A sandpile model with tokamak-like enhanced confinement

phenomenology

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Abstract

Confinement phenomenology characteristic of magnetically confined plasmas emerges naturally from a simple sandpile algorithm when the parameter controlling redistribution scalelength is varied. Close analogues are found for enhanced confinement, edge pedestals, and edge localised modes (ELMs), and for the qualitative correlations between them. These results suggest that tokamak observations of avalanching transport are deeply linked to the existence

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of enhanced confinement and ELMs.

52.55.Dy, 52.55.Fa, 45.70.Ht, 52.35.Ra
The introduction of the sandpile paradigm [1]-[3] into magnetized plasma physics (fusion [4]-[10], magnetospheric [11]-[13], and accretion disk [14]-[16]: for recent reviews see Ref. [17,18]) has opened new conceptual avenues. It provides a framework within which observations of rapid nondissipative nonlocal transport phenomena can be studied; recent examples include analyses of auroral energy deposition derived from global imaging [13], and of electron temperature fluctuations in the DIII-D tokamak [10], both of which involve avalanching. Insofar as such phenomena resemble those in experimental sandpiles or mathematically idealized models thereof, they suggest that the confinement physics of macroscopic systems (plasma and other) may reflect unifying underlying principles.

In this paper we present results suggesting that this unity may extend to some of the most distinctive features of toroidal magnetic plasma confinement: enhanced confinement regimes (“H-modes”), edge localized modes (“ELMs”), steep edge gradients (“edge pedestal”), and their observed phenomenological and statistical correlations – for recent quantitative studies, see for example Refs. [19–21] and references therein. An important question is whether the L to H transition necessarily reflects a catastrophic bifurcation of confinement properties, or can be associated with a monotonic change in the character of the turbulence [21]. We show that key elements of the observed phenomenology emerge naturally from a simple one-dimensional sandpile model, that of Chapman [22] (hereafter CDH), which incorporates other established models [1,23] as limiting cases. The centrally fueled (at cell \(n = 1\)) CDH model’s distinctive algorithmic feature relates to the local redistribution of sand at a cell (say at \(n = k\)) where the critical gradient \(z_c\) is exceeded: the sandpile is flattened behind the unstable cell over a “fluidization length” \(L_f\), embracing the cells \(n = k - (L_f - 1), k - (L_f - 2), ..., k\); and this sand is conservatively relocated to the cell at \(n = k + 1\). In Ref. [22] the CDH sandpile is explored for all regimes \(1 < L_f < N\) for both constant and fluctuating critical gradient \(z_c\). Here we consider the dynamics of the more realistic case with random fluctuations in \(z_c\); the system is robust in that once fluctuations are introduced in the critical gradient, the behavior is essentially insensitive to both their level and spectral properties [22], see also Ref. [17]. The limit \(L_f = 1\) is the fixed point corresponding to the centrally
fueled algorithm of Ref. [1] in one dimension. In the limit $L_f = N$ (where $N$ is the number of cells in the sandpile) the sandpile is flattened everywhere behind an unstable cell as in Refs. [23,17]. A real space renormalization group approach [24] shows that the robust scale free dynamics for the limiting case $L_f = N$ corresponds to a nontrivial (repulsive) fixed point (see e.g. Ref. [3]). The essential result of Ref. [22] is that different regimes of avalanche statistics emerge, resembling a transition from regular to intermittent dynamics reminiscent of deterministic chaos. The control parameter is the normalized redistribution scalelength $L_f/N$ which specifies whether the system is close to the nontrivial $L_f = N$ fixed point.

Height profiles for a CDH sandpile with 512 cells, time averaged over many thousands of avalanches, are shown in Fig.1 for three different values of the fluidization length $L_f$ in the range $50 < L_f < 250$. The sandpile profile shape, stored gravitational potential energy, and edge structure (smooth decline or pedestal) correlate with each other and with $L_f$. As $L_f$ is reduced, the edge pedestal steepens and the time averaged stored energy rises; multiple “barriers” (regions of steep gradient) are visible in trace (a) and to some extent trace (b) of Fig.1. Time evolution of the CDH sandpile for $L_f = 50, 150, \text{ and } 250$ respectively is quantified in Figs.2-4. The top traces show total stored energy; the middle traces show the position of the edge of the sandpile (the last occupied cell); and the bottom traces show the magnitude and occurrence times of mass loss events (hereafter MLEs) in which sand is lost from the system by being transferred beyond the 512th cell. Time is normalized to the mean inter-avalanche time $\Delta \tau$ (proportional to the fueling rate). The CDH sandpile is fueled only at the first cell, so that the great majority of avalanches terminate before reaching the 512th cell (these are classified as internal). While internal avalanches result in energy dissipation (recorded in the upper traces of Figs.2-4), and may alter the position of the edge of the sandpile, they do not result in an MLE; there are corresponding periods of quiescence in the middle and lower traces of Figs.2-4. Conversely the MLEs are associated with sudden inward movement of the sandpile edge, and in this important sense appear to be edge localised. However, MLEs and the associated inward edge movement are in fact the result of systemwide avalanches triggered at the sandpile center (cell $n = 1$). The character
of the MLEs changes with $L_f$. In Fig.2, where the mean and peak stored energy are greatest, the MLEs are similar to each other and occur with some regularity. The regularity of MLE occurrence in Fig.3 is less marked, the magnitude of the largest MLEs is greater than in Fig.2, and there is greater spread in MLE size. This trend continues in Fig.4, which also has the lowest stored energy. These effects correlate with the underlying dynamics of the CDH sandpile. Figure 5 plots the relation between average stored energy and $L_f$ for the $N = 512$ system and much larger $N = 4096$ and 8192 systems (normalized to the system size $N$). The curves coincide, demonstrating invariance with respect to system size, with an inverse power law with slope close to $-2$ for $L_f/N < 1/4$, and a break at $L_f/N \sim 1/4$. These two regimes yield the quasi-regular and quasi-intermittent dynamics in Figs.2-4 (see also the plot of avalanche length distribution against $L_f$ in Fig.8 of Ref. [22]). The parameter $L_f/N$ is a measure of proximity of this high dimensional system to the $L_f = N$ nontrivial fixed point. This determines both the apparent complexity of the timeseries in Figs.2-4 and the underlying statistical simplicity described below, which is also invariant with respect to system size.

