PHOTON SPLITTING AND PAIR CREATION IN HIGHLY MAGNETIZED PULSARS

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ABSTRACT

The absence of radio pulsars with long periods has led to the popular notion of a high \( P \) “death line.” In the standard picture, beyond this boundary, pulsars with low spin rates cannot accelerate particles above the stellar surface to high enough energies to initiate pair cascades, and the pair creation needed for radio emission is strongly suppressed. In this paper we explore the possibility of another pulsar “death line” in the context of polar cap models, corresponding to high magnetic fields \( B \) in the upper portion of the period-period derivative diagram, a domain where few radio pulsars are observed. The origin of this high \( B \) boundary, which may occur when \( B \) becomes comparable to or exceeds \( B_{cr} \approx 4.4 \times 10^{13} \) Gauss, is also due to the suppression of magnetic pair creation, but primarily because of ineffective competition with magnetic photon splitting. Threshold pair creation also plays a prominent role in the suppression of cascades. We present Monte Carlo calculations of the pair yields in photon splitting/pair cascades which show that, in the absence of scattering effects, pair production is effectively suppressed, but only if all three modes of photon splitting allowed by QED are operating in high fields. This paper describes the probable shape and position of the new “death line,” above which pulsars are expected to be radio quiet, but perhaps still X-ray and gamma-ray bright. The hypothesized existence of radio-quiet sources finds dramatic support in the recent discovery of ultra-strong fields in Soft Gamma-ray Repeaters and Anomalous X-ray Pulsars. Guidelines for moderate to high \( B \) pulsar searches at radio wavelengths and also in the soft and hard gamma-ray bands are presented.

Subject headings: gamma rays: theory — radiation mechanisms — magnetic fields — stars: neutron — pulsars: general

1. INTRODUCTION

Most current theories for the generation of coherent radio emission in pulsar magnetospheres (see summations in Michel 1991; Melrose 1993; Lyutikov et al. 1999) require formation of an electron-positron pair plasma. Such a pair plasma has been shown to develop via electromagnetic cascades along the open magnetic field lines as a result of particle acceleration to TeV energies, followed by curvature, synchrotron radiation and one-photon pair production in the strong magnetic field, \( \gamma \rightarrow e^+ e^- \), near the neutron star surface (e.g. Sturrock 1971; Ruderman & Sutherland 1975, Arons & Scharlemann 1979). Numerical simulations of pair cascades above a pulsar polar cap show that \( 10^3 \sim 10^4 \) pairs are produced for each accelerated primary electron (Daugherty & Harding 1982) in a typical young pulsar. But if any of the necessary conditions for development of a pair cascade, \( i.e. \) production of \( \gamma \)-ray photons (requiring particle acceleration to high energy) or production of sufficient pairs, is absent the pulsar should not, according to theory, emit detectable radio emission. The existence of an observed “death line” in the \( P - \dot{P} \) distribution of known radio pulsars (see Figure 5 below), to the right of which no pulsars were detected, until the recent discovery (Young, Manchester & Johnston 1999) of the 8.5 second pulsar PSR J2144-3933, provided some circumstantial evidence that pairs are required for radio emission. The slope of this putative boundary fits a line of constant voltage across the open field lines, \( V \propto P^{-3/2} \dot{P}^{1/2} \), suggesting that radio emission ceases when the particle acceleration is not high enough to sustain pair production. The radio emission from some long period pulsars like PSR J2144-3933 can be explained through production of pairs by photons from inverse-Compton scattering (Arons 2000, Zhang, Harding & Muslimov 2000), a process which does not require as high a voltage as does curvature radiation.

In this paper, we study another area of pulsar phase space, that of very high magnetic fields, where the character of pair cascades is significantly altered by the process of photon splitting and other effects such as ground state pair creation and positronium formation. Magnetic photon splitting, \( \gamma \rightarrow \gamma \gamma \), a QED process in which a single photon splits into two lower-energy photons (Adler 1971, Baring & Harding 1997), operates efficiently and competes effectively with pulsar pair production only in magnetic fields above \( \sim 10^{13} \) Gauss (Harding, Baring & Gonthier 1997, hereafter HBG97). This region of high magnetic field strength lies in the upper-right part of the \( P - \dot{P} \) diagram. We will explore whether photon splitting may be the explanation for why there are no classical radio pulsars detected with derived magnetic fields above \( \sim 10^{14} \) Gauss. Our initial results (Baring & Harding 1998) have suggested that photon splitting could indeed explain the absence of very high-field radio pulsars. This paper considers in more detail the underlying assumptions made in that calculation, explores the physics of high-field pair cascades and makes predictions for high-energy searches for high-field pulsars. Although pulsars in this region of phase space may be radio-quiet, those that are to the left of the death line are still prolifically accelerating particles to high energies. Their pulsed emission may therefore be detectable in the X-ray and gamma-ray wavebands, making this relatively-
unexplored region of pulsar phase space extremely interesting.

There has recently been growing evidence for another class of pulsars with ultra-strong magnetic fields. Observations at X-ray energies have yielded detections of both long periods and high period derivatives in two types of sources, anomalous X-ray pulsars (AXPs) and soft γ-ray repeaters (SGRs), which suggest dipole spin-down fields in the range $10^{14}$ to $10^{15}$ Gauss. The AXPs are a group of seven or eight pulsating X-ray sources with periods in the range 6–12 seconds, and are continuously spinning down (Vasisht & Gotthelf 1997). The SGRs are a type of γ-ray transient source that undergoes repeated bursts; four (possibly now five: see Cline et al. 2000 for SGR 1801-23) are currently known to exist. A detailed discussion of these sources and recent discoveries pertaining to them is presented in Section 4.3. While conventional radio pulsars tap their rotational energy to power their emission, if such a new class of ultra-magnetized neutron stars or “magnetars” does exist, their long periods indicate that their emission cannot be rotationally-powered: they may instead be driven by their magnetic energy. With the possible exception of the recent report (Shitov 1999; Shitov, Pugachev & Kutuzov 2000) of a low frequency pulsed detection of a counterpart (PSR J1907+0919 at 111 MHz) to the soft gamma repeater SGR 1900+14, which has not been confirmed by pulsation searches in Aricebo data at higher frequencies (Lorimer & Xilouris 2000) none of these sources has detectable pulsed radio emission.

In several previous papers, we have computed the photon splitting attenuation lengths of photons emitted parallel, or at small angles, to the magnetic field near the surface of strongly magnetized neutron stars and compared them to the attenuation lengths for one-photon pair production (Harding, Baring & Gonthier 1996; 1997). Both processes depend on the photon energy $\omega$ and the angle $\theta_B$ of photon propagation to the local field. However, since photon splitting has no threshold, it may occur before the photon reaches the threshold for pair production at $\omega \sin \theta_B = 2m_e c^2$. We found that the photon splitting attenuation lengths are lower than those for pair production in fields higher than $\approx 3 \times 10^{13}$ Gauss, for emission in the open field region near the neutron star surface. One must therefore incorporate photon splitting in cascade models of highly-magnetized pulsars. This has been done in the case of PSR1509-58 (Harding, Baring & Gonthier 1997), which has a field of $3 \times 10^{13}$ Gauss as estimated using Equation (9). We considered both the case where all linear polarization modes of photon splitting allowed by CP invariance operate, and the case where only the $\perp \rightarrow \perp$ mode (our polarization convention is defined near the beginning of Section 2) allowed by selection rules (Adler 1971) in the weakly dispersive limit operates. The latter case, we argued, was most appropriate for pulsars. In both cases, we found that photon splitting attenuation is a plausible explanation for the very low spectral cutoff from 10 and 30 MeV in PSR1509-58 (Kuiper et al. 2000). Since the rates of these attenuation mechanisms are very sensitive to the polarization states of the incoming photons, it is evident that the polarization properties of the radiated photons are very important in modeling both the output spectrum and pair suppression in high-field pulsars.

In ultra-strong magnetic fields, the vacuum dispersion increases to the point where the selection rules derived for weak linear dispersion (appropriate for $B \ll B_c$) may no longer be valid, and higher order non-linear contributions to such dispersion may be significant. We presently do not know which modes of photon splitting operate in supercritical fields and thus do not know which photon polarization modes are allowed to split. We will therefore explore several possibilities for photon splitting mode behavior in our study of high-magnetic field pair cascades. In Section 2.1, we discuss the physics of pair suppression in high magnetic fields, both by photon splitting and by pair production in low-lying Landau states, as well as the possible alternative mechanism of pair suppression by bound-state pair creation, discussed by Usov & Melrose (1996). In Section 3, we present results of detailed simulations of splitting/pair cascades which explore the conditions for suppression of pair creation in a pulsar magnetosphere. We compute the boundary in the pulsar $P - \dot{P}$ distribution where the escape energies for photon splitting and magnetic pair production for photons of $\perp$ polarization are equal. This condition then defines a putative radio-quiescence boundary, posited in Baring & Harding (1998), above which pulsars may not produce the dense pair plasmas required for radio emission. For reasonable assumptions about the location of particle acceleration and the angles of the emitted photons in high-field pulsars, the computed radio-quiescence boundary lies at $\dot{P}$ above those in the known radio pulsar population, potentially explaining the absence of radio pulsars with fields above $\approx 10^{14}$ Gauss. Our cascade simulations indicate that such a boundary is viable only if photons of both polarizations can split; otherwise pair creation is generally postponed a generation rather than suppressed. In the latter case, significant reductions in pair production can ensue only if the maximum energy of primary photons is below or not much above the pair creation escape energy. If pair suppression is rife at these high $\dot{P}$, it suggests the possible existence of a new class of radio-quiet, ultra-magnetized pulsars that may be related to the emerging class of magnetars.

2. PHYSICAL EFFECTS INFLUENCING PAIR CREATION

The most important competitor to pair creation $\gamma \rightarrow e^+e^-$ at high magnetic field strengths is a mechanism for attenuating photons in pulsar magnetospheres is magnetic photon splitting $\gamma \rightarrow \gamma \gamma$. Hence this process motivated the suggestion (Baring & Harding 1998) that splitting can potentially suppress pair creation and inhibit radio emission in highly-magnetized pulsars; accordingly it forms the centerpiece of the discussion of this Section. Yet, the creation of pairs in low Landau levels or the ground state can also suppress subsequent pair creation, so this effect is addressed in Section 2.2. We also include a brief discussion of the role of positronium formation, since the residence of pairs in neutral bound states can inhibit a range of coherent mechanisms for producing radio emission. The section concludes with a discussion of how the combined effects of photon splitting and threshold pair creation change the nature of pair cascades as the magnetic field increases.

At this point, it is appropriate to identify conventions adopted throughout this paper. The photon linear polarizations are such that $\perp$ refers to the state with the photon’s electric field vector parallel to the plane containing the magnetic field and the photon’s momentum vector, while $\parallel$ denotes the photon’s electric field vector being normal to this plane. Furthermore, all photon and electron energies will be written in dimensionless form, being scaled by $m_e c^2$. Since general relativistic effects will play an important role in our considerations, we adopt the convention of labelling photon energies in the local inertial frame of reference by $\omega$, when a connection to quantum processes is most salient, and photon energies that an observer at infinity would measure will be denoted by $\varepsilon$. By the same token, $B$ will be used to denote local fields, and $B_0$ will represent the surface field that
would be inferred at the pole in flat spacetime, i.e. a neutron star of radius $R$ with a pure dipole field has a magnetic moment of $\mu_B = B_0 R^3/2$.

