Dilepton production in proton-proton collisions at BEVALAC energies

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The dilepton production in elementary $pp \rightarrow e^+e^-X$ reactions at BEVALAC energies $T_{lab} = 1 \div 5$ GeV is investigated. The calculations include direct $e^+e^-$ decays of the vector mesons $\rho^0$, $\omega$, and $\phi$, Dalitz decays of the $\pi^0$, $\eta$, $\rho$, $\omega$, and $\phi$-mesons, and of the baryon resonances $\Delta(1232)$, $N(1520)$, $\ldots$. The subthreshold vector meson production cross sections in $pp$ collisions are treated in a way sufficient to avoid double counting with the inclusive vector meson production. The vector meson dominance model for the transition form factors of the resonance Dalitz decays $R \rightarrow e^+e^-N$ is extended to ensure correct asymptotics which are in agreement with the quark counting rules. By such a modification unified and consistent description of both $R \rightarrow N\gamma$ radiative decays and $R \rightarrow N\rho(\omega)$ meson decays is obtained without introducing new parameters. This provides evidence for the validity of the quark counting rules in the baryon resonance sector. The effect of multiple pion production on the experimental efficiency for the detection of the dilepton pairs is studied. We found the dilepton yield in reasonable agreement with the experimental data for the set of intermediate energies whereas at the highest energy $T_{lab} = 4.88$ GeV the number of dilepton pairs is likely

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to be overestimated experimentally in the mass range $M = 300 \div 700$ MeV.
We thus cannot exclude that the origin of the so called "DLS puzzle" can be
traced back to the elementary $pp$ level and therefore is not a specific feature
of heavy-ion collisions. Further the modification of the $pp \to e^+e^-X$ cross
section due to a decoherence of the vector meson propagation inside a hot
and dense nuclear medium is discussed.

**keywords**: pp collisions, transition form factors, dileptons

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I. INTRODUCTION

Relativistic heavy-ion collisions present an unique possibility to create nuclear matter at high densities and temperatures where the hadron properties become different and the phase transition to quark matter with signatures of the deconfinement and restoration of chiral symmetry is expected. The change of the nucleon mass in the nuclear matter was implemented into the Walecka model [1,2] already in 1970’s in the framework of the effective hadron field theory. Later on this effect was put on the firmer grounds on the basis of a partial restoration of the chiral symmetry and finite-density QCD sum rules [3,4], the Nambu-Jona-Lasinio model [5], and effective meson field theory [2,6,7].

Dileptons ($e^+e^-, \mu^+\mu^-$) are the most clear probes of the high density nuclear matter. The reason is that the dileptons interact with the matter only by electromagnetic forces and can therefore leave the heavy-ion reaction zone essentially undistorted by final state interactions. They provide valuable information on the in-medium properties of hadrons and, hence, on the state of matter.

The dilepton spectra from heavy-ion collisions have been measured at two drastically different energies: by the CERES and HELIOS-3 Collaborations at SPS [8,9] (a few hundreds GeV per nucleon) and by the DLS Collaboration at BEVALAC [10] (a few GeV per nucleon). In the CERES and HELIOS-3 experiments and in the BEVALAC experiment the production of dileptons with invariant masses between $300 \div 700$ MeV is found to be enhanced as compared to estimates based on the theoretically known dilepton sources when in-medium modifications of hadron properties are neglected.

The data on the total photoabsorption cross section on heavy nuclei [11] give an evidence for the broadening of nucleon resonances in the nuclear medium [12]. The physics behind is the same as in the collision broadening of the atomic spectral lines in hot and dense gases, discussed by Weisskopf in the early 1930’s [13] (for the present status of this field, see, e.g., [14]). This rather general effect which takes its origin from the atomic spectroscopy should also lead to a broadening of the $\rho$-mesons in heavy-ion collisions. Indeed, it was found to be
sufficient to account for the CERES and HELIOS-3 data [15,16].

In the DLS experiment a different temperature and density regime is probed. The enhancement of the dilepton spectra due to the reduction of the $\rho$-meson mass and the $\rho$-meson broadening is not sufficient to bring theoretical estimates in coincidence with the available experimental data [17]. The in-medium $\rho$-meson scenarios that successfully explain the dilepton yield at SPS energies fail for the DLS data. This phenomenon was called the "DLS puzzle". It worthwhile to notice that the final data from the DLS Collaboration have changed by about a factor of $5 - 7$ as compared to the initially reported results. The future HADES experiment at GSI will study the dilepton spectra at the same energy range in greater details [18].

A possibility to clarify the origin of the DLS puzzle has appeared since data from elementary $pp$ ($pd$) collisions at $T = 1 \div 5$ GeV ($T$ is the kinetic energy of the incident proton in the laboratory frame) became available from the DLS Collaboration [19]. The elementary cross sections enter as an input into the transport simulations of heavy-ion collisions, so their better understanding is of great value.

The dilepton spectra in the $pp$ collisions at $T = 1 \div 5$ GeV are calculated in Refs. [20–22]. In Ref. [20] the agreement achieved with the DLS data is generally good at low energies where the subthreshold production of nucleon resonances is important. When the energy increases and the inclusive production becomes dominant, the dilepton yield is underestimated at the same mass range $300 \div 700$ MeV as in the heavy-ion collisions. A signature for this effect exists also in the calculations of Refs. [21,22]. This can be interpreted to mean that the studies [20–22] revealed, apparently, the reoccurrence of the DLS puzzle on the elementary level of the nucleon collisions. They leave, therefore, a doubt on the quality of the experimental data and/or the reliability of the accepted theoretical schemes.

This paper is devoted to a further going theoretical analysis of the elementary dilepton production cross sections.

In the next Sect., the production mechanisms are critically revisited. The subthreshold production cross sections for the vector mesons are treated such that no double counting
appears with the inclusive processes. The effect of multiple pion production on the experimental detection efficiency for the dilepton pairs is also studied. We demonstrate that the detector efficiency is sensitive to the number of pions produced and propose a simple model to account for the multiple pion production effects.

In the Dalitz decays of the nucleon resonances, $R \to Ne^+e^-$, the Vector Meson Dominance (VMD) model is usually applied for description of the resonance transition form factors. However, the naive VMD which takes the $R \to N\rho$ data as an input fails to reproduce the radiative decay branchings. In Sect.3, it is extended to ensure the correct asymptotic behavior of the transition form factors in agreement with the quark counting rules. Such a modification is sufficient to achieve an unified description of both, $R \to N\gamma$ radiative decays and $R \to N\rho(\omega)$ meson decays without the introduction of new parameters. This provides evidence for validity of the quark counting rules in the baryon resonance sector. Our estimates of the subthreshold cross sections rely therefore on the two essentially different sets of experimental data, $R \to N\gamma$ and $R \to N\rho$. In Sect.4, we discuss a modification of the $pp \to e^+e^-X$ cross section, connected to the decoherence of the propagating vector mesons in a hot and dense nuclear medium.

The numerical results are discussed in Sect.5. We found that the above improvements do not eliminate the discrepancy with the DLS data at $T = 4.88$ GeV. Moreover, the results for the lowest energy, $T = 1.04$ GeV, also require an additional study from the experimental and/or theoretical side.

**II. $PP \to E^+E^-X$ REACTION**

The dilepton production in nucleon collisions goes through the production of virtual photons which decay subsequently into $e^+e^-$ pairs. According to the VMD model, the virtual photons are coupled to vector mesons $V = \rho^0, \omega$, and $\phi$. The dilepton production can therefore be calculated using the inclusive vector meson production cross sections:
\[
\frac{d\sigma(s, M)_{pp-e^+e^-X}}{dM^2} = \sum_{V} (1 + n_V) \frac{d\sigma(s, M)_{pp-VX}}{dM^2} B(M)_{V\rightarrow e^+e^-}.
\]

Here, \(s\) is the square of the invariant mass of two colliding protons, \(M\) the invariant mass of the dilepton pair, \(d\sigma(s, M)_{pp-VX}/dM^2\) is the differential inclusive vector meson production cross section, and \(n_V\) is the average number of additional vector mesons \(V\) in the state \(X\).

In the energy range of interest, \(T = 1 \div 5\) GeV, where \(T\) is the kinetic energy of the proton in the laboratory system, \(n_V = 0\). The branching ratio

\[
B(M)_{V\rightarrow e^+e^-} = \frac{\Gamma(M)_{V\rightarrow e^+e^-}}{\Gamma_{tot}(M)}
\]

corresponds to the direct \(V \rightarrow e^+e^-\) decays, with \(\Gamma_{tot}(M)\) being the total meson decay width.

