model gives the vector part of the hadronic weak neutral current which is the main contribution to P-odd effects in $e + N \rightarrow e + N + \pi$.

The reduced weak asymmetry $A_0$ varies very little as a function of the two basic kinematical variables, $k^2$ and $W$. In our approach this appears naturally from the fact that the isoscalar content of the electromagnetic current for $\gamma + N \rightarrow N + \pi$ is very small in the considered kinematical region. It is of the same sign and size as the $ep$ elastic PV asymmetry, and will, therefore, not much contribute to the experimental uncertainty of the former.

A straightforward extension of the present model would open a way to use P-odd observables in elastic and inelastic electron-proton scattering for the study of the axial contributions. We plan to discuss this in a forthcoming paper.

VI. APPENDIX

A. Born contribution: s-channel

The scalar amplitudes, for $\gamma^* + p \rightarrow p + \pi^0$ are defined as

$$f_{1s} = f_{3s} = -\frac{g}{W - m (E_1 + m)(E_2 + m)} \left[ F_{1p}(k^2) + F_{2p}(k^2) \frac{W + m}{2m} \right],$$

$$f_{2s} = \frac{g}{W - m} \left[ F_{1p}(k^2) - F_{2p}(k^2) \frac{W - m}{2m} \right],$$

$$+ \frac{g}{W - m (E_1 + m)(E_2 + m)} \left[ F_{1p}(k^2) + F_{2p}(k^2) \frac{W + m}{2m} \right],$$

$$f_{4s} = 0,$$

$$f_{5s} = \frac{g}{(W + m)(E_1 + m)} \left[ -F_{1p}(k^2) + F_{2p}(k^2) \frac{E_1 + m}{2m} \right],$$

$$f_{6s} = \frac{g}{(W - m)(E_2 + m)} \left[ \frac{\vec{k}}{|\vec{q}|} \right] \left[ -F_{1p}(k^2) + F_{2p}(k^2) \frac{E_1 - m}{2m} \right],$$

with $|\vec{k}| = \sqrt{E_1^2 - m^2}$ and $|\vec{q}| = \sqrt{E_2^2 - m^2}$. 

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\[ f_{1u} = \frac{g|\vec{k}||\vec{q}|}{u - m^2} \left\{ F_{1p}(k^2) \frac{W + m}{(E_1 + m)(E_2 + m)} \right\} \\
- \frac{F_{2p}(k^2)}{2m(E_1 + m)} \left\{ W + m + \left( m + \frac{m^2 - k^2}{W} \right) \frac{E_\pi}{E_2 + m} + \frac{2\vec{k} \cdot \vec{q}}{E_2 + m} \right\}, \]

\[ f_{2u} = \frac{g}{u - m^2} \left\{ F_{1p}(k^2) \left( W - m + \vec{k} \cdot \vec{q} \right) \frac{W + m}{(E_1 + m)(E_2 + m)} \right\} \\
- \frac{F_{2p}(k^2)}{2m} \left\{ (E_1 - m)(W - m) + \tilde{k}_0 \left( m + \frac{m^2 - m_\pi^2}{W} \right) \right\}, \]

\[ \left( -2\vec{k} \cdot \vec{q} + \left( m + \frac{m^2 - k^2}{W} \right) \left( m + \frac{m^2 - m_\pi^2}{W} \right) \frac{\vec{k} \cdot \vec{q}}{(E_1 + m)(E_2 + m)} \right), \]

\[ f_{3u} = \frac{g}{u - m^2} \left\{ F_{1p}(k^2) \left( \frac{W + m}{E_2 + m} \left( -m + \frac{m^2 - m_\pi^2}{W} \right) \right) \right\} \\
- \frac{F_{2p}(k^2)}{2m} \left\{ -W - m + \left( m + \frac{m^2 - k^2}{W} \right) \frac{E_\pi}{E_2 + m} + \frac{2\vec{k} \cdot \vec{q}}{E_2 + m} \right\}, \]

\[ f_{4u} = \frac{g}{u - m^2} (E_2 - m) \left\{ -2F_{1p}(k^2) + F_{2p}(k^2) \left( -1 + \frac{W}{m} \right) \right\}, \]

\[ f_{5u} = -\frac{g}{(u - m^2)(E_1 + m)} \left\{ \left( -F_{1p}(k^2) + F_{2p}(k^2) \frac{E_1 + m}{2m} \right) \left( m + \frac{m^2 - m_\pi^2}{W} \right) \right\} \\
+ F_{2p}(k^2) \frac{\vec{k} \cdot \vec{q}}{m}, \]