Photons of $\perp$ polarization dominate the photon population generated by the principal primary and secondary emission processes, namely cyclotron/synchrotron radiation, curvature emission and resonant Compton scattering. Synchrotron and curvature radiation are central to the hard X-ray and gamma-ray emission of polar cap models (e.g. Daugherty and Harding 1982; 1996), and are essentially identical in their polarization properties in the classical picture. For monoenergetic electrons, polarization levels $P = |\bar{n}_\perp - n_\parallel|/|\bar{n}_\perp + n_\parallel|$ of between 50% and 100% are achieved (e.g. see Fig. 6.7 of Beker 1966), while for power-law electrons, the degree of polarization is generally in the 60%–70% range (e.g. Beker 1966; Rybicki and Lightman 1979), with $\perp$ photons dominating in both cases. At $B \gtrsim B_{\text{cr}}$, there is some degree of quantum depolarization, though in reality this is generally small since photon angles with respect to local fields are always small. The dominance of $\perp$ photons applies also to the products of resonant Compton scattering, which has more recently been considered (e.g. Sturmer and Dermer 1994) as a primary emission mechanism. In the Thomson limit, the cross-sections for scatterings of polarized photons can be derived from the results of Herold (1979), indicating that $\perp$ photons are produced at 3 times the rate of $\parallel$ photons.

2.1. Competition with Magnetic Photon Splitting

2.1.1. Photon Splitting

The relevance of photon splitting $\gamma \rightarrow \gamma\gamma$ to neutron star environments was emphasized by Adler (1971), Mitrofanov et al. (1986) and Baring (1988). In the context of pulsars, it has been discussed by Baring (1993), who proposed it as a mechanism for suppressing the appearance of $e^+e^-$ annihilation lines, and by Baring, Baring and Gonthier (1996, 1997) and Chang, Chen and Ho (1996), who focused on its action in attenuating gamma-ray pulsar continua. Splitting is a third-order QED process with a triangular Feynman diagram. Though it is permitted by energy and momentum conservation, when $B = 0$ it is forbidden by a charge conjugation symmetry of QED known as Furry’s theorem (e.g. Jauch and Rohrlich 1980), which states that ring diagrams that have an odd number of vertices with only external photon lines generate interaction matrix elements that are identically zero. This symmetry, which pertains to the electron/positron propagators, is broken by the presence of an external field. The splitting of photons is therefore a purely quantum effect, and has appreciable reaction rates only when the magnetic field is at least a significant fraction of the quantum critical field $B_{\text{cr}}$.

It is practical to restrict considerations of splitting to regimes of weak dispersion, where manageable expressions for its rates are obtainable, but are still complicated triple integrations (e.g. see Adler 1971; Stoneham 1979; Baier, Mil’shtein, & Shaisultanov 1996; Adler & Schubert 1996) or triple summations (Baring 2000). Further specialization to either low magnetic fields ($B \ll B_{\text{cr}}$) or low photon energies ($\omega \ll 2$) therefore proves expedient, and palatable results for splitting rates were first obtained in such regimes by Bialynicka-Birula and Bialynicki-Birula (1970), Adler et al. (1970) and Adler (1971). A compact presentation of these rates (i.e. for $\omega \ll 2$) for the three polarization modes of splitting permitted by CP (charge-parity) invariance in QED, namely $\perp\rightarrow\parallel\parallel$, $\perp\rightarrow\parallel\perp$ and $\parallel\rightarrow\parallel\perp$, is

\[ T^\parallel_{\perp\rightarrow\parallel\parallel}(\omega) = \frac{\alpha_e^3}{60\pi^2} \frac{1}{\lambda} \left( \frac{B}{B_{\text{cr}}} \right)^6 \omega^5 \mathcal{M}_1^2 = \frac{1}{2} T^\parallel_{\parallel\rightarrow\parallel\perp} \]

\[ T^\parallel_{\perp\rightarrow\parallel\perp}(\omega) = \frac{\alpha_e^3}{60\pi^2} \frac{1}{\lambda} \left( \frac{B}{B_{\text{cr}}} \right)^6 \omega^5 \mathcal{M}_2^2 \]

\[ T^\parallel_{\parallel\rightarrow\parallel\perp} = \frac{1}{2} T^\parallel_{\perp\rightarrow\parallel\parallel} \]

in the frame where photons propagate perpendicular to the field, where

\[ \mathcal{M}_1 = \left( \frac{B}{B_{\text{cr}}} \right)^{-4} \int_0^\infty ds \frac{d^2}{s} e^{-sB_{\text{cr}}/B} \times \left\{ \left( -\frac{3}{4s} + \frac{s}{6} \right) \cosh s \sinh s + \frac{3 + 2s^2}{12\sinh^2 s} + \frac{s\cosh s}{2\sinh^2 s} \right\} ; \]

\[ \mathcal{M}_2 = \left( \frac{B}{B_{\text{cr}}} \right)^{-4} \int_0^\infty ds \frac{d^2}{s} e^{-sB_{\text{cr}}/B} \times \left\{ \frac{3\cosh s}{4s\sinh s} + \frac{3 - 4s^2}{4\sinh^2 s} - \frac{3s^2}{2\sinh^2 s} \right\} ; \]

and $\alpha_e$ is the fine structure constant and $\lambda = \hbar/(m_e c)$ is the Compton wavelength of the electron divided by $2\pi$. At low fields, $\mathcal{M}_1$ and $\mathcal{M}_2$ are independent of $B$, but at high fields possess $\mathcal{M}_1 \propto B^{-3} \mathcal{M}_2 \propto B^{-4}$ dependences. These are the expressions used in this paper because of their broad applicability to the pulsar problem. Deviations from this low energy limit near pair creation threshold are presented in detail by Baier, Mil’shtein, & Shaisultanov (1996) and Baring & Harding (1997), and are mentioned below where appropriate. An analytic approximation to the total rate in the $B \gg B_{\text{cr}}$ limit is presented in Baring and Harding (1997), and $B \gg B_{\text{cr}}$ analytic forms for other splitting modes can be found in Baring (2000).

The birefringence of the magnetized vacuum implies an alteration of the kinematics of strong field QED processes (Adler 1971), admitting the possibility of non-collinear photon splitting. Hence, while the splitting modes $\perp\rightarrow\parallel\parallel$, $\parallel\rightarrow\perp\perp$ and $\parallel\rightarrow\parallel\perp$ are forbidden by CP invariance in the limit of zero dispersion, dispersive effects guarantee a small but non-zero probability for the $\perp\rightarrow\parallel\perp$ channel. Extensive discussions of linear dispersion in a magnetized vacuum are presented by Adler (1971) and Shabad (1975); considerations of plasma dispersion (e.g. see the analysis of Bulik 1998) are not relevant to classical gamma-ray pulsars because of the relatively low densities present in the magnetosphere, but may be quite pertinent to soft gamma repeaters. Adler (1971) showed that in the limit of weak linear vacuum dispersion (roughly delineated by $B \sin\theta_{\text{KB}} \lesssim B_{\text{cr}}$), where the refractive indices for the polarization states are very close to unity, energy and momentum could be simultaneously conserved only for the splitting mode $\perp\rightarrow\parallel\parallel$ (of the modes permitted by CP invariance) below pair production threshold. This kinematic selection rule was demonstrated for linear dispersion, a regime that applies to subcritical fields. Therefore, it is probable that only the one mode ($\perp\rightarrow\parallel\parallel$) of splitting operates in normal pulsars. However, this constraint may not hold in supercritical fields where strong vacuum dispersion arises, thereby requiring a revised assessment using the generalized vacuum polarization tensor (i.e. including quadratic and higher order contributions to the vacuum polarization).
2.1.2. Attenuation Lengths

As pair creation is a first order QED process, whereas splitting is third-order in the fine structure constant $\alpha = e^2/(\hbar c)$, it is not immediately obvious that $\gamma \rightarrow \gamma\gamma$ can ever dominate $\gamma \rightarrow e^+e^-$ in pulsar environs. Yet it is the propagation of the photons at small angles to the field through the pulsar magnetosphere that affords photon splitting an opportunity to compete effectively with pair creation when $B \gtrsim B_\text{cr}$, since the pair threshold $\omega_B \sin \theta_{kB} = 2$ is always crossed from below. An approximate assessment of the relative importance of magnetic photon splitting and pair creation $\gamma \rightarrow e^+e^-$ was performed by Harding, Baring and Gonthier (1997) by computing attenuation lengths $L$ for each of these processes. These are the scalings for attenuation in the neutron star magnetosphere, and are defined to be path lengths over which the optical depth

$$\tau(\Theta, \varepsilon, l) = \int_0^l T(\theta_{kB}, \omega)ds$$

is unity, i.e. $\tau(\Theta, \varepsilon, L) = 1$. In computing such attenuation lengths, it is essential to fully include the effects of general relativity. The reason for this is that the quantum transition rates for both splitting and pair creation are strong functions of the photon energy and angle and the magnitude of the magnetic field strength in the local inertial frame, all of which are strongly influenced by curved spacetime. Such an analysis is confined to the Schwarzschild metric because the dynamical timescales for gamma-ray pulsars are considerably shorter than their period (e.g. $P = 0.15\text{ sec.}$ for PSR1509-58), so that rotation effects in the Kerr metric, for example those due to frame-dragging, can be neglected in our photon attenuation analysis. Throughout this paper, we assume a neutron star mass, $M = 1.4 M_\odot$ and radius, $R = 10^6\text{ cm}$.

Values for $L$ are computed assuming that test photons are emitted on or above the neutron star surface at some polar colatitude $\Theta$ (usually chosen to be at the polar cap rim) and propagate outward, initially at a specified angle $\theta_{kB,0}$ to the gravitationally-modified dipole magnetic field; a depiction of the geometry is presented in HBG97. Frequently in this paper, a surface origin of the photons is chosen to provide a concise and representative presentation of the attenuation properties of the pulsar magnetosphere. Such attenuation lengths possess power-law behavior at high energies, with $L \propto \varepsilon^{-5/7}$ for photon splitting, and $L \approx 2/\varepsilon$ for pair creation just above threshold (HBG97; proportionality terms that hold in both curved and flat spacetime). Furthermore, they display the property that the attenuation length declines with colatitude of emission, a consequence of the associated increase in curvature of the field lines. At low energies, the attenuation lengths diverge and photons escape the magnetosphere without attenuation. The $L \rightarrow \infty$ asymptotes define escape energies (HBG97), below (above) which spectral transparency (opacity) is effectively guaranteed. The existence of such escape energies is a consequence of the $r^{-3}$ decay of the dipole field: there are always sufficiently low photon energies for which the field declines before the photons have had sufficient time to attenuate in their passage through the magnetosphere.

2.1.3. Escape Energies

For each photon trajectory through the magnetosphere, corresponding to a specific set of values for the colatitude $\Theta$ and $\theta_{kB,0}$ of emission at the surface, and for a particular dipole surface field $B_0$, the escape energy $\varepsilon_{\text{esc}}$ is uniquely defined and cleanly delineates the energy ranges of source opacity and transparency. The escape energy also depends on the radius of emission $R_0$ (see Figure 5). The dependence of $\varepsilon_{\text{esc}}$ on such parameters is shown in Figure 1 for photons that are initially of polarization $\perp$. Escape energy plots for unpolarized photons are given in Figures 3 and 5 of HBG97. The feature of Figure 1 that is most salient for the considerations of this paper can be obtained by comparing the plots for splitting and pair production. For low fields, pair production escape energies are below those for splitting, but the situation is reversed in high fields; the escape energies are roughly equal for a narrow band of fields around $B_0 \sim 0.5B_\text{cr}$. For field strengths $B_0 \gtrsim B_\text{cr}$, photon splitting can attenuate photons well below pair threshold at significant colatitudes. Hence splitting can be expected to dominate $\gamma \rightarrow e^\pm$ in supercritical fields for the attenuation of photons of $\perp$ polarization.