The cross section entering into Eq.(1) can be decomposed into pole and background parts:

\[
d\sigma(s, M)_{VX} = d\sigma(s, M)_{VX}^P + d\sigma(s, M)_{VX}^B.
\]

The distribution over the meson mass \(M\) in the pole part of the cross section has a Breit-Wigner form corrected to the available phase space for the final state \(VX\).

At moderate energies, the state \(X\) is dominated by two nucleons and pions, so one can write

\[
d\sigma(s, M)_{VX}^P = \sigma(s)_{VX}^P \frac{1}{\pi (M^2 - m^2_{V})^2 + (M\Gamma_{tot}(M))^2} \sum_{n=0}^{N} w_n C_n \Phi_{3+n}(\sqrt{s})
\]

where

\[
\Phi_{3+n}(\sqrt{s}) = \Phi(\sqrt{s}, m_N, m_N, M, \mu_\pi, ..., \mu_\pi)
\]

is the \((3 + n)\)-body phase space of the final state (two nucleons with masses \(m_N\), one vector meson \(V\) with mass \(m_V\), and \(n\) pions with masses \(\mu_\pi\)). The value \(N_\pi = \lfloor(\sqrt{s} - 2m_N - m_V)/\mu_\pi\rfloor\) is the maximal number of pions allowed by energy conservation, \([x]\) denotes the integer value of \(x\), and the values \(w_n\) are the probabilities for the production of \(n\) pions, with
The normalization factor $C_n$ is given by

$$C_n^{-1} = \int_{\mu_0^2} \frac{1}{\pi (M^2 - m_V^2)^2 + (M \Gamma_{\text{tot}}^V(M))^2} \Phi_{3+n}(\sqrt{s}...)(7)$$

where $\mu_0$ is the physical threshold for vector meson decays ($\mu_0 = 2\mu_\pi$ for the $\rho$-meson).

Notice that the cross section (3) vanishes at values $M < \mu_0$. However, the total width $\Gamma_{\text{tot}}^V(M)$ entering into the denominator of the branching ratio (2) at $M < \mu_0$ vanishes as well, so that the cross section (1) is actually finite everywhere above the two-electron mass.

In the zero-width limit, $\Gamma_{\text{tot}}^V(M) = 0$, Eq.(4) simplifies to give

$$d\sigma(s, M)_P^{VX} = \sigma(s)_P^{VX} \delta(M^2 - m_V^2) dM^2.$$  

(8)

The finite-width effects are important for the $\rho$-meson and less important for $\omega$- and $\phi$-mesons.

Experimental data on the exclusive cross sections $\sigma(s)_P^{VX}$ with $X = n\pi NN$ at $n \geq 1$ are not available. Here we assume that the probabilities $w_n$ are described by a binomial distribution

$$w_n = \frac{N_\pi!}{n!(N_\pi - n)!} p^n (1 - p)^{N_\pi - n}.$$  

(9)

To fix all probabilities it is sufficient to know the ratio between the exclusive vector meson production cross section $\sigma(s)_P^{VNN}$ and the inclusive cross section $\sigma(s)_P^{VX}$. These two cross sections are experimentally known [23]. The value $N_\pi$ is defined as above by the energy conservation while the value $p$ can be extracted from the relation

$$\frac{\sigma(s)_P^{VNN}}{\sigma(s)_P^{VX}} = (1 - p)^{N_\pi}.$$  

(10)

The pion multiplicity equals

$$n_\pi = \sum_{n=0}^{N_\pi} n w_n = p N_\pi.$$  

(11)
In the case of the $\rho$-meson, the cross section $\sigma(s)^{V_X}_p$ determines the pole behavior of the total cross section $d\sigma(s, M)^{\pi^+\pi^-X}$ in the vicinity of the $\rho$-meson peak. Like for vector mesons, Eq.(3), the total cross section $d\sigma(s, M)^{\pi^+\pi^-X}$ for the $2\pi$ production can be decomposed into pole and background parts

$$d\sigma(s, M)^{\pi^+\pi^-X} = d\sigma(s, M)^{\rho^0X}_P + d\sigma(s, M)^{\pi^+\pi^-X}_B.$$  \(12\)

This decomposition is not unique. The $\rho$-meson contribution, however, can always be parametrized in a reasonable way. The two terms on the right side of Eq.(12) are thus fixed, but the total cross sections with two pions or a $\rho$-meson in the final state must not be equal, since the $\pi^+\pi^-$ quantum numbers are not necessarily equal to the quantum numbers of the $\rho$-meson (this is the case at the $\rho$-meson peak only and this is why with $V = \rho^0$ the first terms in Eq.(3) and Eq.(12) coincide). Since the $\rho$-meson is always detected via $2\pi$ final states (or via $3\pi$ and $2K$ final states for $\omega$- and $\phi$-mesons, respectively), the inclusive cross section for the production of vector mesons which enters into Eq.(1) cannot be uniquely determined from the experimental side. Instead, one needs a model for the calculation of the background part of the cross section in Eq.(3). A subtle problem of double counting in the total dilepton production cross section appears in this way. We discuss and propose a phenomenological solution for it.

The background term $d\sigma(s, M)^{\rho^0X}_B$ at $X \neq NN$ can be saturated, at least partially, by considering the production of light mesons: $pp \to \eta X \to \rho^0\gamma X \to \pi^+\pi^-\gamma X$, $pp \to \omega X \to \rho^0\pi^0 X \to \pi^+\pi^-\pi^0 X$, etc., similarly for the $\omega$- and $\phi$-mesons. The $\pi^+\pi^-$ invariant masses are small here, so these processes contribute to the $\pi^+\pi^-$ background. Eq.(1) can be rewritten as follows

$$d\sigma(s, M)^{e^+e^-X}_e = d\sigma(s, M)^{e^+e^-X}_P + d\sigma(s, M)^{e^+e^-X}_B \big|_{X=NN} + d\sigma(s, M)^{e^+e^-X}_B \big|_{X\neq NN}.$$  \(13\)

The first term is the same as in Eq.(1), i.e. with the sum running over all vector mesons, however, keeping only the pole part of the cross section. In Eq.(13) the background is divided into contributions from direct decays of intermediate vector mesons, which are off-shell and
typically below their physical thresholds \((X = NN)\) and from decays of intermediate mesons, \(M\), to multi-particle final states \((X \neq NN)\). In the latter case, the cross section for the production of the intermediate meson has to be folded over its branching ratio to the final state under consideration:

\[
\frac{d\sigma(s, M)^{e^+e^-X}}{dM^2} \bigg|_{X \neq NN} = \sum_{M} \int d\mu^2 (1 + n_M) \frac{d\sigma(s, \mu)^{MX'}}{d\mu^2} \frac{dB(\mu, M)^{M \rightarrow e^+e'^X}}{dM^2}.
\]

The sum runs over the mesons \(M = \pi, \eta, \rho, \omega, \) and \(\phi\). Here, \(n_M\) is the average number of mesons \(M\) in the state \(X'\). The value \(\mu\) in the last equation describes the distribution over the off-shell masses of the mesons. For pseudo-scalar mesons, the cross sections due to their small widths are proportional to the delta-function \(\delta(\mu^2 - m^2_M)\) (cf. Eq.(8)) and the expression reduces to a sum over the on-shell mesons decaying to the states \(e^+e'^X''\) with \(X'' \neq \emptyset, X = X' + X''\). For vector mesons entering the sum of Eq.(14) one should use the cross sections (3) whose pole components are well defined.

The contribution to the background part of the cross section (1) with \(X = NN\) can be calculated assuming that it results from subthreshold decays of baryon resonances \(R = \Delta(1232), N(1520), \ldots\) produced in \(pp\) collisions, which decay into nucleons and vector mesons, \(R \rightarrow NV\) [22]. In terms of the branching ratios for the Dalitz decays of the baryon resonances, the cross section can be written as follows

\[
\frac{d\sigma(s, M)^{e^+e^-X}}{dM^2} \bigg|_{X = NN} = \sum_{R} \int \frac{(\sqrt{s} - m_N)^2}{(m_N + M)^2} d\mu^2 \frac{d\sigma(s, \mu)^{pp \rightarrow pR}}{d\mu^2} \sum_{V} dB(\mu, M)^{R \rightarrow VP \rightarrow e^+e^-p} \cdot \frac{dM^2}{dM^2} S_V(M^2).
\]

Here, \(\mu\) is the running mass of the baryon resonance \(R\) with the cross section \(d\sigma(s, \mu)^{pp \rightarrow pR}\), \(dB(\mu, M)^{R \rightarrow VP \rightarrow e^+e^-p}\) is the differential branching ratio for the Dalitz decay \(R \rightarrow e^+e^-p\) through the vector meson \(V\).