\[ f_{6u} = \frac{g}{(u - m^2)(E_2 + m)} \frac{|\vec{q}|}{|\vec{k}|} \left\{ \left( F_{1p}(k^2) + F_{2p}(k^2) \frac{E_1 - m}{2m} \right) \left( m + \frac{m^2 - m_\pi^2}{W} \right) + F_{2p}(k^2) \frac{\vec{k} \cdot \vec{q}}{m} \right\}, \]

where

\[ u - m^2 = k^2 - 2\tilde{k}_0E_2 - 2\vec{k} \cdot \vec{q}, \quad \tilde{k}_0 = \frac{W^2 + k^2 - m^2}{2W}, \quad \text{and} \quad E_\pi = W - E_2. \]
$$f_{1V} = g_{V \pi^*} (k^2) \frac{g}{m_V (t - m_V^2)} \frac{|\vec{k}| |\vec{q}|}{m_V (t - m_V^2)}$$

$$f_{2V} = g_{V \pi^*} (k^2) \frac{g}{m_V (t - m_V^2)} \frac{|\vec{k}| |\vec{q}|}{m_V (t - m_V^2)}$$

$$f_{3V} = g_{V \pi^*} (k^2) \frac{g}{m_V (t - m_V^2)} \frac{|\vec{k}| |\vec{q}|}{m_V (t - m_V^2)}$$

$$f_{4V} = -g_{V \pi^*} (k^2) \frac{g}{m_V (t - m_V^2)} \frac{|\vec{k}| |\vec{q}|}{m_V (t - m_V^2)}$$

$$f_{5V} = g_{V \pi^*} (k^2) \frac{g}{m_V (t - m_V^2)} \frac{|\vec{k}| |\vec{q}|}{m_V (t - m_V^2)}$$

$$f_{6V} = -g_{V \pi^*} (k^2) \frac{g}{m_V (t - m_V^2)} \frac{|\vec{k}| |\vec{q}|}{m_V (t - m_V^2)}$$

where

$$t - m_V^2 = m_\pi^2 - m_V^2 - 2\vec{k}_0 E_\pi + 2\vec{k} \cdot \vec{q} + k^2.$$ 

D. One pion contribution: t-channel

$$f_{1t} = f_{2t} = 0,$$
\[ f_{3t} = \frac{2\hat{k}|\hat{q}|}{t - m_{\pi}^2} F_{\pi}(k^2) \]
\[ f_{4t} = -2g \frac{E_2 - m}{t - m_{\pi}^2} F_{\pi}(k^2), \]
\[ f_{5t} = -\frac{g}{t - m_{\pi}^2} F_{\pi}(k^2) 2E_{\pi} - k_0 \frac{E_1 + m}{E_1 + m}, \]
\[ f_{6t} = -\frac{g}{t - m_{\pi}^2} F_{\pi}(k^2) \frac{|\hat{k}||\hat{q}|}{E_2 - m} \frac{2E_{\pi} - k_0}{E_2 + m}. \]

where
\[ F_{\pi}(k^2) = \frac{1}{1 - k^2/m_{\rho}^2} \]
is the pion electromagnetic form factor, given in the framework of VDM, \( m_{\rho} = 0.77 \text{ GeV} \) is the \( \rho \)-meson mass.

**E. Calculation of the isoscalar amplitudes** \( f_i^{(s)}(\gamma^*p \rightarrow p\pi^0) \)

The isoscalar amplitudes are:
\[ f_i^{(s)}(\gamma^*p \rightarrow p\pi^0) = -f_i^{(s)} - f_i^{(u)} - f_i^{(\rho)} \]
where the contributions \( f_i^{(s)} \) and \( f_i^{(u)} \) are determined by the corresponding formulas, with the following substitutions:
\[ F_{1p} \rightarrow F_{1s} = \frac{F_{1p} + F_{1n}}{2}, \]
\[ F_{2p} \rightarrow F_{2s} = \frac{F_{2p} + F_{2n}}{2}, \]
with
\[ F_{1n} = \frac{G_{E_n} - \tau G_{M_n}}{1 - \tau}, \quad F_{2n} = \frac{-G_{E_n} + G_{M_n}}{1 - \tau}. \]

**F. Gauge invariance of the suggested model**

In the framework of the considered model, for the process of neutral pion electroproduction, \( e + p \rightarrow e + p + \pi^0 \), the corresponding hadronic electromagnetic current is conserved: \( k \cdot J_{em}^{(\gamma^*p \rightarrow p\pi^0)} = 0 \) for any form factor in \( \gamma^*NN, \gamma^*\pi\pi, \gamma^*V_{\pi}, \) and \( \gamma^*N\Delta \)-vertices.

