A smoother approach to scaling by suppressing monopoles and vortices

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Suppressing monopoles and vortices by introducing large chemical potentials for them in the Wilson action for the $SU(2)$ lattice gauge theory, we study the nature of the deconfinement phase transition on $N^3_\sigma \times N^\tau$ lattices for $N^\tau = 4, 5, 6$ and $N^\sigma = 8 - 16$. Using finite size scaling theory, we obtain $\omega \equiv \gamma/\nu = 1.93 \pm 0.03$ for $N^\tau = 4$, in excellent agreement with universality. The critical couplings for $N^\tau = 4, 5, 6$ and 8 lattices exhibit large shifts towards the strong coupling region when compared with the usual Wilson action, and suggest a lot smoother approach to scaling.

1. INTRODUCTION

Universality of lattice gauge theories implies that actions which differ merely by irrelevant terms in the parlance of the renormalization group lead to the same continuum quantum field theory. While many numerical simulations have been performed for the Wilson action for pure gauge theories, other choices, some motivated by the desire to find a smoother continuum limit, have also been employed. This usually has lead to results in accord with universality. Investigations of the deconfinement phase transition for mixed actions, obtained by extending the Wilson action by addition of an adjoint coupling term, showed [1,2], however, surprising challenges to the above notion of universality. Several finite temperature investigations have established the presence of a second order deconfinement phase transition for the Wilson action. Its critical exponents have been shown [3] to be in very good agreement with those of the three dimensional Ising model, as conjectured by Svetitsky and Yaffe [4]. This verification of the universality conjecture strengthened our analytical understanding of the deconfinement phase transition which, however, came under a shadow of doubt by the results for the mixed actions. For two different actions, it was found [1,2] on a range of temporal lattice sizes that for large adjoint couplings a) the deconfinement transition became definitely of first order and b) no other separate bulk transition existed. In fact, simulations on large symmetric lattices even suggested [5] a lack of a bulk phase transition at an adjoint coupling where a first order deconfinement transition for a lattice of temporal size four was clearly seen.

Recently it was shown [6,7] that suppression of some lattice artifacts such as $Z_2$ monopoles and vortices do restore universality for one of the two actions: no first order deconfinement transition was found in the entire coupling plane in that case. Here we address this question for the other action in the same manner and find that one gains an additional bonus. The approach to scaling seems to become smoother than that for the original Wilson action.

2. ACTION AND RESULTS

Both actions [8,9] have an adjoint coupling $\beta_A$ in addition to the usual Wilson coupling $\beta$. The $\beta = 0$ axis in each case describes an $SO(3)$ model which has a first order phase transition. At $\beta_A = \infty$, the theories reduce to a $Z_2$ lattice gauge theory with again a first order phase transition at $\beta_{\text{crit}} \approx 0.44$. The first order transitions extend into the $(\beta, \beta_A)$ coupling plane, meet at a triple point, and continue as a single line of first order transitions which ends at an apparently critical point, say, D in both cases. The proximity of D
to the $\beta_A = 0$ line has commonly been held responsible for the abrupt change from the strong coupling region to the scaling region for the Wilson action. Following Refs. [6,7], one can also suppress $Z_2$ monopoles and $Z_2$ electric vortices for the Bhanot-Creutz action with suitable chemical potentials to restore universality in its entire coupling plane. Using $\text{sign}(\text{Tr}_P U_P)$ to identify the $Z_2$ monopoles and vortices, one can define the corresponding action for such suppressions as below:

$$S_{BC,S} = \sum_p \beta \left(1 - \frac{\text{Tr}_E U_P}{2}\right) +$$
$$\sum_p \beta_A \left(1 - \frac{\text{Tr}_A U_P}{3}\right) + \lambda_M \sum_c (1 - \sigma_c)$$
$$+ \lambda_E \sum_l (1 - \sigma_l),$$

where $\sigma_c = \prod_{p \in \partial c} \text{sign}(\text{Tr}_P U_P)$ and $\sigma_l = \prod_{p \in \partial l} \text{sign}(\text{Tr}_P U_P)$. It is clear that the above theory flows to the same critical fixed point in the continuum limit for all $\beta_A$, $\lambda_M$ and $\lambda_E$ and has the same scaling behavior near the critical point. It has to be stressed though that universality has to be tested afresh for eq.(1) to be sure that the above naive argument about the $\lambda_M$ and $\lambda_E$ terms being irrelevant is correct. This is what we do in the following by determining a critical index of the deconfinement phase transition. We then check whether the passage to scaling is affected by studying the deconfinement transition as a function of the temporal lattice size. We chose $\beta_A = 0$, $\lambda_M = 1$ and $\lambda_E = 5$ throughout.