The escape energies in Figure 1 generally decline with $\Theta$ and are monotonically decreasing functions of $B_0$ for the range of fields shown. Consider first the dashed curves in each panel, corresponding to the initial propagation of photons along the field. The divergences as $\Theta \rightarrow 0$ are due to the divergence of the field line radius of curvature at the poles. The maximum angle $\theta_{kB}$ achieved before the field falls off and inhibits attenuation is proportional to the colatitude $\Theta$. For photon splitting, since the rate in equation (1), and therefore also the inverse of the attenuation length $L$, is proportional to $\omega_B^2 \sin^6 \theta_{kB}$, it follows that the escape energy scales as $\varepsilon_{\text{esc}} \propto \Theta^{-6/5}$ near the poles. At fields $B_0 \gtrsim 0.3B_\text{cr}$, there is a diminishing dependence of $B$ in the attenuation coefficient (e.g. see Adler 1971; Baring & Harding 1997 for graphical illustrations) so that a saturation of the photon splitting attenuation lengths and escape energies arises in highly supercritical fields. For pair production, the behaviour of the rate (and therefore $1/L$) is exponential in $1/(\omega_B \sin \theta_{kB})$ (e.g. see Daugherty & Harding 1983), which then quickly yields a dependence $\varepsilon_{\text{esc}} \propto \Theta^{-3}$ near the poles for $B_0 \lesssim 0.1B_\text{cr}$. This behavior extends to higher surface fields because production then occurs at threshold, which determines $\varepsilon_{\text{esc}} \sim 2/\theta_{kB} \propto \Theta^{-1}$. The pair production escape energy curves are bounded below by the pair threshold $2/\sin \theta_{kB}$ at the point of pair creation (not photon emission), and for high $\Theta$ the pair approach the threshold pair production ($1 + \sqrt{1 + 2B_0/B_\text{cr}}/\sin \theta_{kB}$ with $\sin \theta_{kB} \sim 1$ determined by geometry), blueshifted by the factor $(1 - 2GM/R^2)^{-1/2} \sim 1.3$. The field dependence in this saturation energy arises because the creation of pairs by $\perp$-polarized photons cannot leave both the electron and positron in the lowest Landau level (e.g. see Daugherty & Harding 1983); the minimum energy configuration requires one member of the pair to be in the first excited state.

Since pulsar cascade high energy emission from curvature radiation, inverse Compton or synchrotron by relativistic particles with Lorentz factor $\gamma_e$ will not beam the photons precisely along the magnetic field, but within some angle $\sim 1/\gamma_e$ to the field, it is important to illustrate the effect on the escape energies of a non-zero angle of emission $\theta_{kB,0}$ of the photons relative to $B$. This is done via the solid curves in Figure 1, where $\theta_{kB,0} = 10^{-2}$ (i.e. $0.57^\circ$) is taken towards the dipole axis; qualitatively and quantitatively similar behavior arises when $\theta_{kB,0} = 10^{-2}$ is taken away from this axis. Curvature radiation-initiated cascades generally have $\gamma_e \sim 10^7$ (e.g. Daugherty & Harding 1989; see also Harding & Muslimov 1998), while inverse-Compton seeded pair cascades yield $\gamma_e \sim 3 \times 10^5 - 10^6$ (e.g. Sturmer
In Fig. 1b, the same effect is seen for pair creation, but this convergence is a consequence of the field being almost uniform and in Figure 1 aids clarity of illustration. Angles, the escape energy decreases and the curves flatten below the solid curves, at low magnetic colatitudes $\Theta$, and diverge near $\Theta = 0$, where the field line radius of curvature becomes infinite. For the range of parameters shown. The escape energies for each process are monotonically decreasing functions of $B$ for the range of parameters shown. The $\theta_{B,0}$ curves have slopes of $-6/5$ (splitting) and -1 (pair creation) at small $\Theta$, as discussed by Harding, Baring & Gonthier (1997), and diverge near $\Theta = 0$, where the field line radius of curvature becomes infinite. For the solid curves, at low magnetic colatitudes $\Theta \lesssim 10\theta_{B,0}$, the field curvature is so low that photon attenuation is insensitive to the value of $\Theta$ and is well described by the uniform field results in Eqs. (1) and (2). The escape energies are presented for surface photon emission, $R_0 = R$.

Fig. 1.— The escape energy (i.e. where $L \to \infty$) for (a) photon splitting and (b) pair production as a function of magnetic colatitude for photon emission both along $B$ (dashed curves) and at angle $\theta_{B,0} = 0.01$ radians ($= 0.57^\circ$) to the field (solid curves). The escape energies for each process are monotonically decreasing functions of $B$ for the range of parameters shown. The $\theta_{B,0}$ curves have slopes of $-6/5$ (splitting) and -1 (pair creation) at small $\Theta$, as discussed by Harding, Baring & Gonthier (1997), and diverge near $\Theta = 0$, where the field line radius of curvature becomes infinite. For the solid curves, at low magnetic colatitudes $\Theta \lesssim 10\theta_{B,0}$, the field curvature is so low that photon attenuation is insensitive to the value of $\Theta$ and is well described by the uniform field results in Eqs. (1) and (2). The escape energies are presented for surface photon emission, $R_0 = R$.

1995; see also Harding & Muslimov 1998), both yielding angles $\theta_{B,0}$ very much smaller than the example in the figure. The situation is similar for synchrotron radiation from first generation secondary electrons, where $\gamma_e \sim 10^2 - 10^4$; such Lorentz factors result from pairs created by primary photons (curvature or resonant Compton) in the 1 GeV–10 GeV range. However, as the cascade proceeds, higher generations of pairs achieve lower Lorentz factors, resulting in a cumulative power-law spectrum between $\gamma_e \sim 10^2$ and $\gamma_e \sim 10^4$ (Daugherty & Harding 1982). This behaviour applies to pulsars with low to moderate magnetic fields, $B_0 \lesssim 0.2B_{cr}$, a domain that is well-studied in the literature. For more highly-magnetized pulsars, the restriction of pair creation to a single state, discussed at length in Section 2.2, will inhibit the creation of second and higher generation pairs, implying that the dominant synchrotron signal will sample angles to the field much smaller than $\theta_{B,0} \sim 10^{-2}$. Hence, in summation, we expect that $\theta_{B,0} \lesssim 10^{-4} - 10^{-3}$ will be representative for the considerations of this paper. The $\theta_{B,0} = 10^{-2}$ choice in Figure 1 aids clarity of illustration.

The general behaviour of the $\theta_{B,0} = 10^{-2}$ curves in Figure 1 can be simply understood. For the most part, the escape energy is insensitive to the emission angle for $\Theta \gtrsim 10\theta_{B,0}$. For small angles, the escape energy decreases and the curves flatten below the $\theta_{B,0} = 0$ curves, converging as $\Theta \to 0$ to an energy that is proportional to $(B_0 \sin \theta_{B,0})^{-6/5}$ when $B_0 \lesssim B_{cr}$. This convergence is a consequence of the field being almost uniform and tilted at about angle $\theta_{B,0}$ to the photon path for trajectories that originate near the pole; this behaviour at low colatitudes was first noted, in the case of pair creation in flat spacetime, by Chang, Chen and Ho (1996). These “saturation” energies follow from the dependence of the photon splitting rate on $B$ and $\Theta$, and have a declining sensitivity to $B_0$ when $B_0 \gtrsim 0.3B_{cr}$. In Fig. 1b, the same effect is seen for pair creation, but this time the “saturation” is at the redshifted threshold energy for $\gamma \to e^\pm$, which varies as $(1 + \sqrt{1 + 2B/B_{cr}})/\sin \theta_{B,0}$. Hence, instead of the pair creation escape energies always being monotonically decreasing functions of $B_0$, they experience an inversion at small colatitudes and increase with $B_0$. Such an effect was not present in the polarization-averaged escape energies presented in HBG97 because the $\parallel$ state could always access the true pair threshold, i.e. $2/\sin \theta_{B,0}$. Note also, that while splitting escape energies always drop when $\theta_{B,0}$ is increased from zero to $10^{-2}$, opposite behaviour is seen in high fields for pair creation at moderate colatitudes, due to subtleties concerning the sudden onset of pair creation (HBG97).

It is important to emphasize that general relativistic effects are crucial to these calculations. While the dependence of the escape energy on emission colatitude and surface polar field strength is similar in curved and flat spacetime, the introduction of the Schwarzschild metric results in decreases of the escape energies for both process by factors of between 2 and 4, as is illustrated in Figure 4 of HBG97. Moreover, these decreases differ for splitting and pair creation, making it imperative to carefully account for propagation in curved spacetime when comparing attenuation characteristics for the two processes. The largest effects of the incorporation of general relativity are due to the increase of the surface dipole field strength by roughly a factor of 1.4 above the dipole spin-down estimate of $B_0$, and the correction for the gravitational redshift of the photon, which increases the photon energy by roughly a factor 1.2–1.3 in the local inertial frame at the neutron star surface compared to the energy measured by the observer in flat spacetime at infinity. The influence of these modification factors is amplified by the sensitivity of the rates for $\gamma \to \gamma \gamma$ and $\gamma \to e^+e^-$ to $B$ and $\varepsilon$ so that 20%-30% deviations from flat spacetime parameters map over to the factors of 2–4 in the escape energies. Near the polar cap, the curvature of the photon trajectory in a Schwarzschild metric does not affect the escape energies, to first order, since it is generally compensated for by relativistic modifications to the curvature of the magnetic field (e.g. see Gonthier & Harding...
This equality defines curves in the $(\Theta, B)$ plane, and therefore specify functional dependences of the dipole spin-down magnetic field on the colatitude of emission. These functions are displayed in Figure 2 for different values of the initial angle $\theta_{kB,0}$ of the photons relative to the field. These solutions define the basis for our considerations of the suppression of pair creation in the context of radio pulsars in Section 3 below.

To understand the behaviour of the curves, first consider the case where $\theta_{kB,0} = 0$. The escape energies plotted in Figure 1 have the approximate dependence $\varepsilon_{esc} \propto B_0^{3/4} \Theta^{-1}$, with $\alpha \approx 3/4$ for $B_0 \lesssim B_{cr} \lesssim 4 B_{cr}$ and $\beta = 6/5$ for splitting, and $\beta = 1$ for pair creation. The dependence of the pair creation escape energy on field strength for $B_0 \gtrsim B_{cr}$ is quite weak ($\alpha \approx 0$) so that solutions to Eq. (4) must yield escape energies approximately proportional to $\Theta^{-1}$. This can be immediately folded into the photon splitting proportionality $\varepsilon_{esc} \propto B_0^{3/4} \Theta^{-6/5}$ to yield the magnetic field dependence $B_0 \propto \Theta^{-4/15}$ in Figure 2, an approximation that is applicable for $B_{cr} \lesssim B_0 \lesssim 4 B_{cr}$, the range of fields of interest to the focus of this paper. The variation of $B_0$ with $\Theta$ actually increases very slowly at smaller colatitudes due to the diminishing dependence of the photon splitting rate for the $\perp \rightarrow \parallel$ mode on field strength in highly supercritical fields. The non-zero $\theta_{kB,0}$ solutions for $B_0$ are monotonically decreasing functions of $\theta_{kB,0}$, and eventually become independent of colatitude when $\Theta \lesssim 10^4 \theta_{kB,0}$, as is expected from the horizontal branches of the escape energy plots in Figure 2. Since the escape energy solutions trace the behaviour of the pair creation $\varepsilon_{esc}$, small increases with $\theta_{kB,0}$ are exhibited at moderate to high colatitudes, even though for $\Theta \lesssim 10^4 \theta_{kB,0}$, the solutions scale as $(\theta_{kB,0})^{-1}$, as expected.