With increasing energy, the vector mesons in Eq.(15) are produced above their physical masses. In such a case, the subthreshold processes contribute to the pole part of the cross section \(d\sigma(s, M)^{e^+e^-X}\), so that the basic requirement of a saturation of the background from the subthreshold production is violated. Expression (15) for the background part of the cross
section (1) with $X = NN$ is valid as long as the energy of the colliding protons is low enough, $T \lesssim 1.85$ GeV. The inclusive cross sections by definition accounts for all possible sources for the appearance of on-shell vector mesons, so that a naive extension of the subthreshold cross section to higher energies would result in a double counting. We thus introduce in Eq.(15) under the sign of the integral a phenomenological suppression factor

$$S_V(M^2) = \tanh \left( \frac{(M^2 - m_V^2)^2 + (m_V \Gamma_{tot}(m_V))^2}{m_V^2 \Gamma_{tot}(m_V)} \right). \quad (16)$$

In the vicinity of the pole, its role reduces to the elimination of the vector meson Breit-Wigner peak. This apparent source of the double counting is thus excluded. Away from the pole, the suppression factor is close to unity. It does not change therefore results in the region $T \lesssim 1.85$ GeV where the inclusive cross section is zero and the double counting problem does not exist.

The meson multiplicities $n_M$ in Eq.(14) are set equal to zero except for the neutral pions. For pions, we assume $n_{\pi^0} = \frac{1}{2} n_\pi$ where $n_\pi$ is given by Eq.(11). The factor $\frac{1}{2}$ appears statistically in the limit $n \to \infty$ from charge conservation which implies that in every channel the total number of the neutral-pion pairs $\pi^0 \pi^0$ must be equal to the number of the charged-pion $\pi^+ \pi^-$ pairs.

There are no experimental data on the pion production inclusive cross section $\sigma(s, \mu_\pi)_{pp \rightarrow \pi^0 X}$ at $T = 1 \div 5$ GeV. There exist, however, experimental data on the two-pion production cross sections $\sigma(s, \mu_\pi)_{pp \rightarrow \pi \pi NN}$ [24]. The parameter $p$ of the pion number distribution can therefore be estimated from equation

$$\frac{\sigma(s)_{\pi^0 \pi NN}}{\sigma(s)_{\pi^0 pp}} = \frac{pN_\pi}{1 - p}. \quad (17)$$

It can further be used to find the total inclusive cross section $\sigma(s, \mu_\pi)_{pp \rightarrow \pi^0 X}$ using the same relation as in Eq.(10). The results for the pion multiplicities produced in reactions in accordance with different mesons $M$ are summarized in Table 1 for those values of kinetic energies, $T$, at which the dilepton cross sections are measured by the DLS collaboration. We give there also the cross sections obtained from the interpolation and/or extrapolation
between the available experimental points. The accuracy of these estimates is rarely better than 20%. For the pion, we give an estimate for the inclusive cross section. Notice that it is in a reasonable agreement with the prediction from the UrQMD transport [20]. For mesons $M = \eta, \rho, \omega, \phi$, estimates for the cross sections $pp \rightarrow M\pi NN$ are given as derived from the distribution (9).

Table 1. The maximum number of pions, $N_\pi$, the pion multiplicities, $n_\pi$, and the cross sections $\sigma^{M\pi NN}, \sigma^{MNN},$ and $\sigma^{MX}$ for the production of mesons $M = \pi^0, \eta, \rho, \omega, \phi$ in the proton-proton collisions for the set of energies $T = 1.04, 1.27, 1.61, 2.09, 4.88$ GeV, at which the dilepton production cross section has been measured by the DLS Collaboration. The numbers marked by a symbol "#" are predictions based on the statistical distribution (9).

<table>
<thead>
<tr>
<th>$T$ [GeV]</th>
<th>1.04</th>
<th>1.27</th>
<th>1.61</th>
<th>1.85</th>
<th>1.85</th>
<th>2.09</th>
<th>2.09</th>
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<td>4</td>
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<td>2</td>
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<td>6</td>
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</tr>
<tr>
<td>$n_\pi$</td>
<td>0.09</td>
<td>0.20</td>
<td>0.56</td>
<td>0.74</td>
<td>0.26</td>
<td>1.07</td>
<td>0.55</td>
<td>1.80</td>
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<td>1.67</td>
<td>1.36</td>
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<td>$\sigma^{M\pi NN}$ [mb]</td>
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<td>0.9</td>
<td>2.7</td>
<td>3.4</td>
<td>0.05 (#)</td>
<td>4.9</td>
<td>0.11 (#)</td>
<td>5.8</td>
<td>0.34 (#)</td>
<td>0.28 (#)</td>
<td>0.35 (#)</td>
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<tr>
<td>$\sigma^{MNN}$ [mb]</td>
<td>4.5</td>
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<td>3.9</td>
<td>3.8</td>
<td>0.13</td>
<td>3.6</td>
<td>0.14</td>
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<tr>
<td>$\sigma^{MX}$ [mb]</td>
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<td>5.2 (#)</td>
<td>7.3 (#)</td>
<td>8.5 (#)</td>
<td>0.17</td>
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<td>0.27</td>
<td>19 (#)</td>
<td>1.19</td>
<td>0.94</td>
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</table>

For the mesonic dilepton decays of mesons $M \rightarrow e^+e^-X$, the experimental data exist in the most cases. The radiative decays $M \rightarrow \gamma X$ can further be used to calculate the dilepton decays when the experimental data are not available. For details of the determination of the various branching ratios see Ref. [25]. The inclusive cross sections for the meson production and the branching ratios for the mesonic decays to dileptons can be combined to estimate the dilepton yield from Eq.(13).

The differential decay branchings $dBR^{N_{e^+e^-}(\mu, M)}/dM^2$ are calculated in Ref. [22] in a non-relativistic approximation for the multiple decays with the emission of a massive vector particle. We follow a similar approach here, however, with a modification of transition
form factors for the nucleon resonances, \( R \), needed to bring their asymptotic behavior in the coincidence with the quark counting rules and to provide an unified description of the radiative and vector meson decays, \( R \to N \gamma \) and \( R \to N \rho(\omega) \).

III. TRANSITION FORM FACTORS, QUARK COUNTING RULES, AND RADIATIVE AND VECTOR MESON DECAYS OF NUCLEON RESONANCES

The description of the resonance decays \( R \to N \gamma^*, \gamma^* \to e^+e^- \) is usually based on the VMD model which provides transition form factors \( R N \gamma \) of a monopole form. The pole corresponds to the masses of the \( \rho \)- and \( \omega \)-mesons. This model should give, in principle, an unified description of the radiative \( R N \gamma \) and the mesonic \( R N V \) decays. However, a normalization to the radiative branchings \( (R N \gamma) \) strongly underestimates the mesonic branchings \( (R N V) \) as we discuss below.

The resonance \( N(1520) \) is a case for which both, the \( N(1520) \to N\rho \) and \( N(1520) \to N\gamma \) widths are known with a relatively high precision: \( B(N(1520) \to N\rho) = 15 \div 25\% \), \( B(N(1520) \to N\gamma) = 0.46 \div 0.56 \% \) (\( p\gamma \) mode), \( 0.30 \div 0.53 \% \) (\( n\gamma \) mode). The branching ratios of the proton and neutron modes are equal within the experimental errors. This can be interpreted to mean that the radiative mode is dominated either through the \( \rho \)-meson or the \( \omega \)-meson. The same conclusion is reasonable for other \( N^* \) resonances: \( B(N(1440) \to N\gamma) = 0.035 \div 0.048 \% \) (\( p\gamma \) mode), \( 0.009 \div 0.032 \% \) (\( n\gamma \) mode); \( B(N(1535) \to N\gamma) = 0.15 \div 0.35 \% \) (\( p\gamma \) mode), \( 0.004 \div 0.29 \% \) (\( n\gamma \) mode), etc. The \( \Delta \) decays, on the other hand, proceed exclusively through the \( \rho \)-meson.

