EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-PS DIVISION

CERN/PS 2000-069 (AE)

**Collective Effects and Final Bunch Rotation in a 2.2 GeV – 44 MHz
Proton Accumulator – Compressor for a Neutrino Factory**

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NUFACT'00, Monterey, California, USA
22-26 May 2000

Geneva, Switzerland
29 September 2000

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1 INTRODUCTION

The proton driver for a neutrino factory must provide megawatts of beam power at a few GeV, with nanosecond long bunches. Such beam powers are within reach of a high-energy linac, but the required time structure cannot be provided without accumulation and compression. The option of a linac-based 2.2 GeV – 44 MHz proton driver has been studied at CERN [1]. This paper describes the collective effects in the accumulator ring where the beam is accumulated during 2.2 ms, and the final bunch rotation, which is performed in the compressor ring in about 30 μ s.

2 COLLECTIVE EFFECTS IN THE ACCUMULATOR

2.1 Transverse space charge

The horizontal and vertical incoherent tune shifts of the particle located at the centre of a transversally Gaussian and longitudinally parabolic bunch, is given by (neglecting the wall effects) [2]

$$\Delta Q_{inc,x0} = -\frac{2r_p I_p \beta_x R}{ec \beta^3 \gamma^3 a_x (a_x + a_y)}, \quad \Delta Q_{inc,y0} = -\frac{2r_p I_p \beta_y R}{ec \beta^3 \gamma^3 a_y (a_x + a_y)}. \quad (1)$$

Making the numerical computations (see Appendix), one obtains* $\Delta Q_{inc,x0}^{\mu b} = -0.065$, $\Delta Q_{inc,y0}^{\mu b} = -0.060$, $\Delta Q_{inc,x0}^{MbA} = -0.076$, $\Delta Q_{inc,y0}^{MbA} = -0.070$, $\Delta Q_{inc,x0}^{MbC} = -0.210$, $\Delta Q_{inc,y0}^{MbC} = -0.197$. These

* μb stands for the micro-bunches injected from the Linac into the Accumulator, MbA and MbC stand for the macro-bunches at the end of accumulation and compression, respectively.

numbers are within the classical synchrotron limit of about -0.3 .

2.2 RF matching

Without space charge

Neglecting space charge, the peak RF voltage needed to match a bunch with a longitudinal emittance ε_l in a stationary bucket, is given by [3]

$$\hat{V}_{RF,0} = \varepsilon_l^2 \times \frac{2\pi}{256} \times \frac{h^3 |\eta| c^2}{E R^2} \times \left(\frac{\sin[\pi/2.255]}{\sin[\pi f_{RF} \tau_b / 2.255]} \right)^4. \quad (2)$$

It yields, for the macro-bunch at the end of accumulation, $\hat{V}_{RF,0}^{MbA} = 137.2$ kV. The (small amplitudes) synchrotron frequency is given by

$$f_s = f_0 \times \sqrt{\frac{\hat{V}_{RF,0} h |\eta|}{2\pi \beta^2 E}}, \quad (3)$$

which gives $f_s^{MbA} = 2.933$ kHz. The acceptance of the bucket is computed from

$$A_{bucket} = \varepsilon_l \times \left(\frac{\sin[\pi/2.255]}{\sin[\pi f_{RF} \tau_b / 2.255]} \right)^2, \quad (4)$$

and the maximum bunch energy spread from

$$\Delta E_{spread}^{bunch} = \pm \frac{2\pi f_{RF}}{8} \times A_{bucket} \times \sin[\pi f_{RF} \tau_b / 2]. \quad (5)$$

One obtains $A_{bucket}^{MbA} = 0.13$ eVs and $\Delta E_{spread}^{MbA} = \pm 4.14$ MeV, or using $\Delta E = \beta^2 \gamma E_0 \times \Delta p / p_0$, the maximum relative bunch momentum spread is $(\Delta p / p_0)_{MbA} = \pm 1.45 \times 10^{-3}$.