2.2. Threshold Pair Production

As discussed in the previous sections, pair production by photons initially emitted at small angles to the field occurs very near threshold in magnetic fields exceeding $0.1 B_{cr}$. In this case, there are only a small number of kinematically available electron and positron Landau states. In very high fields, the accessible number of excited pair states diminishes, and the pairs created by most photons in the primary particle spectrum will occupy the ground state (for photon polarization $\parallel$) or the first excited state (for photon polarization $\perp$). Pairs in the ground state cannot produce synchrotron photons. Even pairs in the first excited state radiate photons of energy $\omega \approx \sqrt{1 + 2 B_{cr}/B_0} - 1$ for electron momentum $p_\perp$ parallel to the field, which will generate very few pairs (Harding & Daugherty 1983), and certainly none if $p_\perp^2 < 4(1 + \sqrt{1 + 2 B_{cr}/B_0})$ for photons of $\perp$ polarization. This phenomenon thus provides another mechanism for suppressing the number of pairs produced by pulsar cascades. Although pairs in the ground state could be excited to higher Landau levels through inverse-Compton scattering of soft X-ray photons (e.g. Zhang & Harding 2000), we leave discussion of the potential importance of this effect to the end of this section.

First, we examine the effect of threshold pair production. The threshold photon energy in the center-of-momentum (CM)
Analyze the text to identify key points and transformations:

- The number of accessible pair states, \( N_{\text{states}}(\omega_{\text{CM}}, B) \), at the point of pair creation, is given by:
  \[
  N_{\text{states}}(\omega_{\text{CM}}, B) \approx \frac{1}{\pi} \frac{(B_{\text{cr}})^2}{B_{\text{cr}}} \omega_{\text{CM}}(\omega_{\text{CM}} + 4) (\omega_{\text{CM}} - 2)^2,
  \]
  where \( \omega_{\text{CM}} = \omega \sin \theta_{\text{KB}} \) is the energy in the CM frame and \( B_{\text{cr}} \) is the critical magnetic field.

- Photon energy and pair creation: The condition for pair creation is
  \[
  \frac{B}{B_{\text{cr}}} < 0.1
  \]
  where \( B_{\text{cr}} \) is the critical magnetic field. For \( B_{\text{cr}} < 0.1 B_{\text{cr}} \), photons at all energies create pairs in the ground state or the first excited state.

- The number of accessible pair states is further limited by the field strength and the energy of the photon, with significant decreases observed at very low and very high fields.

- The attenuation length \( L \) is given by:
  \[
  L \propto \rho_c^{-1/2} \approx \frac{B}{B_{\text{cr}}},
  \]
  where \( \rho_c \) is the cyclotron radius.

- The escape energy, \( \omega_0 = B/B_{\text{cr}} \), is crucial for calculating the number of photons that escape.

- The analysis above is valid for relatively low fields, with significant deviations observed at very low and very high fields.

- Inverse-Compton scattering contributes to the production of high-energy photons, which are crucial for understanding the radiation from pulsars.

- The condition for pair creation is determined by the field strength and the energy of the photon, with significant decreases observed at very low and very high fields.

- The attenuation length is a key parameter for calculating the number of photons that escape.

- The escape energy is a crucial parameter for understanding the radiation from pulsars.

- The analysis above is valid for relatively low fields, with significant deviations observed at very low and very high fields.

- Inverse-Compton scattering contributes to the production of high-energy photons, which are crucial for understanding the radiation from pulsars.
are fairly relativistic. The mean-free path of these particles to scattering will then be
\[
\lambda_s \simeq \frac{1}{\eta \sigma_T n_e} = 7.5 \times 10^4 \eta^{-1} T_6^{-3} \text{ cm},
\]
where \( T_6 \equiv T/10^6 \) K and \( \eta \) is a suppression factor for the scattering cross section \( \sigma = \sigma_T \) in the Klein-Nishina regime. Such mean free paths are short enough to suggest that excitation through scattering is potentially important. However, these photons scattering above the resonance in the electron rest frame will not form the bulk of the produced pairs, because the resonant cross section (and thus the scattering rate) is several orders of magnitude larger than for non-resonant cross section.

2.3. Positronium Formation

Another mode of pair creation exists, namely the formation of pairs in a bound state, i.e. positronium. This has been proposed as an effective competitor to the production of free pairs (Shabad & Usov 1985, 1986; Herold, Ruder & Wunner 1985; Usov & Shabad 1985; Usov & Melrose 1995) because the binding energy lowers the threshold slightly (\( \ll 1 \% \)) below the value for production of free pairs. Positronium formation fundamentally alters the magnetosphere: it amounts to the suppression of electron or positron currents that can screen the induced electric fields in the rotator. Furthermore, depending on how long the pairs remain bound in their passage along open field lines to the light cylinder, such a presence of neutral positronium will help to suppress any collective plasma modes that might spawn radio emission. Hence positronium formation is a viable alternative means of inhibiting the radio signal from pulsars.

The relevance of positronium formation to pulsars is potentially great if the bound state is stable for considerable times. Positronium is subject to destruction by three main mechanisms: free decay, electric field ionization, and photo-ionization. Using the spontaneous two-photon decay rate (in the positronium rest frame) of \( 3.5 \times 10^{14} (B/B_{cr}) \text{ sec}^{-1} \) computed by Wunner & Herold (1979), Bhatia, Chopra & Panchapakesan (1987) obtained positronium decay lengths considerably shorter (by two orders of magnitude for bulk Lorentz factor \( \gamma_p = 10^3 \)) than the stellar radius for \( B \sim 0.2B_{cr} \); the length is a strongly declining function of \( B \). Hence it appears that positronium is unlikely to be long-lived in the magnetosphere except for a narrow range of fields, or for sufficiently high positronium Lorentz factors, i.e. from the attenuation of photons above 10 GeV. Approximate estimates of when field ionization becomes important were obtained by Herold, Ruder & Wunner (1985) and Usov & Melrose (1996): for pulsar periods \( P \lesssim 0.5(10B_0/B_{cr})^{-2/3} \text{ sec} \), ionization due to \( E_\| \) should convert positronium into free pairs. This corresponds to a sizeable portion of the phase space of interest for the considerations of this paper. In addition, photo-ionization can destroy positronium, forming free pairs. Herold, Ruder & Wunner (1985) claimed that this was highly likely due to collisions with thermal radiation emanating from the stellar surface for temperatures greater than around \( 10^8 \) K. Usov & Melrose (1995) used the calculations of Bhatia, Chopra & Panchapakesan (1992) to argue that photo-ionization timescales exceed dynamical ones for surface temperatures \( \sim 10^8 \) K. This discrepancy is yet to be resolved.

2.4. Pair Cascades in High Magnetic Fields

We have discussed several of the physical mechanisms capable of suppressing pair production when the local magnetic fields approach and exceed the critical field. To more realistically measure pair suppression in high magnetic fields, one must examine not only the propagation of single high-energy photons through the neutron star magnetosphere (i.e., the escape energies) but also the transport of the higher generations of photons and particles, which are produced by the primary photons that do not escape. Even though the first generation of photons split instead of producing pairs, the second generation of photons may create pairs. It is therefore necessary to investigate pair yields of the pair/photon cascades which are initiated by the primary photons. In Section 3.2, we will present results of numerical simulations of such pair cascades. In this section, we will qualitatively discuss how we expect the nature of these cascades to change as the magnetic field increases.

![Fig. 4.— The nature of pair cascades in increasing magnetic fields.](image)

- Low Field Cascades: \( B < 0.1 B_{cr} \)
  - No Splitting
  - Splitting Only
  - 3 Splitting Modes
- High Field Cascades: \( B > 0.1 B_{cr} \)
  - \( \perp \rightarrow \| \) Splitting Only
  - \( \perp \rightarrow \perp \) Splitting Only
  - \( \perp \rightarrow \| \perp \) Splitting Only
At fields $B \gtrsim 0.5B_{\text{cr}}$, the photon splitting attenuation lengths become shorter than pair attenuation lengths. At this point, the nature of the pair cascades will depend on which modes of splitting are operating at what field strengths. In fields $B \lesssim B_{\text{cr}}$, it is probable that only the $\perp - ||$ mode operates (see Section 2.1.1). Thus, primary $||$ mode photons can produce pairs, but with both members in the ground state. The primary $\perp$ mode photons above photon splitting escape energy will split into two $||$ mode photons, each of which can produce a pair in the ground state. Thus, each polarization branch of the cascade ends after, at most, one pair generation. The action of photon splitting in one mode therefore will lower the cascade pair yield above $0.5B_{\text{cr}}$, but only moderately.

If, as was discussed in Section 2.1, other modes of photon splitting open up in magnetic fields above $B_{\text{cr}}$ due to increased vacuum dispersion, then the character of pair cascades undergo a further transition. If both photon polarization modes can split (as is the case when all three modes permitted by QED are operating), then it is possible to completely prevent the production of pairs. A pure photon splitting cascade then ensues, in which photons split repeatedly until they can escape the magnetosphere. The only question then becomes whether the splitting cascade can degrade the photon energy fast enough so that the photons escape without pair conversions in lower fields. Since the photons are splitting below pair threshold in high fields, this would seem very likely.

3. PAIR SUPPRESSION IN HIGHLY-MAGNETIZED PULSARS

Pair creation is obviously pertinent to the hard X-ray and gamma-ray emission of pulsars. Yet it is also relevant to the discussion of coherent emission in radio pulsars, since it is commonly assumed that a plentiful supply of pairs is a prerequisite for, and maybe also a guarantee of, coherent radio emission at observable flux levels. Such a connection is the premise of standard models for radio pulsars (e.g. Sturrock 1971; Ruderman & Sutherland 1975; Arons & Scharlemann 1979), a relationship founded in the simplicity of its explanation (Sturrock, Baker & Turk 1976) for the extinction of radio pulsars beyond the conventional death line at long periods. Detailed discussions of the relationship of pairs to the production of radio emission can be found in Michel (1991) and Melrose (1993), though we note that there are dissenting views to this popular connection (e.g. Weatherall & Eilek 1997).

The pre-eminent consequence of any suppression of pair creation in pulsars is that the emission of radio waves should be strongly inhibited. Hence the issue of possible quenching at high $B_0$ by the mechanisms discussed above forms the focus of this Section. First we compute the putative radio quiescence boundaries in the $\dot{P} - P$ diagram using the approximate criterion for the suppression of pair creation by photon splitting defined in Section 2.1.3, namely when the escape energy for the splitting of $\perp$ photons is less than or equal to that for pair creation $\perp - e^\pm$. Essentially this provides a detailed derivation of the boundary introduced in Baring & Harding (1998). We then model the physics of pair suppression near pulsar polar caps by means of a Monte Carlo simulation of pair cascades in high fields, treating both photon polarization states. These detailed calculations identify the requirements for effective pair suppression which must be met to justify the simplified criterion for pair suppression of Section 2.1.3.

3.1. Radio Quiescence Boundaries in the $P - \dot{P}$ Diagram

In Section 2.1.3, the condition for the equality of photon splitting and pair creation escape energies established a region in $(B_0, \Theta)$ space where pair suppression would probably occur. This space can be easily transformed into a region in the radio astronomer’s traditional pulsar phase space, which consists of the measurables $P$, the pulsar period, and $\dot{P}$, the period derivative. The first part of this transformation derives from the relationship between the period and the size of the polar cap $\Theta$. Such a relation follows from the definition of the cap as the portion of the stellar surface that anchors open field lines, i.e. those that are not closed within the light cylinder. In flat spacetime, for a rotating dipole field, the angle $2\Theta$ subtended by the polar cap can be expressed via (e.g. Manchester and Taylor 1977) $\sin \Theta = [2\pi R/(Pc)]^{1/2}$. Due to general relativistic distortions of the dipole in a Schwarzschild metric, this formula is modified to the form (Muslimov and Harding 1997)

$$\sin \Theta = \left\{ \frac{2\pi R}{Pc} \mathcal{F} \left( \frac{R_s}{R} \right) \right\}^{1/2},$$

$$\mathcal{F}(x) = -\frac{3}{3} \left[ \log_e(1 - x) + \frac{x}{2} (x + 2) \right]^{-1}$$

that is used in this paper. Here $R_s = 2GM/c^2$ is the Schwarzschild radius, and the result for any radius $r$ can be obtained by the substitution $R \rightarrow r$. Clearly, the strong gravitational field reduces the size of the polar cap ($\mathcal{F}(x) \approx 1 - 3x/4$ for $x \ll 1$), arising in conjunction with its intensification of the magnetic field. Note that distinctly speaking, the polar cap connects to field lines that are not closed within the Alfvén radius $r_A$, however, the intense pulsar fields yield relativistic Alfvén speeds so that $r_A$ is generally close to the light cylinder radius $Pc/(2\pi)$. The loading of the field with plasma can have a significant impact on $r_A$ and $\Theta$ in soft gamma repeaters, as discussed in Section 4.3 below.