However, now the standard VMD model as it has been used in [22] leads to a severe inconsistency: Using the coupling constant \( f_{N(1520)N\rho} = 7.0 \) extracted from the mesonic \( N(1520) \to N\rho \) decay, the branching ratio for the radiative decay can be found to be two to three times greater than the experimental value. Analogous overestimations are observed almost for all other \( N \) and \( \Delta \) resonances for which the experimental \( N\rho \) and \( N\gamma \) data are available. Table 2 summarizes the results.
Table 2. The coupling constants $f_{RN\rho}$ derived from the $R \to N\rho$ mesonic decays are compared to the coupling constants $f_{RN\rho}^\gamma$ fixed from the radiative $R \to N\gamma$ decays. The numerical values $f_{RN\rho}$ are taken from Ref. [16], with exception of the $\Delta(1232)$ resonance for which the theoretical value from [15] is given and of the $N(1440)$ and $N(1535)$ resonances where the results of our calculations are given.

<table>
<thead>
<tr>
<th>$R$</th>
<th>$N(1440)$</th>
<th>$N(1520)$</th>
<th>$N(1535)$</th>
<th>$N(1650)$</th>
<th>$N(1680)$</th>
<th>$N(1720)$</th>
<th>$\Delta(1232)$</th>
<th>$\Delta(1620)$</th>
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<td>$\frac{3}{2}^+$</td>
<td>$\frac{3}{2}^+$</td>
<td>$\frac{1}{2}^-$</td>
<td>$\frac{3}{2}^-$</td>
<td>$\frac{5}{2}^+$</td>
</tr>
<tr>
<td>$f_{RN\rho}$</td>
<td>$&lt; 26$</td>
<td>$7.0$</td>
<td>$&lt; 2.0$</td>
<td>$0.9$</td>
<td>$6.3$</td>
<td>$7.8$</td>
<td>$15.3$</td>
<td>$2.5$</td>
<td>$5.0$</td>
<td>$12.2$</td>
</tr>
<tr>
<td>$f_{RN\rho}^\gamma$</td>
<td>$1.3$</td>
<td>$3.8$</td>
<td>$1.8$</td>
<td>$&lt; 0.8$</td>
<td>$3.9$</td>
<td>$2.2$</td>
<td>$10.8$</td>
<td>$0.7$</td>
<td>$2.7$</td>
<td>$2.1$</td>
</tr>
</tbody>
</table>

The standard VMD predicts a $1/t$ asymptotic behavior for the transition form factors. However, quark counting rules require a stronger suppression at high $t$. It is known from the nucleon form factors, the pion form factor, and the $\omega\pi\gamma$ and $\rho\pi\gamma$ transition form factors that the quark counting rules start to work experimentally at moderate $t \sim 1 \text{ GeV}^2$. One can assume that an appropriate modification of the standard VMD which takes the correct asymptotics of the $RN\gamma$ transition form factors into account can provide a more accurate description of the radiative decays of the nucleon resonances.

We propose the following solution of the inconsistency between the $RN\nu$ and $RN\gamma$ decay rates: Let radial excitations of the $\rho$-meson, the $\rho(1450)$-meson and $\rho(1700)$-meson, interfere with the $\rho$-meson in radiative processes. However, we know neither the couplings of the $\rho(1450)$ and $\rho(1700)$ to the resonances ($f_{RN\rho}$, $f_{RN\rho}'$) nor the couplings of the $\rho(1450)$ and $\rho(1700)$ to a photon ($g_{\rho}$, $g_{\rho}'$). Thus in the sum

$$M(M^2) = \sum_{i=1}^{3} M_i = \frac{f_{RN\rho}}{m_\rho} \frac{m_\rho^2}{g_{\rho}} \frac{1}{m_\rho^2 - M^2} + \frac{f_{RN\rho}'}{m_\rho'} \frac{m_\rho'^2}{g_{\rho}'} \frac{1}{m_\rho'^2 - M^2} + \frac{f_{RN\rho}''}{m_\rho''} \frac{m_\rho''^2}{g_{\rho}''} \frac{1}{m_\rho''^2 - M^2},$$

(18)

where $\tilde{m}_k^2 = m_k^2 - iM_k$ with $k = \rho, \rho'$, and $\rho''$, $\rho'$ and $\rho''$ refer to $\rho(1450)$- and $\rho(1700)$-mesons, respectively, the coefficients $\frac{f_{RN\rho}}{m_\rho} \frac{m_\rho^2}{g_{\rho}}$ and $\frac{f_{RN\rho}'}{m_\rho'} \frac{m_\rho'^2}{g_{\rho}'}$ are unknown. According to
the quark counting rules [26,27], for large and negative $M^2$ the form factors of the $RN\gamma^*$ amplitudes decrease like $1/M^6$. On the phenomenological level we can attribute such a behavior to a cancellation between the $\rho$, $\rho'$, and $\rho''$-mesons. The constants $f_{RN\rho} m_\rho^2/g_\rho$ and $f_{RN\rho'} m_{\rho'}^2/g_{\rho'}$ are then fixed and we obtain

$$\mathcal{M}(M^2) = \frac{f_{RN\rho} m_\rho^2}{m_\rho g_\rho} \frac{1}{\bar{m}_\rho^2 - M^2} \left( \frac{\bar{m}_{\rho'}^2 - \bar{m}_\rho^2}{\bar{m}_{\rho'}^2 - M^2} \right) \left( \frac{\bar{m}_{\rho''}^2 - \bar{m}_{\rho'}^2}{\bar{m}_{\rho''}^2 - M^2} \right). \quad (19)$$

The last two factors in Eq.(19) give the desired modification of the $\rho$-meson contribution to the radiative decays of the baryon resonances, as compared to the naive VMD model:

$$d\Gamma(R \rightarrow N e^+ e^-)(\mu, M) = d\Gamma(R \rightarrow N e^+ e^-)(\mu, M)^{(\text{naive VMD})} F_\rho(M^2). \quad (20)$$

The mass-dependent correction factor is given by

$$F_\rho(M^2) = \left| \frac{\bar{m}_{\rho'}^2 - \bar{m}_\rho^2}{\bar{m}_{\rho'}^2 - M^2} \right|^2. \quad (21)$$

The same modification applies to the $R \rightarrow N\gamma$ decays. The reduction factor in the amplitude $R \rightarrow N\gamma$ equals $\sqrt{F_\rho(M^2 = 0)} = 0.56$. It is seen from Table 2 that a reduction of about $\frac{1}{2}$ is just what one needs for a consistent description of both, the $\rho$-meson and the radiative decay of the $N(1520)$. In all other cases the reduction factor also improves the agreement.

The unobserved $RN\rho$ mode of the resonance $N(1535)$ decay implies apparently that the $RN\gamma$ decay goes over an intermediate $\omega$-meson. In the case of the $\Delta(1905)$ resonance, the large difference between $f_{RN\rho}$ and $f_{RN\rho}^\gamma$ can be attributed to a further suppression of the amplitude $A(M^2)$ due to the quark counting rules which require a $1/M^8$ behavior of the $RN\gamma^*$ vertex for the $\frac{3}{2}^+ \rightarrow \frac{1}{2}^+$ transition.

It is seen from Table 2 that the resonances $N(1440)$ and $N(1535)$ do not have experimentally observed $N\rho$ modes. We use therefore the $N\gamma$ decays to fix the $N\rho$ coupling constants $f_{N(1440)N\rho} = 1.3/0.56$ and $f_{N(1535)N\rho} = 1.8/0.56$. Since $f_{N(1535)N\rho}$ is greater than 2, we set $f_{N(1535)N\rho} \approx 0$ and introduce the $\omega$-meson coupling $f_{N(1535)N\omega} = \frac{2\omega}{g_\rho} 1.8/0.56$. Taking into account that the production cross sections of the $N(1440)$ and $N(1535)$ resonances are significant we expect a noticeable contribution from these two resonances.
The \(\Delta(1232)\) resonance is treated in the same way as the other resonances. Finally, we include 10 resonances listed in Table 2. The numerical results demonstrate that besides the \(N(1520)\) and \(\Delta(1232)\) resonances, the \(N(1535)\) and \(\Delta(1620)\) have considerable contributions. It can be seen from Figs.1 and 2 where the resonance contributions are shown for two proton kinetic energies \(T = 1.61\) GeV and \(T = 2.09\) GeV. At moderate invariant masses \(M \lesssim 0.5\) GeV of the dilepton pair, the resonance contributions are dominated by the \(\Delta(1232)\). At larger masses \(M \gtrsim 0.5\) GeV, contributions from the heavier resonances become dominant. In the case of the \(N(1535)\)-resonance, the narrow \(\omega\)-peak reflects the coupling to the \(\omega\)-meson.