With space charge

Taking into account space charge, using Hofmann-Pedersen formalism, i.e. considering a local elliptic energy distribution, the space charge voltage is given by [4]

$$\hat{V}_{SC} = \frac{2\pi h^2 I_0 \text{Im}[Z_l^{SC}(p)/p]}{\sin \phi_2 - \sin \phi_1 - (\phi_2 - \phi_1) \times (\cos \phi_1 + \cos \phi_2) / 2}, \quad (6)$$

where the space charge impedance is given by [5]

$$\frac{Z_l^{SC}(p)}{p} = -j \frac{Z_0}{2\beta\gamma^2} \times \left[1 + 2\ln\left(\frac{b}{a}\right) \right], \quad (7)$$

with $p = \omega / \omega_0$. Notice that $Z_{l,\mu b}^{SC}(p) / p \approx -j \times 150 \, \Omega$, $Z_{l,MbA}^{SC}(p) / p \approx Z_{l,MbC}^{SC}(p) / p \approx -j \times 70 \, \Omega$. Applying Eq. (6) for the macro-bunch at the end of accumulation yields $\hat{V}_{SC}^{MbA} = -106 \, \text{kV}$. The RF voltage needed for the matching at the end of accumulation, taking into account space charge, is thus $\hat{V}_{RF}^{MbA} = \hat{V}_{RF,0}^{MbA} + |\hat{V}_{SC}^{MbA}| = 243.2 \, \text{kV}$, i.e. $\sim 250 \, \text{kV}$.

2.3 Microwave instability

Coasting-beam formalism without momentum spread

The instability rise-time, neglecting momentum spread and using the well-known coasting-beam formalism with the bunch peak intensity (which should be valid for bunched beams when the wave length of the unstable modes is much smaller than the bunch length), is given by [6]

$$\tau_{\mu W} = \frac{1}{p_r^{BB} \omega_0} \times \text{Im} \left[\sqrt{-j \frac{Z_l(p)}{p} \times \frac{\eta I_p}{2\pi \beta^2 E/e}} \right], \quad (8)$$

where $p_r^{BB} \approx \omega_r^{BB} / \omega_0$ and $Z_l = Z_l^{SC} + Z_l^{BB}$, with

$$Z_l^{BB}(\omega) = R_{r,l}^{BB} / \left[1 - j Q_r^{BB} \left(\frac{\omega_r^{BB}}{\omega} - \frac{\omega}{\omega_r^{BB}} \right) \right]. \quad (9)$$

The plot of the rise-time for the macro-bunch at the end of accumulation vs. the broadband impedance is shown in Figure 1. It can be seen that for $|Z_l^{BB}(p)/p| = 1 \, \Omega$, the rise-time is about 0.6 ms.

Figure 1: Microwave-instability rise-time (neglecting momentum spread) for the macro-bunch at the end of accumulation (using the coasting-beam formalism with the bunch peak current) vs. the broadband impedance $|Z_l^{BB}(p)/p|$.

Coasting-beam formalism with momentum spread

The instability rise-time, taking into account momentum spread (by assuming for instance a parabolic distribution), can be obtained from the parametric equations [6]

$$\Lambda \times \operatorname{Re} \left[\frac{Z_l(p)}{p} \right] = \operatorname{Im} [J^{-1}], \quad \Lambda \times \operatorname{Im} \left[\frac{Z_l(p)}{p} \right] = -\operatorname{Re} [J^{-1}], \quad (10)$$

with

$$\Lambda = \frac{3I_p}{2\pi\eta(E/e)\beta^2 \left(\frac{\Delta p}{p_0} \right)_{\text{FWHH}}^2}, \quad J = \int_{x=-1}^{x=1} \frac{x dx}{x + \frac{\sqrt{2} \omega_{c,p}^l}{p \omega_0 |\eta| \left(\frac{\Delta p}{p_0} \right)_{\text{FWHH}}}}, \quad (11)$$

where $\omega_{c,p}^l$ is the coherent frequency to be determined. The curve with constant rise-time $-1/\operatorname{Im}[\omega_{c,p}^l] = 0.7 \text{ ms}$ (corresponding to the working point with $|Z_l^{BB}(p)/p| = 1 \Omega$) for the macro-bunch at the end of accumulation is shown in Figure 2.