The second half of the transformation of the $(B_0, \Theta)$ space to the $(P, \dot{P})$ phase space arises from the commonly assumed spin-down relationship between the dipole spin-down field $B_0$ and the measurables. For radio pulsars, the increase in period is usually attributed to magnetic dipole radiation that taps the rotational kinetic energy of the neutron star. This couples $B_0$ to $P$ and $\dot{P}$ through equating the rotational energy loss to the dipole radiation energy loss which is specified in terms of the total magnetic moment $\mu_0$. The radio pulsar community has commonly adopted the approximation $\mu_0 \sim B_0 R^2$ (e.g. see Manchester and Taylor 1977). The actual dipole moment of a star-centered dipole, regardless of internal field configuration, is $\mu_0 = (B_0 R^2)/2$ (e.g. see Shapiro and Teukolsky 1983, Usov and Melrose 1995). This choice, which we adopt throughout this paper, leads to a dipole radiation loss rate of $dE/dt = -B_0^2 R^4 \Omega^2/(6c^3)$ for an orthogonal rotator of angular frequency $\Omega = 2\pi/P$, and

$$B_0 = 6.4 \times 10^{19} \sqrt{P \dot{P}} \text{ Gauss}$$
in the understanding of the global field structure are sufficient that the use of Eq. (9) is justified for the purposes of this paper. Note that curved spacetime does not impact the determination of $B_0$, a flat spacetime quantity, but just increases the value of the field in the local frame at the pole over $B_0$; this increase in magnetic field energy density can be viewed as a general relativistic “redshifting” of the Poynting flux of the rotator.

By inverting Eq. (9) to solve for $\dot{P}$ in terms of $B_0$ and $P$, the solutions $B_0(\Theta)$ to Eq. (4) that demarcate the domain of possibly strong suppression of pair creation by photon splitting are mapped onto the $P-\dot{P}$ diagram in Figure 5. Curves are depicted for three of the four values of $\theta_{KB,0}$ that are addressed in Eq. 2.4, specifically for the case of surface photon emission, and also for a $\theta_{KB,0} = 0$ case of emission above the surface. Much of the extent isolated radio pulsar population is also exhibited (only those with positive $\dot{P}$), namely those sources listed in the Princeton Pulsar Catalogue (Taylor, Manchester & Lyne 1993, with 541 pulsars: see http://pulsar.princeton.edu/), with a recent update from the Parkes Multi-Beam Survey (adding 122 new pulsars with presently archived $P$ and $\dot{P}$: see http://www.atnf.csiro.au/~pulsar/psr/psrcat/pmsurv/pmsurv/).

Note that from the discussion in Section 2.1.3, the $\theta_{KB,0} = 0$ boundary for surface emission ($R_0 = R$) corresponds roughly to $B_0 \propto \Theta^{-4/15}$ and hence to $\dot{P} \propto P^{-11/15}$; specifically, we find the approximation

$$\dot{P} \approx 7.9 \times 10^{-13} \left(\frac{P}{\text{ms}}\right)^{-11/15},$$

the slope of which depends on the how strongly the rate of photon splitting depends on $B$ in this regime of fields. It should also be noted that while such putative boundaries of radio quiescence have been labelled by specific values of $\theta_{KB,0}$ for the purpose of illustration, in fact the value of this initial angle of photon propagation with respect to the field is a weak function of the colatitude. This is due to the coupling between the Lorentz factor of accelerated electrons and therefore $\theta_{KB,0}$ and the polar cap size $\Theta$, a connection discussed below in Section 4.2. Also depicted in Figure 5 is a fiducial positioning (adapted by Taylor, Manchester & Lyne 1993) of the conventional death line, with $\dot{P} \propto P^3$ (i.e. corresponding fixed open field line voltage, adopted), to the right of which there is just one detected radio pulsar (PSR J2144-3933). It is a well-known problem that the computed position of the pulsar “death line”, assuming pairs are produced only by curvature radiation photons, lies at smaller periods than the observed “death line”, for the assumption of the standard polar cap size (e.g. Arons & Scharlemann 1979). However, increasing the polar cap size to less than twice that of the standard brings the computed “death line” into agreement with virtually the entire radio pulsar population, the notable exception being the recently “re-discovered” 8.5 second Parkes pulsar (Young, Manchester & Johnston 1999) that is conspicuous on the right hand side of Fig. 5. However, pair production by inverse Compton-scattered photons can explain radio emission from this pulsar (Zhang et al. 2000).

The Princeton catalog by itself clearly rules out boundaries of radio quiescence due to photon splitting that correspond to $\theta_{KB,0} \gtrsim 3 \times 10^{-4}$ for surface emission. The fact that the $\theta_{KB,0} = 0$, $R_0 = R$ boundary was comfortably located above the entire Princeton collection of radio pulsars was an attractive feature that underpinned the suggestion of Baring & Harding (1998) that this particular case marked the approximate location of the boundary of radio quiescence, where photon splitting could strongly inhibit pair creation. However, this conclusion has been challenged by the dramatic increase in the observed population due to the exciting new Parkes Multi-Beam survey (Camilo et al. 2000; D’Amico et al. 2000; Kaspi et al. 2000), with three new pulsars (PSRs J1119-6127, 1726-3530 and 1814-1744, all with high dispersion measures in the 700–850 range) having been discovered that are obviously at $\dot{P}$ above the $\theta_{KB,0} = 0$, $R_0 = R$ boundary.

Fig. 5.— The conventional depiction of pulsar phase space, the $P-\dot{P}$ diagram, with filled circles denoting the locations of 541 members (those with $\dot{P} > 0$) of the latest edition (Taylor, Manchester & Lyne 1995; see also http://pulsar.princeton.edu/) of the Princeton Pulsar Catalogue, and open circles marking 122 pulsars in the recent Parkes Multi-Beam survey [http://www.atnf.csiro.au/~pulsar/psr/psrcat/pmsurv/pmsurv/]. Three of the handful of gamma-ray pulsars, namely the Crab, Vela and PSR 1509-58 are highlighted, with point styles as indicated in the inset. The dotted diagonal lines denote constant field strength, as inferred from Eq. (9). A fiducial positioning of the conventional death line, from $P-\dot{P}$ diagrams at http://pulsar.princeton.edu/, is depicted as the dashed line on the right. The heavy solid curves give a variety of choices for the boundary of radio quiescence for surface emission, depending on the value of $\theta_{KB,0}$, the initial angle of photons relative to the field. In addition, the heavy dashed curve is a similar boundary for $\theta_{KB,0} = 0$ and emission point half a stellar radius above the surface (i.e. $R_0 = 1.5R$). Each of these curves, defined by solutions to Eqs. (4)–(9), purportedly partitions the $P-\dot{P}$ diagram into regions of radio loud (below the curve) and radio quiet (above) pulsars, the latter being where photon splitting suppresses pair creation for the $\perp$ polarization state. Values $\theta_{KB,0} \lesssim 10^{-3}$ and $r \gtrsim 1.2R$ can comfortably accommodate the current radio pulsar population.
Although the boundary of radio quiescence for surface emission (chosen for illustrative purposes by Baring & Harding 1998) neatly falls between the population in the Princeton catalog and the handful of known anomalous X-ray pulsars and soft gamma repeaters (see the blow-up of the $P - \dot{P}$ diagram in Figure 9), most (or perhaps all) of which are radio quiet, clearly the new Parkes survey data questions this assumption of surface emission. The quiescence boundary rapidly moves up on the $P - \dot{P}$ diagram for emission at higher altitudes, which might naturally be expected for traditional curvature radiation-initiated cascades in pulsars with Crab-like surface fields (e.g. around $R$ above the surface for Daugherty & Harding’s 1996 modeling of the Vela light curve). However, for the high $B$ fields sampled by the $\theta_{B,0} < 10^{-4}$ curves, inverse Compton is probably more relevant to the acceleration region (see Sturmer 1995; Harding & Muslimov 1998) and low altitude emission is much more probable, as discussed in Section 4.2 below. One might then expect that the mean radius of emission satisfies $r \lesssim 1.5R$. Accordingly, we have computed a $\theta_{B,0} = 0$, $R_0 = 1.5R$ boundary and depicted it in Figure 5; it clearly lies above even the highly-magnetized Parkes Multi-Beam survey pulsars. Intuitively, it might be anticipated that the location of the boundary as a function of emission radius $R_0$ should scale simply by the relationship between field strength and radius for a dipolar structure, so that $R_0 \propto r^3$. This appears to be borne out near the stellar surface, however the radial dependence of the location of the quiescence boundary weakens at altitudes $\sim R$ (in spite of the reduced effects of general relativity at $r > R$), principally because of changes in the flaring of field lines sampled by photons in their flight away from the neutron star. The present Parkes survey data constrain computed quiescence boundaries to the $r \gtrsim 1.2R$ range; in the light of the present indeterminacy in the actual locale of the acceleration region (addressed in Section 4.2), this is not a serious problem.

3.2. Simulations of Photon Splitting/Pair Cascades

Using the equality of photon splitting and pair creation escape energies for $\perp$ polarization as the criterion for when splitting starts to strongly suppress the production of pairs is a simple choice, motivated largely by the predominance of production of $\perp$ photons in the relevant emission processes. It is necessary to determine whether and under what conditions this choice is an appropriate one.

To assess the applicability of the radio quiescence boundaries, we have studied pair suppression in high magnetic fields by means of Monte Carlo simulations of photon splitting/pair cascades in a neutron star magnetosphere. This calculation generalizes that of HBG97 by including the cyclotron and synchrotron radiation from the electron-positron pairs. This improvement is necessary in order to compute the pair yields from all generations of the cascade. Note that we omit resonant Compton scattering from our simulations since, while it is important as a radiation process for primary and secondary particles, it will not significantly impact the spatial transfer of photons in normal radio pulsars due to the low densities and small angles; it may, however, be significant in soft gamma repeaters. We inject photons parallel to the local magnetic fields at the neutron star surface with specified magnetic colatitude $\Theta$ and four-momentum $k$. The photon energies are sampled from a power-law distribution,

$$N(\varepsilon) = N_0 \varepsilon^{-\alpha}, \quad \varepsilon_{\text{min}} < \varepsilon < \varepsilon_{\text{max}},$$

where we generally set $\varepsilon_{\text{min}} = 1$, and let $\varepsilon_{\text{max}}$ be a free parameter. In the results presented in this paper, we generally take $\alpha = 1.6$ as a spectral index representative of either curvature radiation with energy losses (giving $\alpha = 5/3$) or resonant inverse Compton emission which is relatively hard. Polarization is chosen randomly such that the average distribution has 75% in the $\perp$ mode and 25% in the $\parallel$ mode, reflecting the polarization produced by the radiation mechanisms discussed at the beginning of Section 2. The path of each input photon is traced through the magnetic field, in curved spacetime, accumulating the survival probabilities for splitting, $P_{\text{surv}}$, and for pair production, $P_{\text{pair}}$, independently:

$$P_{\text{surv}}(s) = \exp\left\{ -\tau(\Theta, \varepsilon, l) \right\}$$

where $\tau(\Theta, \varepsilon, l)$ is the optical depth as defined in Eqn. (3). Each photon may split, produce pairs or escape, based on a combination of the running survival probabilities for splitting and pair production (see HBG97). If a photon splits, the energies and polarizations of the final photons are sampled from the distribution from the branching ratios given in HBG97. Each final photon is then followed in the same way as the parent photon. If a photon produces pairs, the total energy, Landau state and parallel momentum of the electron and positron are determined. Each member of the pair is assumed to have half the energy and the same direction of the parent photon (technically this is only appropriate for high Landau states), except when the pair is produced at threshold (i.e. in the [0,0] state for $\parallel$ polarization and [0,1] or [1,0] for $\perp$ polarization), in which case we use the full kinematic equations to determine the energy and momentum of each pair member. Each member of the pair occupying an excited state emits a sequence of cyclotron or synchrotron photons. The method used to simulate the cyclotron/synchrotron emission is similar to that of Daugherty & Harding (1996). If the particle Landau level is larger than 20, the high-energy limit of the quantum synchrotron transition rate (Sokolov & Ternov 1968) is used, in which case we assume that the photons are emitted perpendicular to the magnetic field in the particle rest frame (high-energy limit). When the particle Landau level is smaller than 20, the exact QED cyclotron transition rate (Harding & Preece 1987) is used, in which case the angles of the emitted photons are sampled from a distribution. In both cases, the emitted photon polarizations are sampled from the corresponding polarization distributions. Each emitted photon is propagated through the magnetic field from its emission point until it splits, produces pairs or escapes. The cyclotron/synchrotron emission sequence continues until each particle reaches the ground state. The cascade continues until all photons from each branch have escaped. Throughout the cascade, a running tally is kept of the number of pairs produced and of the number of photon splittings. General relativistic effects of a Schwarzschild metric on the photon momentum and dipole magnetic field, are included in the same way as described in HBG97.