In Ref. [21], the \(N(1520)\) was considered as the most important source for the subthreshold dilepton production. We found that the \(N(1520)\) contribution to the cross section is smaller at \(T = 1.61\) GeV by about a factor of 10 as compared to the results of Ref. [21]. The reduction occurs due to the consistent treatment of the radiative and meson decays and, secondly, from the proper use of the Breit-Wigner distribution (see Appendix 1). In the total spectrum, the reduced \(N(1520)\) contribution is compensated by contributions from two other resonances, \(N(1535)\) and \(\Delta(1620)\), which are the dominant modes at large \(M\) for \(T = 2.09\) GeV (see Fig.2).

IV. IN-MEDIUM EFFECT TO THE \(PP \rightarrow E^+E^-X\) CROSS SECTION: DECOHERENCE

It is well known that hadrons change their properties in the medium. Many studies are devoted to this subject (for a review see [7]). In the present context we want to discuss one particular aspect of the in-medium modification of the cross section \(pp \rightarrow e^+e^-X\), connected with the decoherence of vector mesons propagating in a hot and dense nuclear medium.

In the previous Sect., we introduced two radially excited \(\rho\)-mesons \(\rho(1450)\) and \(\rho(1700)\) in the transition form factors \(RN\gamma\) to ensure the correct asymptotic behavior of the amplitudes in line with the quark counting rules. Thereby we required the destructive interference of the ground-state and the excited \(\rho\)-mesons. The net effect of this modification results for
the corresponding branching ratios in the additional factor

\[ F_\rho(M^2) = \left| \sum_i \mathcal{M}_i(M^2) / \mathcal{M}_1(M^2) \right|^2. \]  

(22)

The summation runs over the mesons \( \rho, \rho', \) and \( \rho'' \).

In heavy-ion collisions, the interference between the virtual \( \rho, \rho', \) and \( \rho'' \)-resonances produced in the nucleon resonance decays can be destroyed at least partially by interactions with the hot and dense nuclear medium. In the case of total decoherence, the \( \rho, \rho', \) and \( \rho'' \) contributions to the subthreshold part of the cross section \( pp \rightarrow e^+e^-X \) must be summed up decoherently which leads to the replacement

\[ \left| \sum_i \mathcal{M}_i(M^2) \right|^2 \rightarrow \sum_i \left| \mathcal{M}_i(M^2) \right|^2. \]  

(23)

Thus, the total decoherence results in an enhancement of the resonance contributions due to the presence of the nuclear medium.

It is more adequate, however, to assume that each of the propagated \( \rho \)-mesons radiates \( e^+e^- \) pairs coherently up to the first collision with a nucleon in the medium and incoherently afterwards. This leads to the destruction of the coherence of one meson with the other pair which, itself, may still be in a coherent state.

Now, let’s try to give a more quantitative estimate. Let \( L_R, L_D, \) and \( L_C \) be the radiation length, decay length, and collision length of the meson, respectively. The probability to radiate the \( e^+e^- \) pair before the first collision with a nucleon in the medium is equal to

\[ \int_0^{+\infty} \exp(-x/L_C)\exp(-x/L_D)dx/L_R = \frac{L_C}{L_C + L_D} \frac{L_D}{L_R}. \]  

(24)

Normalazing this probability to the total probability of radiation of the \( e^+e^- \) pair, \( L_D/L_R << 1 \), yields the probability \( w \) that the \( e^+e^- \) pair is emitted before the first collision (coherent radiation):

\[ w = \frac{L_C}{L_C + L_D}. \]  

(25)

The probability \( 1 - w \) corresponds then to incoherent radiation.
The collision length \( L_C \) is defined by the expression \( L_C = 1/(n\sigma v) \), where \( \sigma \) is the cross section, \( n \) the nucleon density, \( v \) the velocity of the meson. The decay length for the resonance with life time \( T_D \) equals to \( L_D = v\gamma T_D \), where \( T_D = 1/\Gamma \), \( \Gamma \) being the meson width. All mesons have in general different values \( L_R, L_D, \) and \( L_C \), so the coherent radiation probabilities are also different.

Thus, for a \( \rho \)-meson family of \( n \) interfering mesons the enhancement factor becomes

\[
E_n(M) = \frac{\left( \prod_i w_i |\mathcal{M}_i|^2 + \sum_j (1 - w_j) \prod_i \left( |\mathcal{M}_i|^2 + \sum_{i \neq j} |\mathcal{M}_i|^2 \right) + \cdots + \prod_i \left( 1 - w_i \right) \sum_i |\mathcal{M}_i|^2 \right)}{\left( \sum_i |\mathcal{M}_i|^2 \right)}.
\]

Here, the first term in the nominator corresponds to the probability that all \( \rho \)-mesons radiate the dileptons pairs coherently. The second term corresponds to the probability that the \( j \)-th meson decays to the dilepton pair after its first collision, while the other mesons radiate before the first collision. Finally, the last term corresponds to the probability for the incoherent radiation from all \( \rho \)-mesons. Each term in Eq.(26) contains the squares of the amplitudes according to the proper interference pattern.

In order to estimate the possible effect of the enhancement factor, let us take \( L_D \approx T_D \) and vary the collision length \( L_C \) from 0 (total decoherence) to \( \infty \) (total coherence). In Figs.3 and 4 we plot the enhancement factors \( E_2(M) \) and \( E_3(M) \) as a functions of the running mass \( M \).

The subthreshold part of the cross section, Eq.(15), is enhanced by \( E_3(M) \). A similar effect exists for the cross section (14). The meson decays \( \mathcal{M} \to e^+e^-X'' \) have also well known constraints to the asymptotic behavior from the quark counting rules and therefore another type of the interference patterns between members of the \( \rho \)-meson family in the \( \mathcal{M} \to e^+e^-X'' \) transition form factors occur.

The decay modes \( P \to e^+e^-\gamma \) where \( P = \pi, \eta \) and \( \rho^0 \to e^+e^-\pi^+\pi^- \) have monopole form factors in the amplitudes. To get a monopole form factor it is sufficient to have just one \( \rho \)-meson. In such a case, no enhancement exists: \( E_1(M) \equiv 1 \). The decay modes \( V \to e^+e^-P \),
\eta \rightarrow e^+e^-\pi^+\pi^-, \rho^0(\omega) \rightarrow e^+e^-\pi^0\pi^0, \text{ with dipole form factors require the existence of at least two } \rho \text{-mesons. In such a minimal case, these modes are enhanced by the } E_2(M). \text{ The in-medium modification of the cross section (14) requires therefore to check all channels entering into this expression and to decide if they belong to the first class of the processes with no enhancement or to the second class of the processes with enhancement. We discussed here the } \rho \text{-meson family. This discussion applies to the } \omega \text{- and } \phi \text{-meson families as well.}

V. NUMERICAL RESULTS

The results for the dilepton spectra are shown in Fig. 3. To compare with the experimental data, the acceptance of the DLS detector with respect to the } e^+e^- \text{ pairs that have invariant mass } M, \text{ transverse momentum } p_T, \text{ and rapidity } y \text{ is taken into account. For each process, the distribution over the } p_T \text{ and } y \text{ is determined by the available phase space of the process and then weighted with the filter function } f(M, p_T, y) \text{ provided by the DLS collaboration. The details of this procedure are described in Appendix 2. Finally, the finite mass resolution of the detector, } \Delta M_{\text{exp}} = \pm 25 \text{ MeV, is taken into account by smearing the spectra with a Gaussian distribution which corresponds to a standard error of } \sigma = 25 \text{ MeV.}

