Incorporating Radial Flow in the Lattice Gas Model for Nuclear Disassembly

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Abstract

We consider extensions of the lattice gas model to incorporate radial flow. Experimental data are used to set the magnitude of radial flow. This flow is then included in the Lattice Gas Model in a microcanonical formalism. For magnitudes of flow seen in experiments, the main effect of the flow on observables is a shift along the $E^*/A$ axis.

25.70.Pq, 24.10.Pa, 64.60.My
It is expected that when nuclei disintegrate after heavy ion collisions, there will be a radial flow in the disintegrating system in addition to chaotic motion which is usually described by thermal motion. This was first proposed for collisions in the Bevalac [1] but is expected and seen in collisions at lower energies as well [2–4]. The amount of radial flow is larger for central collisions.

In this paper we address the issue of incorporating radial flow in statistical models for nuclear disassembly. This is automatically taken into account in models based on transport equations such as BUU (Boltzmann-Uehling-Uhlenbeck) [5]. But in many statistical models such as the SMM (statistical multifragmentation model [6]), thermodynamic model [7] or the microcanonical model [8] the flow can only be included \textit{a posteriori}. The idea here would be that the energy which is lost in radial flow is lost for thermalisation, thus essentially less energy is available for thermal disassembly. While this idea is certainly quite attractive, radial flow may do more than just take away energy. As far as we know this was first pointed out in [9]. It is this aspect that we study quantitatively here.

Some additional insight can be gained by incorporating a radial flow in the Lattice Gas Model (LGM) which is being applied more and more to fit experimental data [10,11]. In the usual formulation of the LGM, equilibrium statistical mechanics is done before composites are calculated [12]. We combine statistical mechanics with radial flow and then examine how it affects the composite production. The important issue here is the relative kinetic energy of two nearest neighbours. If this is less than the attractive bond, the two nearest neighbours will be part of the same cluster. Now, if one has a radial flow which diverges outward from the centre, the flow will affect the relative kinetic energy and hence the composite production. The argument of merely some energy being unavailable for thermalisation is strictly valid when the collective velocity is the same for every nucleon.

We are not claiming that radial flow arises in LGM in a fundamental way. But it can be included with a reasonable prescription. Inclusion of radial flow in a model similar to LGM was considered by Elattari et al [13]. Here we base our calculations on experimental data. The data used here is from a work by Williams et al. [4]. In that paper experimental
data of average radial flow is plotted as a function of excitation energy per nucleon ($E^*/A$). This is converted here to $E_f/A$ against $E^*/A$ (see Fig. 1) where $E_f/A$ is the flow energy per nucleon. Our calculation, by construct, will reproduce this curve. The model we use is this. From Fig. 1 we can also construct a $E_f/A$ against $E_{stat}/A$ where $E_{stat}/A = E^*/A - E_f/A$. The part $E_{stat}/A$ is generated by the LGM. In LGM we generate events which pertain to a $E_{stat}/A$. If there were no flow then from these events we would generate clusters and compare with experiments. But since experiments dictate that there is also energy tied up with flow we impose a flow energy on each event. The amount of flow energy is taken from the experimental $E_f/A$ against $E_{stat}/A$ curve. We take the flow velocity to be proportional to the distance from the center of mass of the exploding nucleus. Since in an event the position of each nucleon is known, this can be done uniquely. This is the principle of this calculation; below we provide some more details.

The most straightforward approach would be to use a microcanonical Lattice gas model. This is as simple as a canonical Lattice gas model calculation (see ref. [14]). Our simulations are done for $A = 84, N = 48, Z = 36$. We use a $6^3$ lattice. The neutron-proton bond is -5.33 MeV; like particle bonds are set at 0. Coulomb interaction between protons is taken into account as in [11]. We use Metropolis Monte-Carlo simulations to obtain microcanonical samplings. We start from a suitable initial lattice configuration which gives an interaction energy $E_{pot}$. The statistical energy $E_{stat}$ is fixed. The statistical kinetic energy for this configuration is then $E_{stat} + E_{ground} - E_{pot} = E_{kin}$. The phase space $\Omega_k(E_{kin})$ available for this kinetic energy is well-known:

\[ \Omega_k(E_{kin}) = \int d^3p_1 d^3p_2 \ldots d^3p_A \delta(E_{kin} - \sum \frac{p_i^2}{2m}) = \left(\frac{2\pi m}{1}^{3A/2}\right) (E_{kin})^{3A/2-1} \]