We have chosen the total number of pairs produced per injected photon as a quantitative measure of the pair yield of the cascade initiated by a particular parent photon spectrum. The free parameters are then surface magnetic field strength, emission colatitude $\Theta$, spectral index $\alpha$, and minimum, $\varepsilon_{\text{min}}$ and maximum, $\varepsilon_{\text{max}}$, energy of the primary photon spectrum. Cascade simulations have been run for the cases where one mode of splitting operates (the $\perp \rightarrow ||$ mode), three modes of splitting operate (the $\perp \rightarrow ||$, $\perp \rightarrow \perp \perp$ and $|| \rightarrow \perp \parallel$ modes permitted by
QED), and where splitting is turned off completely (“no splitting”). Figure 6 shows the cascade pair yield as a function of magnetic field strength and maximum primary photon energy; since the total number of photons and the pair yield depend on the value of $\varepsilon_{\text{min}}$, we hold $\varepsilon_{\text{min}} = 1$ constant for all calculations.

The curves for the “no splitting” case, shown only for $\varepsilon_{\text{max}} = 10^2$ and $\varepsilon_{\text{max}} = 10^6$ to avoid confusion, are of course not physical but are included to illustrate the effect of one splitting mode on the pair yield. When $\varepsilon_{\text{max}}$ is below the pair escape energy (cf. Section 2.1.3), as it is for the $\varepsilon_{\text{max}} = 10^2$ and $\varepsilon_{\text{max}} = 10^6$ curves at lower field strengths, the pair yield is considerably lower than when $\varepsilon_{\text{max}}$ is well above pair escape energy (as is the case for the $\varepsilon_{\text{max}} = 10^6$ and $\varepsilon_{\text{max}} = 10^6$ curves), for which pair yields at the lower field strengths can be quite large due to the pair multiplication effect through successive generations, as described in Section 2.4. Above a field strength of 0.1$B_{\text{cr}}$, in $\varepsilon_{\text{max}} \geq 10^4$ cases the pair yields drop because the pairs are produced in low Landau states at or near threshold, suppressing synchrotron emission and thus the pair multiplication effect. In the absence of photon splitting, the pair yield increases slowly with increasing $B_0 > 0.1B_{\text{cr}}$, because the number of generations of the cascade can increase. The rapid rise of the $\varepsilon_{\text{max}} = 10^2$ case when $B_0 \lesssim 0.2B_{\text{cr}}$ is due primarily to the escape energy for pair production $\frac{\varepsilon_{\text{esc}}}{\varepsilon_{\text{esc}}} - e^\pm e^\mp$ being close to $\varepsilon_{\text{max}}$, so that the optical depth is of the order of unity or less; the pair yield must then necessarily be a strong function of $B_0$ by virtue of the exponential asymptotic form (see Daugherty & Harding 1983) for the pair creation rate. Since the pair escape energy saturates when $B_0 \gtrsim 0.5B_{\text{cr}}$ at approximately the threshold value of $2/\sin \Theta$ (see Figure 1), the pair yield should asymptote to a constant value at high field strengths. This constant is just the fraction of primary photons above pair threshold: when $\varepsilon_{\text{max}} \gg 1$, this fraction is approximately $(\sin \Theta/2)^{\alpha - 1}$, which evaluates to $\approx 0.15$ for a polar cap size of $\Theta = 5^\circ$ and $\alpha = 1.6$. Reducing $\varepsilon_{\text{max}}$ clearly diminishes the proportion of photons above pair threshold.

In the presence of photon splitting, above $B_0 \sim 0.5B_{\text{cr}}$, the cascade is limited to only one pair generation in each polarization mode, amounting to one pair per $\parallel$ mode photon injected above the pair escape energy plus two pairs per $\perp$ mode photon injected above the splitting escape energy (see Figure 4). The results clearly indicate that photon splitting in only one mode does not significantly suppress the pair yield. As long as one photon polarization mode does not undergo splitting, there is always a channel for pair production. In fact, photon splitting in one mode can even increase the pair yield, as is evident in Figure 6 for field strengths between $0.2B_{\text{cr}}$ and $0.7B_{\text{cr}}$, for $\varepsilon_{\text{max}} \geq 10^4$. Consequently, the putative radio quiescence boundaries in Fig. 5 are not borders to domains of significant pair suppression if photons of $\parallel$ polarization cannot split. While splittings $\perp \rightarrow \parallel\parallel$ can compete effectively with pair creation, the two produced $\parallel$ photons can only create pairs if splitting of $\parallel$ photons is forbidden by kinematic selection rules. In such a case, pair creation is prolific unless the second generation $\parallel$ photons are around or below the escape energy for pair production, $\parallel\parallel \rightarrow e^\pm e^\mp$, i.e. the original $\perp$ photon is of energy $\lesssim 2\varepsilon_{\text{esc}}$, i.e. the energy range $\parallel\parallel \rightarrow e^\pm e^\mp \lesssim \varepsilon \lesssim 2\varepsilon_{\text{esc}}$. Therefore, when only $\perp$ mode photons can split, pair creation is inhibited only when $\varepsilon_{\text{max}} \lesssim 2\varepsilon_{\text{esc}}$. The situation changes dramatically if three modes of photon splitting (those allowed by QED) are operating. As shown in Figure 6 for the case of $\varepsilon_{\text{max}} = 10^2$ and $\varepsilon_{\text{max}} = 10^4$ (the other cases not shown behave similarly to this case), the pair yield drops dramatically above $B \sim 0.4B_{\text{cr}}$, depending on $\Theta$, when photons of both polarization modes are allowed to split. Pure photon splitting cascades can now take place, and typically exceed ten photon generations. In fields above $0.4B_{\text{cr}}$, very few pairs are created because the photon splitting cascade degrades the photon energies below the pair escape energy before the cascade reaches a height where pair production can take over. Figure 7 shows the effect of varying the primary photon injection colatitude on the cascade pair yield, as a function of magnetic field strength for cascades where three modes of splitting are allowed. The field strength at which the pair yield begins to drop increases with decreasing colatitude, due to the increasing field line radius of curvature. This effect was seen in the radio quiescence boundary, where $B_0 \propto \Theta^{-4/15}$ was computed in Section 3.1 by equating pair and splitting escape energies. A similar boundary may be predicted by equating the polarization-averaged escape energies for pair creation and photon splitting

![Figure 6](image-url)
with the result being a virtually identical dependence of $B_0$, on $\Theta$, but with a constant of proportionality that is a factor of about 1.7 lower than that in Eq. [10]. Such a polarization-averaged boundary of radio quiescence reproduces the ($B_0$, $\Theta$) phase space corresponding to the precipitous declines observed in Figure 7. Hence it can be concluded that the simplified criterion for pair suppression by splitting is fairly accurate if all three QED-permitted modes of splitting are operating.

![Image of Figure 7](image_url)

**Figure 7.** The cascade pair yield (number of pairs per injected photon) as a function of surface magnetic field strength, $B_0$, in units of the critical field, $B_{cr}$, for different primary photon injection colatitudes, $\Theta$, for the case where three photon splitting modes operate. Pair suppression becomes substantial in near-critical and supercritical fields when pure photon splitting cascades operate. Fluctuation in the curves at the highest field strengths is numerical noise due to low counts. Again, surface emission of photons is assumed.

As a more complete representation of the relevant phase space, contour plots of the pair yield as a function of magnetic field and pulsar period are shown in Figure 8 for the cases of one and three splitting modes. The period is derived from the surface injection colatitude assuming emission at the rim of a polar cap of standard size $\Theta$ as given in Eq. (8). In the left panel of Figure 8, which shows the case for one splitting mode, the most dramatic feature is the sharp fall-off of pair yield at large pulsar periods. This effect is purely a consequence of the polar magnetic field geometry: at large periods (small injection colatitudes) the radius of curvature of the field lines is larger and the pair production attenuation length can exceed a stellar radius, suppressing pair cascades. This defines the “death line” for radio pulsars. There is a weak dependence of the “death line” on magnetic field strength for $B \lesssim 0.1B_{cr}$, because the pair attenuation length saturates due to threshold pair production. With only one splitting mode in operation, the pair yield has a relatively weak variation with magnetic field. However, there is a maximum in pair yield at fields $B \lesssim 0.1B_{cr}$ and short periods, because pairs are produced in high Landau states.

In the right panel of Figure 8, which shows the case where three splitting modes are operating, the most dramatic feature is a sharp fall-off of pair yield at magnetic fields around $B \sim B_{cr}$. This feature is the radio quiescence line due to photon splitting as described in Section 3.1. The period dependence of this line is consistent with that inferred from the escape energy calculation, i.e. $B_0 \propto \Theta^{-4/15} \propto P^{-2/15}$. The radio “death line” is also apparent as a decline in pair yield at large periods.

4. DISCUSSION

4.1. Radio quiescence?

The results of the previous section show that if three modes of splitting operate at high field strengths, then there is nearly complete pair suppression at high field strengths and the approximate condition for radio quiescence proposed in Section 3.1 is valid. If only one mode of splitting (i.e. $\perp \rightarrow ||$) operates, then some other effect must act to reduce the pair yield, otherwise, pair creation in cascades remains prolific at near-critical and supercritical fields, and there is no reason to expect a regime of radio quiescence in the upper portion of the $P-P$ diagram. An answer to the question of how many modes of photon splitting operate in highly dispersive magnetic fields is not presently available and will require detailed investigation into the behavior of the photon splitting rate in this regime.

We have discussed several ways in which a suppression of pair creation at high $B$ can be present if only one splitting mode operates. First, if the effective maximum energy $\varepsilon_{max}$ of primary photons satisfies $\varepsilon_{max} < 2\varepsilon_{esc}$, then the pair yield must be dramatically reduced whenever such photons are mostly of $\perp$ polarization. However, there is no reason a priori to believe that such low values of $\varepsilon_{max}$ should arise when $B_0 > B_{cr}$. For curvature radiation, the spectrum is only dependent on $B_0$ via acceleration properties. For resonant Compton scattering, the spectral dependence on $B_0$ is more complicated and needs to be the subject of future investigation. While the maximum electron energy declines somewhat with $B_0$ (e.g. Harding & Muslimov 1998; see the discussion just below), the primary photon spectrum is flat up to $B \gtrsim 10^{12}$ Gauss. This value is below the spin-down fields of the most magnetic radio pulsars, so that evidently ground state pair creation cannot be primarily responsible for imposing radio quiescence. Yet this physical effect could be a contributory factor, noting that the extent pulsar population is somewhat thinner in the $P-P$ diagram than might be expected at $B_0 > 10^{15}$ Gauss. A related possibility (as discussed in Section 2.3) is that positronium formation may inhibit radio emission without reductions in pair creation. But even if stable positronium formation does occur in pulsar magnetospheres, it cannot account for observed radio quiescence in pulsars because the predicted onset of positronium formation occurs at field strengths $\sim 0.1B_{cr}$, in the middle of the radio pulsar population. Indeed, Usov & Melrose (1996) invoke bound-state pair creation in their model for gamma-ray emitting radio pulsars.