At the lowest initial kinetic energy of the proton, i.e. } T = 1.04 \text{ GeV, the cross section is dominated by the } \pi^0\text{-Dalitz decay below } M \lesssim 100 \text{ MeV and by the Dalitz decays of the nucleon resonances, mainly the } \Delta(1232)\text{-resonance, at } M \approx 200-500 \text{ MeV. Compared to our calculation there is an excess of detected } e^+e^- \text{ pairs at } M \gtrsim 300 \text{ MeV. Earlier calculations } [22] \text{ obtained higher cross sections in this mass range. This is due to the normalization to the } R \rightarrow N\rho \text{ branching ratios within the framework of the naive VMD which overestimates the radiative decay rates } R \rightarrow N\gamma, \text{ as discussed in Sect.3.}

At higher initial proton energies, the agreement with the DLS data is generally very reasonable. The contribution from the } \eta \text{-meson Dalitz decay is dominant at } M \approx 0.2 - 0.4 \text{ GeV, while the Dalitz decays of the baryon resonances dominate at } M \gtrsim 400 \text{ MeV.}

For the proton kinetic energy } T = 2.09 \text{ GeV, the inclusive production of the } \rho \text{- and } \omega-
mesons becomes visible at $M \approx 800$ MeV. The $\omega$-meson peak is rather pronounced, whereas no signature for a peak can be found in the experimental data. In Ref. [20], the $\omega$-peak is not reproduced also, apparently, due to a stronger smearing with a mass-dependent parameter $\sigma = 0.1M$. Remarkably, the three highest experimental points at $T = 2.09$ GeV lie above the kinematical limit $M_{\text{max}} = \sqrt{s} - 2m_N \approx 850$ MeV and the indicated experimental error $\Delta M^{\text{expt}} = \pm 25$ MeV is not sufficient to explain their occurrence. We suppose that the experimental resolution is not good enough to resolve the kinematic threshold and to observe the $\omega$-meson peak.

Finally, at $T = 4.88$ GeV, the contribution from the inclusive production of the $\eta$, $\rho$, and $\omega$-mesons becomes dominant at $M \approx 300 \div 800$ MeV. There is an underestimation of the dilepton yield in the region $M \approx 400 \div 700$ MeV. A similar underestimation was found in [20] both, at $T = 2.09$ GeV and $T = 4.88$ GeV. As proposed in Ref. [22], the existing gap might be filled by the subthreshold dilepton production via the baryon resonances. However, we were not able to match the data using a consistent description of the photoproduction data and the $R \rightarrow N\rho$ meson decay branchings, with the proper application of the Breit-Wigner formula (see details in Appendix 1), and removing possible sources for the double counting. Each of this three aspects leads to a reduction of the dilepton yield. Therefore, one cannot exclude that the origin of the so called ”DLS puzzle” can be traced back to the elementary $pp$ level and is not a specific feature of heavy-ion collisions. New experimental measurements of the dilepton cross section, especially at $T = 1.04$ GeV and $T = 4.88$ GeV, would certainly help to clarify this point.

In this context one should be aware that the comparison to the experimental data is strongly influenced by the acceptance of the DLS detector. In Appendix 2, we discuss the application of the corresponding filter program [19] for the calculation of the experimentally measured cross sections. In Fig.7 the effective detector efficiency, smeared by a Gaussian distribution with the standard deviation $\sigma = 25$ MeV, is shown as a function of the dilepton mass $M$ for decays $\pi \rightarrow \gamma e^+e^-$, $\eta \rightarrow \gamma e^+e^-$, and $\rho^0(\omega) \rightarrow e^+e^-$ at the two highest proton energies $T = 2.09$ GeV and $T = 4.88$ GeV, where the effects of the multiple pion production
are most important. It can be seen that the effective acceptance decreases with increasing energy for a fixed number of pions in the final state. On the other hand, when the number of pions increases, the acceptance increases as well. This can be interpreted to mean that a larger number of the pions reduces the available phase space for mesons decaying to the dilepton pairs, and the decays of such mesons can be detected with better efficiency.

The effect is particularly strong for the $\pi \rightarrow \gamma e^+ e^-$ decay at $T = 4.88$ GeV. While the average pion multiplicity $n_\pi$ is around 2, the effective acceptance is extremely small below $n \approx 6$. The acceptance is not reliable when it is much smaller than unity. In our case this happens at $n \lesssim 6$. While the statistical distribution gives here, as we expect, reasonable estimates, the calculation of the part of the cross section connected to the additional production of $n \lesssim 6$ pions turns out to be unreliable. From the other side, at $n \gtrsim 6$ the filter is well defined, but the binomial distribution gives exponentially small probabilities. The highest part of the pion spectra corresponding to $n \gtrsim 6$ also cannot be calculated accurately. So, we consider the difference between our results and those of Refs. [20,21] by about a factor of 3 and those of Ref. [22] by about a factor of 6 as a conservative estimate for uncertainties inherent in the theoretical calculations for both, the distribution over the pion multiplicities and the experimental filter acceptance in the region of small invariant masses.

At lower energies, these uncertainties practically disappear. In Fig.7, we show a plot for the $\pi \rightarrow \gamma e^+ e^-$ decay at $T = 2.09$ GeV. It can be seen that now already for $n = 0$ the effective acceptance is no more extremely small. For heavier mesons, the calculation of the acceptance is safer, which is again connected with less energy available for the produced mesons and, respectively, a better efficiency for the detection of the dileptons.

For the application of the filter we assumed an isotropic distribution of the particles in the final states in the c. m. frame. This is justified at small energies $T$. With increasing kinetic energy, the distribution acquires a bias towards the beam direction. This is an additional source of uncertainties in the calculations of the filter, which can be important at energy $T = 4.88$ GeV for the pion Dalitz decays.

The many-body phase spaces entering into Eq.(4) are known to be very sharp functions
of the arguments. The Breit-Wigner distribution over the dilepton mass, $M$, gets therefore an enhancement towards small values of $M$. The greater the number of pions in the final state, the more important is this effect. We found that rare processes with probabilities $w_n \lesssim 0.03$ corresponding to large numbers of pions produced in association with the vector mesons, that should in principle give small contributions to the total cross sections, become very important at masses $M \lesssim 200 \div 300$ MeV. The spectral functions of the vector mesons are not known well far away from the vector meson peaks. This effect is thus beyond the scope of the present model. It should be analysed separately. In the present calculations, we apply a 3% criterion to the multiple pion processes: The values $w_n$ are set equal to zero every time when $w_n < 0.03$. This works for the inclusive vector meson production at $T = 4.88$ GeV. At smaller energies pions are not produced in association with vector mesons.

Eq.(15) can be modified in a trivial way to include Dalitz decays of the vector mesons from the subthreshold production. We take into account decays $V \rightarrow P e^+ e^-$ with $P = \pi^0$ and $\eta$. Since the Breit-Wigner peak with respect to the running vector meson masses, $\mu$, is eliminated by the suppression factor $S_V(\mu^2)$, the corresponding integral over the $\mu$ is not constrained as usually around the physical masses of the vector mesons within their widths. The integral takes its values from a broader interval. For this reason, the decays $V \rightarrow P e^+ e^-$ can proceed through the on-shell vector mesons, $V \rightarrow V' P$, $V' \rightarrow e^+ e^-$, since the vector mesons due to the VMD are virtually present in the transition form factors $V P \gamma^*$. This is another source of the double counting if one attempts to sum up the subthreshold production cross section and the inclusive production cross section. To avoid the double counting, we multiply the results by the suppression factor (16) whose argument now is the dilepton mass $M^2$. The first suppression factor is therefore needed to eliminate the peak with respect to the running mass, $\mu$, whereas the second suppression factor is needed to eliminate the peak with respect to the dilepton mass $M$. We found e.g. that for subthreshold $\omega$-mesons the $\omega \rightarrow e^+ e^- \pi^0$ decay is more important than the decay $\omega \rightarrow e^+ e^-$ at $T = 4.88$ GeV and $M \lesssim 700$ MeV (cf. bold dashed curves #4 and #3 in Fig.6). In other cases, Dalitz decays of the subthreshold vector mesons are less important than the direct ones.
VI. CONCLUSION

We have considered the dilepton production in $pp$ collisions at BEVALAC energies. The subthreshold production of vector mesons through the nucleon resonances is described within the extended VMD model which allows to bring the transition form factors in agreement with the quark counting rules and overcome the difficulty of a simultaneous description of the $R \to N\rho$ and $R \to N\gamma$ decays. This leads to a sizeable reduction of the $N(1520)$ contribution to the dilepton spectra. From all other $N$ and $\Delta$ channels which we have taken into account only the $N(1535)$ and $\Delta(1620)$ are relevant for the dilepton production. In this context we considered also the problem of double counting and provided a possible solution. The resulting dilepton spectra are reasonably well described at proton energies ranging form $T = 1.04 \div 2.09$ GeV. At $T = 4.88$ GeV we observe still an underestimation in the region of dilepton masses below the $\rho$-peak ($M \approx 400 \div 700$ MeV). We hope that future theoretical and experimental investigations will clarify this problem.