Figure 2: Curve with constant microwave instability rise-time (0.7 ms, corresponding to the working point with $|Z_l^{BB}(p)/p| = 1 \Omega$ represented by a dot) for the macro-bunch at the end of accumulation, considering a parabolic momentum distribution. The Keil-Schnell approximation corresponds here to a circle with a radius of about 0.5.

The accumulator ring should be carefully designed to have a broad-band impedance $|Z_l^{BB}(p)/p| \leq 1 \Omega$. It can be seen from the above computations that in the case of 1Ω , the rise-time is smaller than 1 ms, which is to be compared with the accumulation time (2.2 ms). Although the microwave instability could be a source of concern at first sight, according to the above computations (using the simple coasting-beam formalism), preliminary simulations show that it should not be a problem and that the value of 1Ω can be considered to be a very conservative estimate [7].

2.4 Transverse instabilities at “low” intensity (below the mode coupling threshold)

Single-bunch instability

The transverse coherent complex frequency shifts of bunched-beam modes are given by [8]

$$\Delta\omega_{m,m}^{x,y} = (|m|+1)^{-1} \frac{j e \beta I_0}{2 m_0 \gamma Q_{x,y} \omega_0 L} \frac{\sum_{k=-\infty}^{k=+\infty} Z_{x,y}(\omega_k^{x,y}) h_{m,m}(\omega_k^{x,y} - \omega_{\xi_{x,y}})}{\sum_{k=-\infty}^{k=+\infty} h_{m,m}(\omega_k^{x,y} - \omega_{\xi_{x,y}})}, \quad (12)$$

where $m = \dots, -1, 0, 1, \dots$ is the head-tail mode number, $\omega_k^{x,y} = (k + Q_{x,y}) \omega_0 + m \omega_s$, with $\omega_s = 2\pi f_s$, and $Z_{x,y}$ are the transverse coupling impedances. The transverse bunch spectra of mode $|m|$ and the resistive-wall impedances are given by

$$h_{m,m}(\omega) = \frac{\tau_b^2}{2\pi^4} (|m|+1)^2 \frac{1 + (-1)^{|m|} \cos[\omega \tau_b]}{\left\{ [\omega \tau_b / \pi]^2 - (|m|+1)^2 \right\}^2}, \quad (13)$$

$$Z_{x,y}^{RW}(\omega) = [Sgn(\omega) + j] \frac{R}{b^3} \sqrt{\frac{2\rho_w}{\epsilon_0 |\omega|}}, \quad (14)$$

where $Sgn(\omega) = 1$ if $\omega > 0$, -1 if $\omega < 0$. The plots of the horizontal power spectrum and the resistive part of the resistive-wall impedance are shown in Figure 3. The same kind of picture is obtained for the vertical plane.

Figure 3: Horizontal power spectra for the head-tail modes $|m| = 0, 1, 2$ (solid lines) and resistive part of the resistive-wall impedance (dotted lines).

Applying Eq. (12) for the macro-bunch at the end of accumulation interacting with the transverse resistive-wall impedances, yields head-tail instabilities $|m|=2$ with rise-times of about 400 ms. The rise-times are long compared to the 2.2 ms of accumulation time.

Coupled-bunch instability

In the case of coupled motion of M (equi-spaced and equi-populated) bunches, Eq. (12) can be used with $k = n_{x,y} + k' M$, where $-\infty \leq k' \leq +\infty$ and $n_{x,y} = 0, 1, \dots, M-1$ are the transverse coupled-bunch modes. For the macro-bunch at the end of accumulation interacting with the transverse resistive-wall impedances (considering the worst case of 146 bunches instead of 140), this yields coupled-bunch instabilities $|m|=1$, with $n_x = 134$ and $n_y = 132$, with rise-times of about 10 ms. The rise-times are again long compared to the 2.2 ms of accumulation time.