Finally, we have argued that suppression of pair creation by the conversion of $\parallel$ polarization photons that are the product of $\perp \rightarrow ||$ splittings back into the $\perp$ state by the resonant Compton scattering process, before they can pair produce, will not
be effective in normal pulsar magnetospheres where the particle densities are low.

While these theoretical issues leave the question of radio quiescence quite open, several observational issues are pertinent to this discussion. These relate to whether or not one expects to observe radio pulsars with high $\dot{P}$, which divides into issues of intrinsic population densities in the $P-\dot{P}$ diagram and observational selection effects. There are several obvious natural biases influencing pulsar observability. First, due to the expectation that pulsars with higher spin-down power (i.e. $B_0^2/\dot{P}^4$) are probably more luminous, a property observed in both the X-ray (Becker & Trümper, 1997) and gamma-ray (e.g. Zhang & Harding 2000) bands, one might anticipate that high-field radio pulsars are more luminous than their less-magnetized counterparts. There is evidence that this trend is present in the observed population: when luminosity distributions for Princeton Pulsar Catalog members are binned in ranges of $B_0$, radio luminosities $L_{\text{rad}}$ increase with spin-down field up to around $2 \times 10^{13}$ Gauss, beyond which the trend appears to reverse and $L_{\text{rad}}$ perhaps begins to decline with $B_0$. While this reversal is suggestive of an onset of radio-quiescence at high fields, it is presently statistically insignificant. The addition of the Parkes Multi-Beam population (for which distances have not yet been established from the dispersion measures) will improve statistics, but perhaps still leave the evidence for a luminosity decline inconclusive. Uncertainty in source distances pervades estimates of pulsar luminosity, and since there is a correlation between distance out of the galactic plane and pulsar age (and therefore period), we caution against over-interpretation of $L_{\text{rad}}$ trends for high $B_0$.

The next factor is the age distribution in the $P-\dot{P}$ diagram. Clearly, evolution (with or without field decay) guarantees a denser population at long periods, and the speed at which the period evolves is a rapidly increasing function of $B_0$. The population density would be constant along diagonal lines of constant age $P/(2\dot{P})$ for a uniform birth rates vs. field distribution. Unless there are many fewer pulsars born with high field strengths, the smaller average age of high field pulsars to the left of the conventional death lines cannot account for the dearth of radio pulsars at these fields. Concomitantly, the expected clustering of pulsars near the death line at subcritical fields is not realized; the beaming of radio emission is obviously a contributor to this effect. The reduction of the polar cap size with period should reduce the solid angle ($\propto P^{-1}$) of the cone of emission to more or less compensate the age clustering effect. Such a property was invoked by Young, Manchester & Johnston (1999) to argue that the 8.5 second Parkes pulsar PSR J2144-3933 was not isolated in its existence, but rather representative of a large, unseen population that challenges conventional theories explaining the death line. Yet the observed thinning of the population near this boundary argues that another factor is influential in determining the period distribution; a reduction in luminosity due to the onset of pair quenching may provide a partial explanation of this dilution. Notwithstanding, the beaming phenomenon impacts the $P$ distribution and not $\dot{P}$ phase space.

There are two main observational selection effects, the first being an intrinsically greater sensitivity at longer periods and lower dispersion measures, basically due to interstellar dispersion and scintillation effects, radiometer noise, and pulse shape and Fourier analysis properties (e.g. see Cordes and Chernoff 1997 for a discussion of sensitivities). The second true selection effect, less evident in the literature, is that pulsar surveys tend to filter out baseline fluctuations on long (i.e. $\gtrsim 5$ second) timescales, thereby selecting against supersecond period pulsars (Cordes, private communication). This bias is partly driven by past needs to focus on the appropriate range of periods of conventional pulsars, and will be counterbalanced by the increasing scientific interest in magnetars, discussed in Section 4.3 below. In summation, in the light of all these observational considerations, without performing an extensive population statistics analysis, it is fairly safe to argue that radio pulsars with $\dot{P}$ in excess of $10^{-11}$ are either rare or non-existent. The confir-

**Fig. 8.** Contour plots of cascade pair yield (number of pairs per injected photon) as a function of surface magnetic field strength, $B_0$, in units of the critical field, $B_{cr}$, and pulsar period for the cases where (left panel) only one mode of photon splitting is allowed, and (right panel) where three photon splitting modes are permitted. Contour levels are equally spaced logarithmic (base ten) intervals.
mation of a possible radio pulsar counterpart to SGR 1900+14 (discussed in Section 4.3), or otherwise, shall play a prominent role in resolving this issue.

4.2. Particle Acceleration Locales

Since the strength of the local magnetic field is one of the key influences on the location of the high-field death line, the site of the high energy emission in the pulsar magnetosphere is critically important. The initial angle at which the high-energy photons are emitted relative to the local magnetic field has also been shown in the previous subsection to be of critical importance (see Figure 5). Both of these factors are determined by the nature of the particle acceleration above the polar cap. Several types of models have studied pulsar acceleration due to charge deficits at different locations in the magnetosphere. Polar cap models consider the formation of a parallel electric field in the open field region near the magnetic poles, while outer gap models consider acceleration in the outer magnetosphere, near the null charge surface (see Mestel 1998 for the most recent and comprehensive review of pulsar electrodynamics).

Polar cap models for pulsar high-energy emission are all based on the idea, dating from the earliest pulsar models of Sturrock (1971) and Ruderman & Sutherland (1975; hereafter RS75), of particle acceleration and radiation near the neutron star surface at the magnetic poles. Within this broad class, there is a large variation, with the primary division being whether or not there is free emission of particles from the neutron star surface. This question hinges on whether the surface temperature $T$ of the neutron star (many of which have now been measured in the range $T \sim 10^{5} - 10^{6}$ K; Becker & Trümper 1997) exceeds the ion, $T_i$, and electron, $T_e$, thermal emission temperatures. If $T < T_i$, a vacuum gap will develop at the surface, due to the trapping of ions in the neutron star crust (RS75, Usov & Melrose 1995). In this case, the particle acceleration and radiation will take place very near the neutron star surface. If $T > T_e$, free emission of particles of either sign of charge will occur. The flow of particles is then limited only by space charge (SCLF case), and an accelerating potential will develop (Arons & Scharlemann 1979; Muslimov & Tsygan 1992) due to an inability of the particle flow all along each open field line to supply the corotation charge (required to short out $E_\parallel$). In space charge-limited flow models, the accelerating $E_\parallel$ is screened at a height where the particles radiate $\gamma$-rays that produce pairs. This so-called pair formation front (FF) (e.g. Arons 1983, Harding & Muslimov 1998) can occur at high altitudes above the polar cap.

Zhang & Harding (2000b) propose that, if both photon polarization modes can undergo splitting at high fields, the radio quiescence line will be much lower for pulsars with vacuum gaps (“anti-pulsars” in the language of RS75), than for pulsars with SCLF gaps (“pulsars”). In the case of vacuum gaps the high energy radiation occurs near the neutron star surface, where the local fields are high, whereas in the case of SCLF gaps, the high energy emission (and subsequent pair creation) may occur at high altitudes, especially when photon splitting prevents pair creation and thus a PFF near the surface. The radio quiescence line for “anti-pulsars” will be the photon splitting death line for surface emission, while the radio quiescence line for “pulsars” will be located around $B_0 = 2 \times 10^{14}$ G (for a surface temperature of $10^6 K$), above which thermionic emission of electrons from the surface is no longer possible and “pulsars” must have vacuum gaps. This could account for the observed existence of radio-loud pulsars such as PSR J1814 and radio-quiet pulsars such as AXP 1E 2259 at the same field strength and in close proximity in the $P - \dot{P}$ diagram.

4.3. Magnetars

The importance of a boundary for radio quiescence to the study of the radio pulsar population is obvious. Yet the absence of radio pulsars in the high-$B_0$, region of phase space has become of even greater importance with the mounting observational evidence at X-ray and $\gamma$-ray energies for the existence of neutron stars with such large fields, which a priori are not discriminated against (for a given age) by known radio selection effects (as discussed in Section 4.1). This evidence includes the detection of spin-down in the growing number of anomalous X-ray pulsars (AXPs). These sources have been known to exist for over a decade (e.g. see the biographical summary of Mereghetti & Stella 1995), though the identification of them as being anomalous was forged slowly with the accumulation of observational data. The designation “anomalous,” coined by van Paradijs, Taam & van den Heuvel (1995), was founded in their periods (6–12 seconds) being relatively short compared with typical accreting X-ray binary systems, combined with their unusually steep X-ray spectra and monotonic increases in periods: short period binary X-ray pulsars usually exhibit $\dot{P} < 0$, i.e. spin-up. A list of AXPs with measured $P$ and $\dot{P}$ is given in Table 1, an adaptation of that in Gotthelf & Vasisht (1998). Furthermore, the association of two of these pulsars (1E 1841-045 and 1E 2259+586) with supernova remnants (which are typically much younger than X-ray binaries) has shifted the focus from accretion torques, as championed by Mereghetti & Stella (1995) and van Paradijs, Taam & van den Heuvel (1995), to electromagnetic dipole torques (e.g. Vasisht & Gotthelf 1997) as the origin of the spin-down. In fact, Vasisht & Gotthelf (1997) argue that it is difficult for accretion torques to spin a pulsar down to $P \sim 10$ seconds on a $10^3$ year timescale unless the pulsar was born a slow rotator.

The electromagnetic dipole interpretation for the spin-down immediately implies immense supercritical ($B_0 > 4.41 \times 10^{13}$ Gauss) fields in these sources: inferred values are given in Table 1. The motivation for such a perspective has been dramatically enhanced by the recent detection of similar periods and period derivatives of soft gamma repeaters (SGRs), the neutron star sources with transient outbursts of soft gamma-ray emission possessing quasi-thermal spectra. Duncan & Thompson (1992) postulated that neutron stars with supercritical magnetizations, magnetars, were responsible for SGR activity based on the observed 8 second periodicity (Mazets et al. 1981) of the 5th March 1979 outburst from SGR 0525-66, combined with its strong directional association (Helfand & Long 1979; Chne et al. 1982) with the supernova remnant N49 in the Large Magellanic Cloud. No spin-down $\dot{P}$ was measurable for this source since the data that unambiguously exhibited periodicity spanned a single range of around two minutes. Hence the surface field of $B_0 \sim 6 \times 10^{14}$ Gauss inferred by Duncan & Thompson (1992) for SGR 0525-66 was purely circumstantial. Nevertheless, their hypothesis was dramatically bolstered by the detection of a supersecond periodicity coupled with a high rate of spin-down in the quiescent counterparts of SGR 1806-20 (Kouveliotou et al. 1998) and of SGR1900+14 (Kouveliotou et al. et al. 1999); parameters for these sources are also listed in Table 1. A giant burst, very similar to that observed from SGR 0525-66, was seen from SGR1900+14, exhibiting the same 5 s period as was detected in the quiescent emission (Hurley et al. 1999), strengthening the magnetar identification for SGRs. While the similar-
and radio emission at high altitudes, as discussed by Zhang &
are “pulsars” with active accelerators allowing pair production
radio detection of SGR 1900+14 is confirmed, is that SGRs
sion is thought to be predominantly of thermal origin from the
higher-B radio pulsars, whose high-energy emission may occur at
high-energy emission may occur at
the radio quiescence boundary and the positions of the pulsars
varoff, Kaspi & Camilo 2000) strongly suggests that a quantity
properties of highly-magnetized radio pulsars and AXPs. Note that
other than
excepts the unconfirmed detection of SGR 1900+14 by Shitov
On the other hand, the spin-down of some magnetars could
An episodic wind, which
in SGR1806-20 would lower the derived
in SGR1806-20 would lower the derived
Harding (2000b, see Section 4.2), while AXPs are radio-quiet
“anti-pulsars”.
Furthermore, the actual magnetic fields of the AXP and SGR
sources are not likely to be those derived from pure dipole spin-
down. If rapid field decay is powering the luminous emission from
these sources (Thompson & Duncan 1996), then it is likely that
transient higher multipoles exist near the surface. Since
higher multipole radiation contributes relatively less to the spin-
down torque, multipole fields significantly stronger than those
derived assuming a pure dipole could be present at the surface
without affecting the spin-down. This would effectively
lower the radio quiescence boundary for the AXPs, both due to
the increase in field strength and decrease in radius of curvature.
Measurement of braking indices larger than 3 (the dipole value)
in these sources may reveal the presence of higher multipoles.
In fact, a marginally significant detection of $\tilde{v}$ in the AXP
1RXS J170849.0-400910 (Kaspi, Chakrabarty & Stein-
berger 1999) would give a braking index in excess of 100! Field
decay alone would not change estimates of the surface field, but
would decrease the age of the pulsar relative to the dipole value
(Colpi, Geppart & Page 1999).