There is systematic correlation between time averaged stored energy $<E>$ and MLE frequency $f_{MLE}$, as shown in Fig.6. To obtain these curves, which are again normalized to system size, we have derived MLE frequencies using a standard algorithm previously used [20] to assign frequencies to ELMs observed in tokamak plasmas in the Joint European Torus (JET). Since the CDH sandpile often generates bursts of mass loss with structure down to the smallest timescales, which might not be resolvable under experimental conditions, we have followed Ref. [20] in applying a (relatively narrow) measurement window of width $450 \Delta \tau$ to obtain $f_{MLE}$. The correlation between $<E>$ and $f_{MLE}$ is a noteworthy emergent property, furthermore Fig.6’s characteristic curve is very similar to that of Fig.6 of Ref. [19], which relates measured energy confinement to ELM frequency in JET. Energy confinement time $\tau_c$ can be defined for the CDH sandpile by dividing the time averaged stored energy $<E>$ by the time averaged energy dissipation rate $<\Delta E>$ (where $\Delta E$ is the energy dissipated in a single avalanche). The embedded plot of Fig. 6 shows $\tau_c$ against MLE frequency $f_{MLE}$. 
Finally, we explore the situation where there is a secular change in the redistribution algorithm: in Fig.7, $L_f$ decreases slowly, continuously, and linearly with time, from one constant value to another over a period encompassing many tens of thousands of avalanches. There is a corresponding time evolution of the energy confinement properties of the sandpile and of the character of the MLEs. Figure 7(top) shows total stored energy as a function of time as $L_f$ changes from 250 at $t = 4 \times 10^4$ to 50 at $t = 1.15 \times 10^5$, while $\sim 10^5$ avalanches occur: over a period of time corresponding to a few tens of MLEs, the system smoothly evolves from low to high confinement. This is accompanied by a gradual change in character of the time variation in the sandpile edge (position of last occupied cell, Fig.7(middle)) and of the MLEs (Fig.7(lower)), from large amplitude to small and from irregular to regular. Figure 7 can perhaps be regarded as the CDH sandpile analogue of, for example, Fig.2 of Ref. [20] or Fig.2 of [21]. The essential point here is that the sandpile apparently freely explores phase space with changing control parameter $L_f/N$. Characteristic properties of the dynamics (whether quasi-regular or quasi-intermittent) and correspondingly, confinement properties (such as stored energy and MLE characteristics) smoothly follow changes in this parameter rather than exhibiting a sudden phase transition or catastrophe.

By varying a single control parameter in the CDH sandpile algorithm, we have shown correlations between: stored energy, confinement times, sandpile profile, sandpile edge structure, and the amplitude, frequency, and dynamical character of mass loss events. We have also seen how slow secular change in the control parameter produces a smooth evolution in confinement properties. If a single control parameter analogous to $L_f/N$ exists for tokamaks, it can in principle be found from experimental data by examining scaling with respect to system size as above.

The existence of such extensive tokamak-like phenomenology, emergent from a very simple system, is a novel discovery. Insofar as the phenomenological resemblance is close, there is more to be learnt. A minimalist interpretation starts from the premise that the CDH sandpile algorithm provides a simple one-parameter model for studying generic nonlocal transport, conditioned by a critical gradient, in a macroscopic confinement system. Chang-
ing the value of the single control parameter $L_I$ then corresponds to altering the range in configuration space over which the transport process operates. It then follows from the results in the present paper that this may be the minimum requirement to generate those aspects of tokamak-like confinement phenomenology described. This is a significant conclusion, but one can consider a more far-reaching one. A possible maximalist interpretation attaches greater weight to recent observations \cite{6,9,10} of avalanching transport in tokamaks and in largescale numerical simulations \cite{7,8} thereof, and therefore regards the avalanching transport that is built into sandpile algorithms as an additional point of contact with magnetically confined plasmas. One would then infer from the present results that tokamak observations of avalanching transport are deeply linked to the existence of enhanced confinement and ELMs.

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REFERENCES

Captions

FIG.1. Time averaged height profiles of 512 cell CDH sandpile for $L_f = (a)50$, (b)150 and (c)250. Inset: edge structure.

FIG.2. Time evolution of 512 cell CDH sandpile with $L_f = 50$: (top) stored energy, (middle) position of last occupied cell, (lower) magnitude and occurrence of mass loss events.

FIG.3. As Fig.2, for $L_f = 150$.

FIG.4. As Fig.2, for $L_f = 250$.

FIG.5. Average stored energy versus $L_f/N$ for sandpiles of $N = 512, 4096, 8192$. Energy is normalized to the $L_f = 1$ case (effectively to $N^2$).

FIG.6. Average stored energy versus MLE frequency, and (inset) $\tau_c$ versus MLE frequency for sandpiles of $N = 512, 4096, 8192$. Energy and MLE frequency are normalized as in Fig 6.

FIG.7. Time evolution of (top) stored energy, (middle) sandpile edge position and (lower) MLEs, as $L_f$ changes slowly and linearly from 250 to 50.