2.5 Transverse instabilities at “high” intensity (mode coupling threshold)

The intensity threshold for the transverse mode coupling instability can be obtained from the following equation [9,10]

$$|\Delta\omega_{m,m+1}^{x,y}| = \frac{1}{2} |\omega_s + \Delta\omega_{m+1,m+1}^{x,y} - \Delta\omega_{m,m}^{x,y}|, \quad (15)$$

with

$$\Delta\omega_{m,m+1}^{x,y} = (|m|+1)^{-1} \frac{j e \beta I_0}{2 m_0 \gamma Q_{x,y} \omega_0 L} \frac{\sum_{k=-\infty}^{k=+\infty} Z_{x,y}(\omega_k^{x,y}) h_{m,m+1}(\omega_k^{x,y} - \omega_{\xi_{x,y}})}{\sum_{k=-\infty}^{k=+\infty} h_{m,m}(\omega_k^{x,y} - \omega_{\xi_{x,y}})}, \quad (16)$$

$$h_{m,m+1}(\omega) = \frac{\tau_b^2}{2\pi^4 j} \sin[\omega \tau_b] \times \frac{(|m|+1)(|m|+2)}{\left\{ [\omega \tau_b / \pi]^2 - (|m|+1)^2 \right\} \left\{ [\omega \tau_b / \pi]^2 - (|m|+2)^2 \right\}}. \quad (17)$$

Applying Eq. (15) for the macro-bunch at the end of accumulation interacting with the broad-band impedances

$$Z_{x,y}^{BB}(\omega) = \frac{\omega_r^{BB}}{\omega} R_{r,t}^{BB} / \left[1 - j Q_r^{BB} \left(\frac{\omega_r^{BB}}{\omega} - \frac{\omega}{\omega_r^{BB}} \right) \right], \quad (18)$$

yields intensity thresholds of about 10^{14} protons per macro-bunch for $|Z_l^{BB}(p)/p| = 1 \Omega$. This is much higher than the chosen number of protons per macro-bunch (1.089×10^{12}). Notice that the transverse broad-band impedance is obtained from the longitudinal one by using the same relation as in the case of the resistive wall $Z_{x,y}^{BB}(\omega) \approx (2c/b^2) \times Z_l^{BB}(\omega)/\omega$, although for any structure other than a smooth pipe this relation is only approximate.

A simplified equation, which gives the threshold number of protons per bunch N_b^{th} for the transverse mode coupling instability, can be obtained by equating the coherent tune shift of mode 0 to $-Q_s$, and is given by

$$N_b^{th} \approx \frac{2\pi b^2 Q_s (E/e)}{ec \beta_{x,y} \left| \frac{Z_l^{BB}(p)}{p} \right|} \times \omega_0 \tau_b. \quad (19)$$

Figure 4 is obtained by applying Eq. (19) for the macro-bunch at the end of accumulation interacting with the broad-band impedance.

Figure 4: Threshold number of protons per bunch N_b^{th} for the transverse mode coupling instability vs. the broad-band impedance $|Z_l^{BB}(p)/p|$.

For $|Z_l^{BB}(p)/p| = 1 \Omega$, the threshold number of protons is about 10^{14} protons per macro-bunch, as found before. Notice that the threshold number of protons per bunch given by Eq. (19) is approximately equal to that which would have been obtained for the longitudinal mode coupling (or microwave instability) considering only the broad-band impedance, and neglecting the potential-well distortion. However, the space-charge impedance has a destabilizing effect on the longitudinal microwave instability, and it has thus to be taken into account on the stability diagram of Figure 2 to have a correct description of the phenomenon. On the other hand, as concerns the transverse mode coupling instability, the threshold number of protons per bunch given by Eq. (19) can be considered to be a conservative estimate, since in this case space charge seems to have a strong stabilizing effect [11].

2.6 Beam breakup instability

Single-bunch instability

The growth parameter, i.e. the ratio between the transverse amplitudes of the tail after n turns performed in the circular machine and of the whole bunch at the beginning of the instability process, is given, e.g. for the vertical plane, by [12]

$$\frac{y_n}{y_0} = \frac{1}{2\sqrt{2\pi}} \times \left(\frac{\Omega L}{\beta c} \right)^{1/4} \times \frac{\beta c}{L\omega_r^{BB}} \times e^{\frac{-\varepsilon L}{\beta c} + \sqrt{\frac{\Omega L}{\beta c}}}, \quad (20)$$

with
$$\frac{\Omega L}{\beta c} = \frac{N_b e \beta c}{\omega_y (E/e)} \times \frac{\omega_r^{BB} R_{r,t}^{BB}}{Q_r^{BB}} \times n, \quad \varepsilon = \omega_r^{BB} / (2Q_r^{BB}). \quad (21)$$

The plot of the vertical growth parameter y_n/y_0 vs. the broad-band impedance $|Z_l^{BB}(p)/p|$ is shown in Figure 5.