We studied the deconfinement phase transition on $N^3 \times N_\tau$ lattices by monitoring its order parameter and the corresponding susceptibility for $N_\tau = 4, 5, 6$ and 8 and $N_\sigma = 8, 10, 12, 14, 15$, and 16. We used the simple Metropolis algorithm and tuned it to have an acceptance rate $\sim 40\%$. The expectation values of the observables were recorded every 20 iterations to reduce the autocorrelations. Errors were determined by correcting for the autocorrelations and also by the jack knife method. The observables monitored were the average plaquette, $P$, and the absolute value of the average of the deconfinement order parameter $|L|$. We also monitored the susceptibilities

| $N_\sigma$ | $\beta$ | $\beta_{c,N_\sigma}$ | $\chi_{|L|}^{\text{max}}$ | $\chi_{|L|}^{\text{max}}$ |
|------------|--------|---------------------|-------------------------|-------------------------|
| 8          | 1.37   | 1.366(7)            | 9.34(7)                 |                         |
| 10         | 1.344  | 1.360(5)            | 14.34(11)               |                         |
| 12         | 1.331  | 1.345(2)            | 20.44(29)               |                         |
| 14         | 1.34   | 1.343(2)            | 27.48(64)               |                         |

for both $|L|$ and $P$: $\chi_{|L|} = N^3_\sigma \langle |L|^2 \rangle - \langle |L| \rangle^2$ and $\chi_P = 6N^3_\sigma N_\tau \langle (P^2) - \langle P \rangle^2 \rangle$.

2.1. $N_\tau = 4$

The deconfinement phase transition on $N_\tau = 4$ lattices was studied by first making short hysteresis runs on the smallest lattice to look for abrupt or sharp changes in the order parameter $\langle |L| \rangle$. In the region of its strong variation, longer runs were made to check whether the $|L|$-susceptibility exhibits a peak. Histogramming technique was used to extrapolate to nearby couplings for doing this. A fresh run was made at the $\chi_{|L|}$ peak position and the process repeated until the input coupling for the run was fairly close to the output peak position of the susceptibility. The same procedure was used for the bigger lattices also but by starting from the $\beta_c$ of the smaller lattice. Typically 2-4 million Monte Carlo iterations were used to estimate the magnitude of the peak height and the peak location for each $N_\tau$. Table 1 lists our final results for all the $N_\sigma$ used. Fitting the peak heights in Table 1 to $AN^2_\sigma$, we obtained $\omega = 1.93 \pm 0.03$, in excellent agreement with the values for both the standard Wilson action [3] and the 3-dimensional Ising model.

2.2. $N_\tau = 5, 6$ and 8

For larger $N_\tau$, we used many longer runs in the region of strong variation of $\langle |L| \rangle$ to obtain the
Same as Table 1 but for on $N^3_\sigma \times 6$ lattices.

| $N_\sigma$ | $\beta$ | $\beta_{c,N_\sigma}$ | $\chi_{|L|}^{\text{max}}$ | $\chi_{|L|}^{\text{predicted}}$ |
|------------|---------|----------------------|--------------------------|--------------------------|
| 12         | 1.75    | 1.735(5)             | 16.34(45)                | -                        |
| 15         | 1.70    | 1.702(2)             | 24.39(41)                | 25.13(70)                |

susceptibility directly and used the histogramming only for the finer determination of the critical coupling. Our results for $\langle |L| \rangle$ as a function of $\beta$ clearly show a deconfinement phase transition for $N_\tau = 5, 6$ and 8. This is also evident in the $\chi_{|L|}$ determinations, shown in Fig. 1 for $N_\tau = 6$.

Tables 2 and 3 list the estimated maxima of $\chi_{|L|}$

Figure 1. The $|L|$-susceptibility as a function of $\beta$ on lattices with $N_\tau = 6$. The continuous lines are extrapolations using the histogramming technique.

for $N_\tau = 5$ and 6 for two different spatial volumes along with the corresponding peak locations. Using our value for $\omega$, and the peak height for the smaller spatial volume, the $\chi^{\text{max}}$ on the bigger lattice can be predicted. These predictions are listed in the respective tables in the last column and can be seen to be in very good agreement with the direct Monte Carlo determinations. We also determined $\omega$ from the peak heights in Tables 2 and 3 and found it to be in good agreement with the $N_\tau = 4$ value. Not only is the universality of the deconfinement phase transition thus verified on three different temporal lattice sizes, but it also confirms that the same physical phase transition is being simulated on them, thus approaching the continuum limit of $a \to 0$ in a progressive manner by keeping the transition temperature $T_c$ constant in physical units.