We now try to switch to a different configuration in the lattice. As a result, the potential energy in this new configuration would change to a new value $E'_{pot}$. In this configuration, since we are doing a microcanonical simulation, the statistical kinetic energy would have to adapt to a new value: $E'_{kin} = E_{stat} + E_{ground} - E'_{pot}$. Correspondingly, the new phase-space will be $\Omega_k(E'_{kin})$. If this is bigger than $\Omega_k(E_{kin})$, the switch is made. Otherwise the switch
is made with a probability $\frac{\Omega_k(E_{kin}')}{\Omega_k(E_{kin})}$. After many such switches (some successful and some not) we accept an event. We have to now assign momenta to the nucleons. Let the total statistical kinetic energy of the $N$ nucleons be $\tilde{E}_{kin}$ for the chosen event. This has to be shared between the nucleons based solely upon phase-space. This can be done following this procedure. Choose a sphere of radius $P$. Do a Monte-Carlo sampling on $N$ nucleons for uniform distribution in this sphere. This means fixing $p, \theta_p$ and $\phi_p$ for each particle from $p = P(x_1)^{1/3}, \cos \theta_p = 1 - 2x_2$ and $\phi_p = 2\pi x_3$ where $x_1, x_2$ and $x_3$ are random numbers in the domain 0 to 1. Finally normalize $P$ so that the total statistical kinetic energy equals $\tilde{E}_{kin}$.

If there were no flow, we would now do cluster decomposition for this event to compare directly with experiment. The standard prescription is that two nearest neighbor nucleons are part of the same cluster if the kinetic energy of relative motion is unable to overcome the attractive bond: $p_r^2/2\mu + \epsilon < 0$. To include flow, we add radial momenta to the statistical momenta already generated, thus get new momenta and then apply the prescription above for composites. The model for flow we adopt is that nucleons will have, apart from thermal motion, a momentum which is proportional to the distance from the centre of mass of the cluster: $\vec{p}_f(i) = c \times (\vec{r}(i) - \vec{r}_{cm})$. Here $\vec{r}(i)$ denotes the position of the nucleon in the lattice and $\vec{r}_{cm} = \frac{\sum \vec{r}(i)}{A}$. The constant $c$ is adjusted so that the total flow energy adds up to the pre-assigned value that we choose from the experimental $E_f$ vs. $E_{stat}$ curve. It should be pointed out that because we have many particles, the vector addition of flow momentum to the thermal momentum just leads to a scalar addition of energy, i.e., after addition of flow momenta the total energy remains, to a very good accuracy, $E_f + E_{stat}$. Thus we automatically satisfy the experimental data.

We obtain clusters both in the model just described and in a standard LGM microcanonical model without flow. To compare we extract an approximate exponent $\tau_Z$ ($Y(Z) \propto Z^{-\tau_Z}$) in both the models and plot it against $E^*/A$ (Fig.2). The main influence of the flow is a shift along the $E^*/A$ axis but this shift is not constant. In the same figure we also plot $\tau_Z$ against $E_{stat}/A$. If the only influence of the flow was to take out some energy but otherwise
leave the cluster production unchanged the two curves in the lower part of Fig. 2 would be on top of each other. The fact that the dashed curve trails the solid curve as a function of $E_{\text{stat}}/A$ supports the conjecture made in [9], this time in a fully microscopic model.

For the same value of $\tau_Z$ the cluster decomposition is basically the same (Fig.3)

In summary, we have incorporated the radial flow in nuclear Lattice Gas model with a reasonable prescription. For flow energy between 30 to 50% of thermal energy, as suggested by experiments, the inclusion of flow does not affect the fragment production in a profound way in intermediate energy heavy-ion collisions. While this calculation establishes the framework to include radial flow in the lattice gas model, in future we hope to do calculations for specific experimental cases including very large systems such as considered in [9].

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REFERENCES


Fig.1: Experimental data on flow energy per nucleon, $<E_f/A>$ plotted as a function of excitation energy per nucleon, $E^*/A$ from [4]. The error bars are not shown.
Fig. 2: The extracted values of the exponent $\tau_Z$, with and without flow, as a function of $E^*/A$ (upper panel). In doing the calculation with flow, we used interpolation between experimental points (Fig. 1) and obtained an analytical expression for $E_f/A$ against $E^*/A - E_f/A$. In the lower panel we plot $\tau_Z$ for both the models but for fixed $E_{stat}/A$. The fact that the two curves are not exactly on top of each other shows that flow does more than just takes away some energy.
Fig. 3: Charge yields of the system $A = 84$ with and without flow, at different excitation energies which have same exponents.
$Y(Z)$

- 0 flow ($E/A=10.30 \text{ MeV}$)
- with flow ($E/A=12.36 \text{ MeV}$)

- 0 flow ($E/A=15.55 \text{ MeV}$)
- with flow ($E/A=18.96 \text{ MeV}$)

$Z$-axis labels:
- $\tau_Z=2.0$
- $\tau_Z=3.9$