Figure 5: Vertical growth parameter y_n/y_0 for the single-bunch beam breakup instability vs. the broad-band impedance $|Z_l^{BB}(p)/p|$, after half the number of injection turns.

The critical value of the broad-band impedance is much higher than the one that should be obtained by a careful design of the accumulator ring (about 1 Ω).

Notice that the beam breakup mechanism is essentially described by the exponential term of Eq. (20). An approximate formula, which gives the time (to within few percent) when the tail particles are lost, can be derived for a circular vacuum chamber of radius b . It is given, e.g. for the vertical plane, by [13]

$$\Delta t = \frac{\pi}{4} \times \frac{Q_y (E/e) \tau_b^2}{N_b e R} \times \frac{b}{|Z_l^{BB}(p)/p|}. \quad (22)$$

It is plotted vs. the broad-band impedance $|Z_l^{BB}(p)/p|$ in Figure 6.

Figure 6: Time when the tail particles are lost due to the single-bunch beam breakup instability vs. the broad-band impedance $|Z_l^{BB}(p)/p|$.

Multi-bunch instability

The vertical growth parameter is given by [14]

$$\frac{y_n}{y_0} = \frac{1}{2\sqrt{2\pi}(M_{\mu b} - 1)} \times \left(\frac{\Omega L_t}{\beta c} \right)^{1/4} \times e^{-\varepsilon t_{sep}^{\mu b} (M_{\mu b} - 1) + \sqrt{\frac{\Omega L_t}{\beta c}}} \times \left[\frac{(e^{\varepsilon t_{sep}^{\mu b}} - 1)^2 + 4e^{\varepsilon t_{sep}^{\mu b}} \sin^2 \left(\frac{\omega_0 t_{sep}^{\mu b}}{2} \right)}{1} \right]^{-1/2}, \quad (23)$$

with

$$\frac{\Omega L_t}{\beta c} = \frac{N_{\mu b} M_{\mu b} e \beta c}{\omega_y (E/e)} \times \frac{\omega_r^{BB} R_{r,t}^{BB}}{Q_r^{BB}} \times n, \quad (24)$$

where the upper and lower parts in the curly bracket of Eq. (23) correspond to the initial conditions where (1) every bunch is injected with the same offset without slope and (2) only the first bunch is displaced and the others are on the axis, respectively. The plots of the vertical growth parameters $(y_n/y_0)_{1,2}$ vs. the broad-band impedance $|Z_l^{BB}(p)/p|$ are shown in Figure 7.

Figure 7: Vertical growth parameters $(y_n / y_0)_{1,2}$ for the multi-bunch beam breakup instability vs. the broad-band impedance $|Z_l^{BB}(p)/p|$, after the number of injection turns.

The critical values of the broad-band impedance are again much higher than the one that should be obtained by a careful design of the accumulator ring (about 1 Ω).

2.7 Longitudinal coupled-bunch instability

The longitudinal coherent complex frequency shifts of bunched-beam modes are given by [15]

$$\Delta\omega_{m,m}^l = \frac{m\omega_s}{|m|+1} \frac{jI_0}{3B^3\hat{V}_{RF}h} \frac{\sum_{p=-\infty}^{p=+\infty} \frac{Z_l(\omega_p^l)}{p} h_{m,m}(\omega_p^l)}{\sum_{p=-\infty}^{p=+\infty} h_{m,m}(\omega_p^l)}, \quad (25)$$

where $\omega_p^l = p\omega_0 + m\omega_s$ and $p = n_l + p'M$ for coupled motion of M (equi-spaced and equi-populated) bunches, with n_l the longitudinal coupled-bunch mode and

$-\infty \leq p' \leq +\infty$. It has been shown above that a peak RF voltage of about 250 kV at the end of accumulation with a RF frequency of about 44 MHz is needed. A good estimate of such a RF cavity is given by the CERN PS 40 MHz RF cavity [16]. It can be seen in Ref. [17] that the shunt impedances for all the modes up to 850 MHz are lower than 500Ω with the exception of 3 higher-order modes (HOM's) at about 394, 732 and 774 MHz. After damping with three antennae terminated by 50Ω , plus additional dampings, the following results collected in Table 1 are obtained.