### 2.3. Scaling of $T_c$

![Figure 2](image)

Figure 2. $1/N_\tau$ as a function of $\beta_c$. The full lines depict the 2-loop asymptotic scaling relation, while the dashed line denotes eq. (2), normalized at $N_\tau = 8$ in all cases.

Fig. 2 shows $aT_c = N_\tau^{-1}$ as a function of the corresponding critical $\beta$ for both our simulations (squares) with suppression of monopoles and vortices and the standard Wilson action (circles). The latter are taken from the compilation of Ref. [10]. The dashed line describes a ‘phenomenological’ scaling equation,

$$aT_c = \frac{1}{N_\tau} \propto \left( \frac{4b_0}{\beta} \right)^{-b_1/b_0^2} \exp \left( -\frac{\beta}{4b_0} \right),$$  \hspace{1cm} (2)
which is similar to the usual scaling relation, shown in Fig. 2 by full lines, but with twice the exponent. All lines were normalized to pass through the $N_\tau = 8$ data points.

One sees deviations from asymptotic scaling for both the Wilson action and our action with suppression of monopoles and vortices. The deviations for the same range of $N_\tau$ seem larger for our action but then one is also considerably deeper in the strong coupling region of the Wilson action where one a priori would not have even expected any scaling behaviour. As the agreement of our results with the dashed line of eq.(2) in Fig. 2 shows, scaling may hold in this region of $\beta$ for the suppressed action, since the relation between $a$ and $\beta$ in this region is similar to the asymptotic scaling relation, differing only in the exponent which will cancel in dimensionless ratios of physical quantities. It is clear that as $\beta \to \infty$, the difference between the two actions must vanish. The data in Fig. 2 do show such a trend although the limiting point is still not reached by $N_\tau = 8$. It seems likely though that the trend of evenly spaced transition points for our action will continue and the dashed line traced by its transition points will merge with the Wilson action by $N_\tau \sim 25$ or so. If this were to be so, a much smoother approach to continuum limit is to be expected after the suppression of monopoles and vortices. In particular, one expects that dimensionless ratios of physical quantities at the deconfinement phase transition couplings should be constant, already from $\beta \sim 1.33$, which is the transition point for the $N_\tau = 4$.

3. SUMMARY

The phase diagram of the Bhanot-Cruetz [8] action in the fundamental and adjoint couplings has been crucial in understanding many properties of the $SU(2)$ lattice gauge theory, like the cross-over to the scaling region from the strong coupling region. Adding extra irrelevant terms to the action one obtains a modified action (1) in which monopoles and vortices can be suppressed by setting the additional couplings to large values. Our finite size scaling analysis of $|L|$-susceptibility for $\beta_A = 0$, $\lambda_M = 1$ and $\lambda_E = 5$, yielded $1.93 \pm 0.03$ for the critical exponent $\omega \equiv \gamma/\nu$ for lattices with $N_\tau = 4$, thus verifying the naive universality. However, as a result of the suppression, the critical coupling is shifted by about unity compared to the Wilson case. Our results on $N_\tau = 5$ and 6 also yielded similar values for $\omega$ albeit with larger errors, confirming that the same physical transition was being studied this way as a function of the lattice cut-off, $a$. While the data for $N_\tau^{-1} = aT_c$ was found to vary slower with $\beta$ than expected from the asymptotic scaling relation for $N_\tau = 4$–8, they did obey a similar relation with a factor of two larger exponent. A straightforward extrapolation suggests that the results from the modified action may merge with those of Wilson action for large $N_\tau$ (of about $\sim 25$). Thus the suppression appears to make the transition from strong coupling to the scaling region much smoother than that for the unsuppressed Wilson action, allowing us to simulate the theory at smaller $\beta$. It will be interesting to see if dimensionless ratios of physical quantities such as glueball masses or string tension with $T_c$ are constant in the range of critical couplings explored here. It will also be interesting to study such suppression in $SU(3)$, and indeed $SU(N)$ lattice gauge theories, since their phase diagrams are similar and the same mechanism is expected to work for them as well.

REFERENCES