The presence of a nuclear medium can enhance the yield form the subthreshold vector meson production through nucleon resonances and the Dalitz decays of mesons. The extended VMD model requires the destructive interference of two to three intermediate $\rho$ mesons which is likely to be destroyed by the surrounding medium. The effect depends on the relative weights of the different channels and the type of the reaction. The possible enhancement of the cross section in the nuclear medium was estimated within a schematic model.

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Appendix 1. Breit-Wigner description of resonances with energy-dependent widths

Let us consider a process with a resonance in the intermediate state. It can be produced either in a two-body collision or as a result of the decay of a particle or another resonance. Let the resonance further decay to some specific channel $i$. The amplitude for the total process, $M^i$, i.e. the amplitude for resonance production, propagation, and subsequent decay to the channel $i$ is a product of the amplitude of its production $M_p$, the resonance propagator, and the amplitude of the resonance decay $M^i_d$: 

$$M^i = M_p \frac{1}{p^2 - m^2 + \Sigma(p^2)} M^i_d$$  \hspace{1cm} (27) 

where $p$ is the momentum of the resonance, $m$ is its pole mass, $\Sigma(p^2)$ is the resonance self energy. The pole mass $m$ is defined such that $Re\Sigma(m^2) = 0$. In general, $Re\Sigma(p^2)$ starts with terms of the order $O((p^2 - m^2)^2)$. These terms are further neglected. The imaginary part of $\Sigma(p^2)$ is equal to 

$$Im\Sigma(p^2) = \frac{1}{2} \sum_i |M^i_d|^2 \Phi_d^i(p^2)$$  \hspace{1cm} (28) 

where $\Phi_d^i(p^2)$ is the phase space for the resonance decay into a channel $i$. We can therefore write either 

$$Im\Sigma(p^2) = \sqrt{p^2} \left( \frac{1}{2} \sum_i |M^i_d|^2 \Phi_d^i(p^2) \right) \equiv \sqrt{p^2} \Gamma^R_{tot}(p^2)$$  \hspace{1cm} (29) 

or

$$Im\Sigma(p^2) = m \left( \frac{1}{2m} \sum_i |M^i_d|^2 \Phi_d^i(p^2) \right) \equiv m\tilde{\Gamma}^R_{tot}(p^2).$$  \hspace{1cm} (30) 

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Both definitions of the total width, $\Gamma_{tot}(p^2)$ and $\tilde{\Gamma}_{tot}(p^2)$, can be used in the relativistic Breit-Wigner formula, but the width should be multiplied by the proper resonance masses, $\sqrt{p^2}$ (running mass) and $m$ (pole mass), respectively.\footnote{In Ref. [21] the combination $m_\rho \Gamma_{e^+e^-}(p^2)$ (physical $\rho$-meson mass and $\Gamma$ without "\) has been substituted into the Breit-Wigner formula. Such a combination leads to an additional factor $m_\rho/m$ in the dilepton production cross section and, consequently, to an overestimation of the dilepton yield below the $\rho$-meson peak. We have brought the attention of the authors of Ref. [21] to this circumstance. Recently, a new paper appeared, Ref. [22], where that inconsistency has been removed. Notice also that the dilepton mass $M$ appears mistakenly in Eq.(25) of Ref. [21], line 3, instead of the $\Delta$-resonance mass (cf. Ref. [28], Eq.(9)).}

The square of the amplitude $M_i^i$, integrated over the phase space of the final particles with momenta, $p^k_p$ and $p'_d$, and normalized by the corresponding factors for the initial particles, gives either a cross section (two initial particles) or a width (one initial particle).

For the scattering problem, the cross section has the form

$$d\sigma = \frac{1}{j2E_1E_2} |M|^2(2\pi)^4 \delta^{(4)}(p_1 + p_2 - \Sigma_k p^k_p - \Sigma_l p'_d) \prod_k \frac{d^3p^k_p}{(2\pi)^3 2E_k^k} \prod_l \frac{d^3p'_d}{(2\pi)^3 2E'_d},$$

(31)

where $j$ is the flux of the incoming particles.

Further, we introduce two $\delta$-functions corresponding to momentum conservation in the processes of production and decay of the resonance and the running mass $M$ of the intermediate resonance

$$\delta^{(4)}(p_1 + p_2 - \Sigma_k p^k_p - \Sigma_l p'_d) = \int \delta^{(4)}(p_1 + p_2 - \Sigma_k p^k_p - p) \delta^{(4)}(p - \Sigma_l p'_d) d^4p \int \delta(M^2 - p^2) dM^2$$

(32)

and obtain

$$\sigma = \int \sigma(M^2) \frac{1}{\pi} \frac{M\Gamma_i(M) dM^2}{(M^2 - m^2)^2 + (M\Gamma_{tot}(M))^2}$$

(33)

where...
Here, one should also use the partial widths $\Gamma_i(M), \tilde{\Gamma}_i(M)$ with the proper masses $M, m$. The similar arguments apply in case of the decay problem.

Appendix 2. Effective filter function

A comparison to the DLS data requires to take the experimental detector efficiency into account. For this purpose a filter function is provided by the DLS collaboration. In particular at large $T$ (e.g. $T = 4.88$ GeV) this filter function is not a small correction to the theoretical calculations but is crucial for the comparison to data. Thus, in this appendix we discuss the influence of the detector filter in our analysis.

In terms of the c.m. frame momentum variables, the filter function can be rewritten as

$$f(p_T, y, M) = f(p^*_T, y^* + y_c, M)$$

(35)

where $p^*_T = p_T$ is the transverse momentum of the dilepton pair, $y_c$ is the rapidity of the c.m. frame $L^*$ with respect to the laboratory frame $L$ of the colliding nucleons,

$$y_c = \frac{1}{2} \ln \left( \frac{\sqrt{s} + \sqrt{s - 4m_N^2}}{\sqrt{s} - \sqrt{s - 4m_N^2}} \right),$$

(36)

$T$ is the proton kinetic energy in the $L$ frame. The distribution of dileptons in the c.m. frame $L^*$ is isotropic. This is a universal feature which does not depend on the specific type of the reactions and is connected to the form of the cross section (4) only. So, we work with the filter function averaged over the angles in the $L^*$ frame:

$$f(p^*, y_c, M) = \int_{-1}^{+1} \frac{d\cos \vartheta}{2} f(p^*_T, y^* + y_c, M)$$

(37)

where

$$p^*_T = p^* \sin \vartheta,$$

$$y^* = \frac{1}{2} \ln \left( \frac{e^* + p^*_{\parallel}}{e^* - p^*_{\parallel}} \right),$$

and
\[ p_\parallel^* = p^* \cos \theta, \]
\[ \epsilon^* = \sqrt{M^2 + p^{*2}}. \]

The problem reduces to finding the dilepton distribution over the dilepton momentum \( p^* \) in the c.m. frame \( L^* \).

The probability distribution of the dilepton momentum in the \( L^* \) frame for the direct decays \( V \to e^+e^- \) is given by
\[
dW(p^*) = \sum_{n=0}^{N_s} w_n D_n \Phi_2(\sqrt{s}, M, M_X) dM_X^2 \Phi_{2+n}(M_X \ldots) \tag{38} \]
where
\[
\Phi_{2+n}(M_X \ldots) = \Phi_{2+n}(M_X, m_N, m_N, \mu_\pi, \ldots, \mu_\pi),
\]
\[
D_n = \Phi_{3+n}^{-1}(\sqrt{s}, m_N, m_N, M, \mu_\pi, \ldots, \mu_\pi). \]

The effective filter function can be calculated as follows
\[
f^\text{eff}(T, M) = \sum_{n=0}^{N_s} w_n f_n^\text{eff}(T, M) = \int dW(p^*) f(p^*, y_c, M). \tag{39} \]

The value of \( M_X \) is integrated out within the limits \( 2m_N + n\mu_\pi \leq M_X \leq \sqrt{s} - M \). The momentum of the dilepton pair \( p^* = p^*(\sqrt{s}, M, M_X) \) is given by
\[
p^*(\sqrt{s}, M, M_X) = \frac{\sqrt{(s - (M + M_X)^2)(s - (M - M_X)^2)}}{2\sqrt{s}}. \]

A 100% detector efficiency would yield \( f(p_T, y, M) = 1 \) and \( f^\text{eff}(T, M) = 1 \) in virtue of Eq.(6). The values \( f_n^\text{eff}(T, M) \) are plotted in Fig.7 for the \( \rho^0(\omega) \to e^+e^- \) decays for different values of \( n \) at energies \( T = 2.09 \) and 4.88 GeV.