Table 1: Damped HOM's of the CERN PS 40 MHz RF cavity.

f [MHz]	Q_{loaded}	$R_{shunt,loaded}$ [Ω]
394.367	100	500
732.224	3000	3000
774.713	5000	6700

Applying Eq. (25) for the macro-bunch at the end of accumulation, yields coupled-bunch instabilities with rise-times of about 100, 20 and 10 ms, respectively. The rise-times are long compared to the 2.2 ms of accumulation time. Notice that these results have been obtained by taking as the resonance frequencies the frequencies closest to those of Table 1 which coincide with lines of the coupled-bunch spectrum, and taking into account 146 bunches instead of 140, which corresponds to the worst case.

3 FINAL BUNCH ROTATION IN THE COMPRESSOR

Once the accumulation process is over, the 140 macro-bunches are transferred to the compressor ring where their length is reduced by bunch rotation. For that purpose the compressor ring is equipped with four cavities at 44 MHz, providing a total voltage of 2 MV, and with one cavity at 88 MHz [18] delivering 350 kV. Prior to beam transfer, RF power is applied to these cavities at a slightly offset frequency, so that the full voltages are present when the beam enters the machine. As soon as beam circulates, a peak beam current of about 15 A flows in the cavities. Since the bunch compression is much faster ($\sim 30 \mu s$) than the cavity filling time ($100 \mu s$), the phase shift due to transient beam loading increases almost linearly with time and can be compensated by an appropriate frequency difference between generator and beam frequencies (~ 20 kHz at 44 MHz). The compression process brings the bunches from the estimated initial rms length of 3.5 ns (with $\Delta p/p(2\sigma)=1.45 \times 10^{-3}$) to the final one of 1 ns (with $\Delta p/p(2\sigma)=4.5 \times 10^{-3}$). The initial and final phase portraits and line densities, as well as the evolutions of the rms bunch length and momentum spread, are shown in Figure 8, and have been obtained using the ESME program [19], taking into account the longitudinal space charge.

Figure 8: Bunch rotation in the compressor ring. The phase portraits are shown in the upper part (initial left, final right). The evolutions of the rms bunch length (in degrees of azimuthal angle θ) and energy spread (in MeV, w.r.t the synchronous energy) are shown in the lower part (1 deg corresponds to 9.21 ns and an energy spread of 1 MeV corresponds to a relative momentum spread of 3.5×10^{-4}).

4 CONCLUSION

The CERN 2.2 GeV – 44 MHz scenario for the proton driver of a neutrino factory looks promising. No major obstacles have been found with the current parameters. More detailed investigations have to be performed to investigate electron cloud effects, halo formation and beam losses in both rings.

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APPENDIX: List of the Accumulator (A), Compressor (C) and beam parameters

$a = (a_x + a_y) / 2$	Average beam radius.	$a^{ub} = 2.16 \text{ mm}.$
$a_{x,y} = \sqrt{2} \left[\epsilon_{x,y}^{rms} \beta_{x,y} + (D_{x,y} \sigma_p / p_0)^2 \right]^{1/2}$	$a^{MbA} = a^{MbC} = 1.98 \text{ cm}$	$\sqrt{2} \times$ the rms horizontal and vertical beam dimensions.
	$a_x^{ub} = 2.25 \text{ mm}, \quad a_y^{ub} = 2.07 \text{ mm}, \quad a_x^{MbA} = a_x^{MbC} = 2.06 \text{ cm},$	
	$a_y^{MbA} = a_y^{MbC} = 1.89 \text{ cm}$	
$b = b_x = b_y = 9 \text{ cm}$	Radius of the circular vacuum chamber of A and C	
$B = f_0 \tau_b$	Bunching factor	