In case of charged pion electroproduction, \( e + p \rightarrow e + n + \pi^+ \), a special contribution must be added to the matrix element:
\[ \Delta M = -\sqrt{2}g \frac{e \cdot k}{k^2} \gamma_5 (F_{1p} - F_{1n} - F_{\pi}) \]
which results in additional contributions to the scalar amplitudes: \( \Delta f_i^{(s)}(\gamma p \rightarrow n\pi^+) \):
\[ \Delta f_1 = \Delta f_2 = \Delta f_3 = \Delta f_4 = 0 \]
\[ \Delta f_5 = \sqrt{2}g(E_1 - m)[F_{1p}(k^2) - F_{1n}(k^2) - F_{\pi}(k^2)]/k^2 \]
\[ \Delta f_6 = -\sqrt{2}g(E_2 - m)[F_{1p}(k^2) - F_{1n}(k^2) - F_{\pi}(k^2)]/k^2 \]
[34] P. Achenbach et al., MAMI proposal A4/1-93, D. V. Harrach, spokesperson.

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TABLE I. Numerical coefficients for the different contributions to the Feynmann diagrams
FIG. 1. Feynman diagrams for $\gamma^*$- and $Z^*$-boson exchanges in the processes $e^- + p \to e^- + N + \pi$. 

Fig. 1

FIG. 1. Feynman diagrams for $\gamma^*$- and $Z^*$-boson exchanges in the processes $e^- + p \to e^- + N + \pi$. 

Fig. 1
FIG. 2. Feynman diagrams for $\gamma^* + p \rightarrow N + \pi^-$ processes.
Fig. 3

The angular dependence of the differential cross sections for the photoproduction processes: (a) and (b) $\gamma^* + p \rightarrow p + \pi^0$; open stars: data from ref. [63], open crosses: data from ref. [64], (c) $\gamma^* + p \rightarrow n + \pi^+$; data are from ref. [62]; the dashed line is the prediction of the present model.
FIG. 4. The $k^2$ and $W$-dependences of the ratios of the total cross sections for the $e^- + p \rightarrow e^- + p + \pi^0$ reaction: (a) $R^{(s)}_L(k^2, W) = \sigma^{(s)}_L(k^2, W)/\sigma_T(k^2, W)$; (b) $R_T^{(s)}(k^2, W) = \sigma^{(s)}_T(k^2, W)/\sigma_T(k^2, W)$; (c) $R_{LT}(k^2, W) = \sigma_L(k^2, W)/\sigma_T(k^2, W)$; (d) $R_{np} = \sigma_T(p\pi^0)/\sigma_T(n\pi^+)$.
FIG. 5. The $k^2$ and $W$-dependences of the ratios of the total cross sections for the $e^- + p \rightarrow e^- + n + \pi^+$ reaction. Same conventions as in Fig. 4.
FIG. 6. The $k^2$ and $W$ dependences of the reduced asymmetry $A_0 = -A/|k^2|$ (where $A$ is the theoretical asymmetry to be compared to experimental data) for $p(\vec{e}, e')X$ at three different values of the virtual photon polarization $\epsilon$: (a) $\epsilon = 0$; (b) $\epsilon = 0.5$; (c) $\epsilon = 1$. 

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FIG. 7. The $W$-dependence of the ratios $R_L^{(s)}(k^2, W)$ and $R_T^{(s)}(k^2, W)$ for fixed values of $k^2$ a) and c) $-k^2 = 0.5$ (GeV/c)$^2$; b) and d) $-k^2 = 1.0$ (GeV/c)$^2$ for the $e^- + p \rightarrow e^- + p + \pi^0$ reaction. The curves represent the full calculation (full line), $\Delta$-contribution only (dashed-dotted line), $\Delta$ + Born terms (dashed line), $\Delta$ + vector mesons (dotted line).
FIG. 8. The same as Fig. 7, for the $e^- p \rightarrow e^- n + \pi^+$ reaction.
FIG. 9. The $W$-dependence of the reduced asymmetry $A_0$ for $\epsilon = 0.5$ and two values of $k^2$: (a) $-k^2 = 0.5$; (b) $1.0$ (GeV/c)$^2$. Same conventions as in Fig. 7.
Fig. 1
Fig. 2
Fig. 3
Fig. 9