For the Dalitz decays \( \mathcal{M} \to \mathcal{M}'e^+e^- \), the probability distribution for the dilepton energy \( \epsilon^* \) in the c.m. frame \( L^* \) can be written as follows
\[
dW(\epsilon^*) = \sum_{n=0}^{N_s} w_n D_n \int_{(2m_N + n\mu_\pi)^2}^{(\sqrt{s} - M)^2} dM_X^2 \int_{-1}^{+1} \frac{d\cos \chi}{2} \delta(\epsilon^* - \eta k) d\epsilon^* \Phi_2(\sqrt{s}, \mu_M, M_X) \Phi_{2+n}(M_X \ldots). \tag{40} \]
Here, $k$ is the four momentum of the dilepton pair, $\mu_M$ is the mass of the meson $\mathcal{M}$ in the reaction $pp \rightarrow \mathcal{M}X$ with the subsequent decay $\mathcal{M} \rightarrow \mathcal{M}'e^+e^-$. In the $L^*$ frame, the vector $\eta$ is defined as $\eta = (1, 0)$. The coefficients $D_n$ are defined as before.

Now we should pass to the rest frame $L^{**}$ of the meson $\mathcal{M}$, where $\eta = \gamma(1, -n\nu)$. The $\gamma$-factor and the velocity $v$ of the meson $\mathcal{M}$ in the $L^*$ frame are determined by equations

$$\gamma\mu_M = \frac{(s + \mu_M^2 - M_X^2)}{(2\sqrt{s})},$$

$$v\gamma\mu_M = p^*(\sqrt{s}, \mu_M, M_X).$$

The unit vector $n$ shows in the direction of the meson velocity in the $L^*$ frame. In the meson rest frame, $L^{**}$, the dilepton pair has momentum $k = (\epsilon^{**}, n'p^{**})$ where

$$\epsilon^{**} = \frac{(\mu_M^2 + M^2 - \mu_{M'}^2)}{2\mu_M},$$

$$p^{**} = p^*(\mu_M, \mu_{M'}, M).$$

The function $p^*(\ldots)$ is defined earlier. The unit vector $n'$ shows in the direction of the dilepton pair momentum in the $L^{**}$ frame. In Eq.(40) the value $\chi$ is the angle between the directions of the meson velocity in the $L^*$ frame and velocity of the dilepton pair in the $L^{**}$ frame, so that $\cos\chi = nn'$ and therefore $\eta k = \gamma(\epsilon^{**}, vp^{**}\cos\chi)$.

In Eq.(40) the integral over the angle $\chi$ is evaluated explicitly, and we obtain

$$\frac{dW(\epsilon^*)}{d\epsilon^*} = \sum_{n=0}^{N_\pi} w_n D_n \frac{\pi\mu_M}{2\sqrt{sp^{**}}} \int \frac{(\sqrt{s}-M)^2}{(2m_N+n\mu)^2} dM_X^2 \theta(\epsilon^*, M_X) \Phi_{2+n}(M_X\ldots).$$

where

$$\theta(\epsilon^*, M_X) = \begin{cases} 1, & \gamma(\epsilon^{**} - vp^{**}) \leq \epsilon^* \leq \gamma(\epsilon^{**} + vp^{**}), \\ 0, & \text{otherwise}. \end{cases}$$

(42)

The effective filter function can now be calculated to be

$$f^{eff}(T, M) = \sum_{n=0}^{N_\pi} w_n f_n^{eff}(T, M) = \int dW(\epsilon^*) f(p^*, y_e, M).$$

(43)

The values $f_n^{eff}(T, M)$ are plotted in Fig.7 for the $\pi^0(\eta) \rightarrow e^+e^-\gamma$ decays for different values of $n$ at energies $T = 2.09$ and 4.88 GeV.
It is now sufficient to multiply the differential cross section (1) with the corresponding effective filter function $f_{\text{eff}}(T,M) < 1$ in order to compare the calculations with the experiment. For the evaluation of the direct contributions one should use expression (39), while for the Dalitz decays one should use expression (43). The function $f_{\text{eff}}(T,M)$ is given by a two-dimensional integral in Eq.(39) and by a three-dimensional integral in Eq.(43).
Figure Captions

**Fig.1**: The dilepton production cross sections \( pp \rightarrow e^+e^-pp \) through the nucleon resonances \( R = \Delta, N^*, \text{and } \Delta^* \) at the kinetic proton energy \( T = 1.61 \text{ GeV} \).

**Fig.2**: The dilepton production cross sections \( pp \rightarrow e^+e^-pp \) through the nucleon resonances at the kinetic proton energy \( T = 2.09 \text{ GeV} \).

**Fig.3**: The enhancement factor for transition form factors with the asymptotics \( 1/t^2 \) due to decoherence in the propagation of the vector mesons in a hot and dense nuclear medium, estimated within a schematic model (see Sect.4).

**Fig.4**: The enhancement factor for transition form factors with the asymptotics \( 1/t^3 \) due to decoherence in the propagation of the vector mesons in a hot and dense nuclear medium, estimated within a schematic model (see Sect.4).

**Fig.5**: The contributions of different channels to the differential dilepton production cross section before applying the experimental filter and before the smearing procedure (see text). The bold curves describe the subthreshold contributions, while the normal curves describe the inclusive contributions to the cross sections. The solid bold and solid normal curves correspond to the \( \rho^0 \)-meson decays \( \rho^0 \rightarrow e^+e^- \) (curves #1) and \( \rho^0 \rightarrow e^+e^-\pi^0 \) (curves #2). The contribution from the inclusive \( \rho^0 \)-mesons decaying to the channel \( \rho^0 \rightarrow e^+e^-\eta \) is also shown at energy \( T = 4.88 \text{ GeV} \) by a solid curve in the lower left part of the plot. The bold dashed and normal dashed curves correspond to the \( \omega \)-meson decays \( \omega \rightarrow e^+e^- \) (curves #3) and \( \omega \rightarrow e^+e^-\pi^0 \) (curves #4). In the subthreshold production (the bold dashed curves), the \( \omega \)-mesons are produced through the \( N^*(1535) \)-resonance. The contribution from the inclusive \( \omega \)-mesons decaying to the channel \( \omega \rightarrow e^+e^-\eta \) is also shown at energy \( T = 4.88 \text{ GeV} \) by a dashed curve without number. The dot-dashed curves correspond to the \( \phi \)-meson decays \( \phi \rightarrow e^+e^- \) (curve #5) and \( \phi \rightarrow e^+e^-\pi^0 \) (curve #6) at energy \( T = 4.88 \text{ GeV} \). The long-dashed curves correspond to the \( \eta \)-meson Dalitz decays \( \eta \rightarrow e^+e^-\gamma \). The dotted curves describe the contribution from the \( \pi^0 \)-meson Dalitz decays \( \pi^0 \rightarrow e^+e^-\gamma \). The value \( M \) is the dilepton mass.
**Fig.6:** The differential dilepton production cross sections as a function of the dilepton invariant mass, $M$, after applying the experimental filter and the smearing procedure (see text). The bold solid curves are the total cross sections, the solid curves correspond to the inclusive production, and the dashed curves correspond to the subthreshold production. The experimental data are from Ref. [10].

**Fig.7:** The effective acceptance of the DLS detector versus the invariant dilepton mass, $M$, for different numbers of pions, $n$, produced in association with the pion $\pi^0$, $\eta$-meson, and $\rho^0(\omega)$-mesons at two highest energies $T = 2.09$ and 4.88 GeV. The numbers over the curves show the numbers $n$ of the pions.


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FIG. 1.
FIG. 2.
FIG. 3.
FIG. 4.
FIG. 5.
FIG. 6.
FIG. 7.