$$c = 2.997925 \times 10^8 \text{ ms}^{-1}$$

$$D_{x,y}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$E = E_0 + E_c = 3.138 \text{ GeV}$$

$$E_0 = 0.938 \text{ GeV}$$

$$E_c = 2.2 \text{ GeV}$$

$$f_0 = 301.536 \text{ kHz}$$

$$f_r^{BB} \approx c / (2\pi b) = 0.53 \text{ GHz}$$

$$f_{\text{RepRate}} = 75 \text{ Hz}$$

$$f_{RF} = h f_0 = 44.0243 \text{ MHz}$$

$$f_{\xi_{x,y}} = (\xi_{x,y} / \eta) Q_{x,y} f_0$$

$$h = 146$$

$$I_p = 3e N_b / 2\tau_b$$

$$I_0 = N_b e f_0$$

$$j = \sqrt{-1}$$

$$L$$

$$L_t \approx (M_{\mu b} - 1) \times t_{sep}^{\mu b} = 11.36 \text{ ns}$$

$$m_0 = 1.67 \times 10^{-27} \text{ kg}$$

$$M$$

$$N_{\text{injections}} = 660$$

$$N_b$$

$$P_{\text{beam}} = E_c N_{MbC} M_{MbC} f_{\text{RepRate}} = 4 \text{ MW}$$

$$Q_r^{BB} \approx 1$$

$$Q_s = f_s / f_0$$

$$Q_{x,y}$$

$$r_p = 1.54 \times 10^{-18} \text{ m}$$

$$R = 151 \text{ m}$$

$$R_{r,l}^{BB} = (f_r^{BB} / f_0) \times |Z_l^{BB}(p) / p|$$

$$R_{r,t}^{BB} = 2c R_{r,l}^{BB} / (2\pi f_r^{BB} b^2)$$

Velocity of light

Average transverse dispersion functions.

$$D_x^A = R / (Q_x^A)^2 = 1.20 \text{ m}, \quad D_x^C = R / (Q_x^C)^2 = 0.51 \text{ m},$$

$$D_y^A = 0 \text{ m}, \quad D_y^C = 0 \text{ m}$$

Elementary charge

Proton total energy

Proton rest energy

Proton kinetic energy

Average revolution frequency in A and C

Resonance frequency of the broadband impedances (cut-off frequency of the vacuum chamber)

Repetition rate of the proton driver

Average RF frequency in A and C

Transverse chromatic frequencies. $f_{\xi_x}^A = 39.93 \text{ MHz}$,

$$f_{\xi_y}^A = 47.29 \text{ MHz}$$

Harmonic number in A and C

Bunch peak current considering a parabolic line density.

$$I_p^{\mu b} = 0.16 \text{ A}, \quad I_p^{MbA} = 15.37 \text{ A}, \quad I_p^{MbC} = 65.34 \text{ A}$$

Bunch average current. $I_0^{\mu b} = 0.000016 \text{ A}$,

$$I_0^{MbA} = I_0^{MbC} = 0.053 \text{ A}$$

Imaginary unit

Total (4σ) bunch length (in meters). $L_{\mu b} = 0.143 \text{ m}$,

$L_{MbA} = 4.863 \text{ m}$ ("estimated" length of a macro-bunch at the end of accumulation), $L_{MbC} = 1.144 \text{ m}$

Length (in meters) of the Linac train

Proton rest mass

Number of bunches. $M_{\mu b} = 5$ (per Linac injection train),

$$M_{MbA} = M_{MbC} = 140$$

Number of injection turns in A

Number of protons per bunch. $N_{\mu b} = 3.3 \times 10^8$,

$$N_{MbA} = N_{MbC} = 1.089 \times 10^{12}$$

Mean beam power

Quality factor of the broad-band impedances

Synchrotron tune

Transverse coherent tunes (in the absence of wake fields).

$$Q_x^A = 11.23, \quad Q_y^A = 13.30, \quad Q_x^C = 17.18, \quad Q_y^C = 16.40$$

Classical proton radius

Average machine radius of A and C

Longitudinal shunt impedance of the broadband resonator

Transverse shunt impedance of the broadband resonator

$$t_{sep}^{ub} = 2.84 \text{ ns}$$

$$T_{accumulation} = T_0 N_{injections} = 2.2 \text{ ms}$$

$$T_0 = 1 / f_0 = 3.32 \text{ } \mu\text{s}$$

$$T_{RF} = 1 / f_{RF} = 22.715 \text{ ns}$$

$$Z_0 = 377 \text{ } \Omega$$

$$\beta = \sqrt{1 - \gamma^{-2}} = 0.9543$$

$$\beta_{x,y} = R / Q_{x,y}$$

$$\gamma = E / E_0 = 3.345$$

$$\gamma_{tr}$$

$$\epsilon_0 = 8.84 \times 10^{-12} \text{ Fm}^{-1}$$

$$\epsilon_l$$

$$\epsilon_{x,y}^{\text{norm}, 1\sigma} = \beta \gamma \epsilon_{x,y}^{\text{rms}}$$

$$\eta = \gamma_{tr}^{-2} - \gamma^{-2}$$

$$\xi_{x,y} \approx -1$$

$$\rho_w = 10^{-6} \text{ } \Omega \text{ m}$$

$$\tau_b$$

$$\phi_2 = -\phi_1$$

$$\omega_0 = 2\pi f_0$$

$$\omega_r^{BB} = 2\pi f_r^{BB}$$

$$\omega_{RF} = 2\pi f_{RF}$$

$$\omega_{x,y} = Q_{x,y} \omega_0$$

$$\omega_{\xi_{x,y}} = 2\pi f_{\xi_{x,y}}$$

$$\Delta p / p_0 = \pm 2 \sigma_p / p_0$$

$$(\Delta p / p_0)_{\text{FWHH}} = 2\sqrt{2} (\sigma_p / p_0)$$

$$\sigma_p / p_0$$

Separation in time between (the centres of) 2 consecutive micro-bunches

Accumulation time

Average revolution period in A and C

Average RF period in A and C

Free space impedance

Relativistic velocity factor

Average transverse betatron functions. $\beta_x^A = 13.45 \text{ m}$,

$\beta_y^A = 11.35 \text{ m}$, $\beta_x^C = 8.79 \text{ m}$, $\beta_y^C = 9.21 \text{ m}$

Relativistic mass factor

Relativistic mass factor at transition energy. $\gamma_{tr}^A = 14.837$,

$\gamma_{tr}^C = 15.089$

Permittivity of free space

Longitudinal bunch emittance. $\epsilon_l^{MbA} = 0.1 \text{ eVs}$

Normalized rms transverse emittances.

$(\epsilon_{x,y}^{\text{norm}, 1\sigma})_{ub} = 0.6 \text{ } \mu\text{m}$, $(\epsilon_{x,y}^{\text{norm}, 1\sigma})_{MbA} = (\epsilon_{x,y}^{\text{norm}, 1\sigma})_{MbC} = 50 \text{ } \mu\text{m}$

Slippage (or off-momentum) factor. $\eta^A = -0.08481$,

$\eta^C = -0.08496$

Transverse (relative) chromaticities in A and C

Vacuum chamber resistivity (stainless steel)

Total (4σ) bunch length (in seconds). $\tau_{ub} = 0.5 \text{ ns}$,

$\tau_{MbA} = 17 \text{ ns}$ (“estimated” length of a macro-bunch at the end of accumulation), $\tau_{MbC} = 4 \text{ ns}$

Bunch half length (in RF phase).

$\phi_2^{MbA} = -\phi_1^{MbA} = 134.7 \text{ deg}$

Transverse betatron angular frequencies

Maximum relative bunch momentum spread

Full width at half height of the bunch momentum distribution (considering a parabolic distribution)

Rms relative bunch momentum spread.

$(\sigma_p / p_0)_{ub} = 4 \times 10^{-5}$, $(\sigma_p / p_0)_{MbA} = 7.25 \times 10^{-4}$,

$(\sigma_p / p_0)_{MbC} = 2.25 \times 10^{-3}$