On the other hand, the spin-down of some magnetars could be
influenced by powerful particle winds, in this case lowering
the estimated surface field strength and increasing the character-
istic age (Harding, Contopoulos & Kazanas 1999, Thompson
et al. 1999) because the wind increases the spin-down rate by
distorting the dipole field inside the light cylinder. This is more
likely to be a factor in SGRs, where there is evidence for par-
ticle emission associated with bursts (Frai, Vasisht & Kulkarni
1997; Frail, Kulkarni & Bloom 1999). Wind luminosities in ex-
cess of $\sim 10^{36} \text{ ergs}^{-1}$ in SGR1806-20 would lower the derived
surface field strength below $10^{14} \text{ G}$. An episodic wind, which
is more likely, would decrease the estimated surface field to a
lesser degree. Possibly for this reason, the dipole surface fields of
the SGRs are higher than those of most of the AXPs and
more than an order of magnitude higher than the most strongly
magnetized radio pulsars.

**Table 1**

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>SNR</th>
<th>Ref.</th>
<th>$P$</th>
<th>$\dot{P}$</th>
<th>$\tau = P/2\dot{P}$</th>
<th>$B$</th>
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<tr>
<td>SGR 1806-20</td>
<td>G10.0-0.3</td>
<td>c</td>
<td>7.4</td>
<td>$8.3 \times 10^{-11}$</td>
<td>$1.4 \times 10^3$</td>
<td>$1.6 \times 10^{15}$</td>
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<td>SGR 1900+14</td>
<td>d</td>
<td></td>
<td>5.16</td>
<td>$1.1 \times 10^{-10}$</td>
<td>$7.8 \times 10^2$</td>
<td>$1.5 \times 10^{15}$</td>
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<tr>
<td>1E 1841-045</td>
<td>Kes 73</td>
<td>e</td>
<td>11.77</td>
<td>$4.7 \times 10^{-11}$</td>
<td>$4.0 \times 10^3$</td>
<td>$1.5 \times 10^{15}$</td>
</tr>
<tr>
<td>1E 1048-5937</td>
<td>f</td>
<td></td>
<td>6.45</td>
<td>$2.2 \times 10^{-11}$</td>
<td>$4.6 \times 10^3$</td>
<td>$7.6 \times 10^{14}$</td>
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<tr>
<td>4U 0142+615</td>
<td>g</td>
<td></td>
<td>8.69</td>
<td>$2.3 \times 10^{-12}$</td>
<td>$6.0 \times 10^4$</td>
<td>$2.9 \times 10^{14}$</td>
</tr>
<tr>
<td>1E 2259+586</td>
<td>CTB 109</td>
<td>g</td>
<td>6.98</td>
<td>$7.3 \times 10^{-13}$</td>
<td>$1.5 \times 10^5$</td>
<td>$1.4 \times 10^{14}$</td>
</tr>
<tr>
<td>1RXS J170849.0-400910</td>
<td>h</td>
<td></td>
<td>11</td>
<td>$2.3 \times 10^{-11}$</td>
<td>$8.0 \times 10^3$</td>
<td>$1.1 \times 10^{15}$</td>
</tr>
</tbody>
</table>

**Note.** (a) Only sources with measured $\dot{P}$ are included in the Table. (b) The magnetic field estimates are obtained using the choice for the dipole spin-down energy loss rate $B = 6.4 \times 10^{15} (P \dot{P})^{1/2}$ Gauss adopted by Usov & Melrose (1995), derived in, for example, Shapiro & Teukolsky (1983). References for observed periods and period derivatives are (c) Kouveliotou et al. (1998), (d) Hurley et al. (1999) for the period and Kouveliotou et al. (1999) for $P$ for SGR 1900+14, (e) Vasisht and Gotthelf (1997), (f) Oosterbroek et al. (1998); here the estimate obtained from their Table 3 using the maximum time baseline is used), (g) Mereghetti & Stella (1995), (h) Israel et al. (1999).
Fig. 9.— The upper region of the $P - \dot{P}$ diagram, with filled circles again denoting the locations of members of the latest edition of the Princeton Pulsar Catalogue, and the open circles marking pulsars in the recent Parkes Multi-Beam survey. The Crab, Vela and PSR 1509-58 gamma-ray pulsars are highlighted as indicated in the lower left inset, together with the positions of five radio-quiet anomalous X-ray pulsars (open square stars) and SGRs 1806-20 and 1900+14 (filled squares) in the upper right of the diagram; their measured $\dot{P}$ and inferred fields $B_0$ are listed in Table 1. The conventional death line (as in Fig. 5) lies at periods longer than all seven of the putative magnetars. The $\theta_{kB,0} = 0$ (midweight solid diagonal curve) and $\theta_{kB,0} = 10^{-3}$ (midweight dashed curve) depictions of the radio quiescence boundary from Fig. 5 are exhibited. The heavy-weight pairs of curves labelled $\varepsilon_{\text{max}} = 10$ MeV and $\varepsilon_{\text{max}} = 10^2$ MeV are contours for the escape energy of photon splitting, $\perp \rightarrow \parallel$, with the solid and dashed line styles denoting $\theta_{kB,0} = 0$ and $\theta_{kB,0} = 10^{-3}$, respectively. They are shown only above the potential radio quiescence boundaries, where splitting dominates pair creation for the $\perp$ polarization, marking contours for the approximate maximum observable energy permitted in highly-magnetized pulsars; to the right of these the magnetosphere is transparent to 10 MeV and 100 MeV photons, respectively.

4.4. Guides for Soft and Hard Gamma-Ray Pulsation Searches

To close this section of discussion, it is appropriate to mention the implications of our analysis for gamma-ray pulsar searches, which are, in a sense, independent of the radio quiescence issue. If the electrons are accelerated to high Lorentz factors in profusion, then primary photons will extend into the EGRET band and beyond. Therefore, short period pulsars with high spin-down power ($\propto B_0^2/P^4$) like the Crab pulsar should be luminous and therefore easily detectable by future experiments such as GLAST. At longer periods near the magnetar regime, the collected observational data and the age distribution bias (see Section 4.1) indicate that spin-powered pulsars will be generally faint unless they are nearby. Yet the observed hard X-ray luminosities of AXP's are well in excess of their spin-down luminosities so that non-rotational energy sources are indicated. Hence luminosity biases against long period pulsars may not be as pronounced for those with alternative power sources such as magnetic field decay, at least in the magnetar regime. The work of this paper clearly underlines the fact that photon attenuation considerations have as profound an influence on gamma-ray observability as do issues of power in the primary radiation.

At near-critical and supercritical fields, the attenuation by pair creation and photon splitting in the EGRET band is dramatic, as is evident from the contours of maximum energy depicted in Figure 9. Basically, photon transparency at a given energy is achieved to the right of and below (i.e. at longer periods and lower $\dot{P}$) a given contour so that the positioning of the $\varepsilon_{\text{esc}} = 100$ MeV splitting escape energy contours indicates that it will not be easy for an experiment like GLAST (depending on its final design) to detect many or most of the known magnetars, and the new Parkes Multi-Beam sources PSR J1119-6127 and PSR J1726-3530. Should magnetars emit in the gamma-rays, they can only do so generally below the EGRET band. This conclusion holds even if the magnetospheric field structure is non-dipolar, since escape energies are pushed to lower values by greater field curvature. A possible evasion of this constraint is that the inferred fields are significant overestimates of the true surface fields, as might occur for wind-aided spin-down in magnetars (see Section 4.3), so that the escape energy contours may possess pathological distortions in the $P - \dot{P}$ diagram. Moreover, such a two-dimensional phase space may be insufficient to account for the magnetospheric and photon attenuation prop-
erties. Hence high energy astrophysicists interested in pulsar searches at high $\dot{P}$ should extend their period range to include supersecond periods to maximize their potential harvest. While this range has historically been given low priority (e.g. see the search in Mattox et al. 1996), the excitement generated by AXP and SGRs in recent years is changing emphases in such search programs.

These attenuation properties emphasize that spectroscopy in a variety of gamma-ray pulsars will probably provide the ability to discriminate between the applicability of polar cap or outer gap models. In the polar cap picture, the analysis of this paper shows that spectral cutoff energies possess a strong inverse correlation with surface field strength $B_0$ and the polar cap size $\Theta$. For Crab-like and Vela-like pulsars such cutoffs are coupled to magnetic pair creation, and appear in the EGRET band. However, for highly-magnetized pulsars such as PSR 1509-58, such properties are ideally explored with a medium energy gamma-ray experiment, and probe the action of photon splitting. Expectations for trends of gamma-ray cutoffs in outer gap models are not as well studied. Yet, it appears that their trends with $B_0$ and pulsar period should differ significantly from those for polar cap models, due to the inherently different physics involved. Cutoffs in the outer gap scenario represent maximum energies of acceleration rather than the effects of photon absorption, and so may actually be independent of or increase with surface field strength, and decline with pulse period, contrary to the indications of the polar gap model. Such distinctive predictions can be probed by significant population datasets such as those to be afforded by the GLAST mission. A related observational diagnostic is spawned by the contention (Chen & Ruderman 1993) that gamma-ray emission is not expected in outer gap models when the pulsar period exceeds a sizeable fraction of a second. Hence detection of gamma-ray pulsations from a source with a supersecond period would clearly favor the polar cap model.

Our studies also indicate that the gamma-ray spectra of pulsars should produce distinctive polarization signatures for the polar cap model. This should enable discrimination from outer gap scenarios in a future era when gamma-ray polarimetry is possible. While the predictions of gamma-ray polarizations might be similar for the continuum spectra in each of these two competing models, principally because their continuum emission processes are similar, the pair creation and photon splitting mechanisms generate spectral cutoff energies that are polarization-dependent (as emphasized by HBG97), and therefore immediately distinguishable from outer gap model turnovers that have no intimate connection to QED processes in strong fields. In particular, when $B_0 \gtrsim B_{cr}$, the dominance of splittings $\perp \rightarrow \parallel$ will guarantee that the cutoff energy for $\parallel$ photons exceeds that for ones of $\perp$ polarization. In contrast, at lower fields strengths, photon splitting is removed from the picture and the escape energy for pair creation by $\parallel$ photons is slightly lower than that for $\perp$ ones, resulting in a dominance of $\perp$ photons near the maximum observable energy. Because the strong dependence of the spectral high-energy cutoff in fields $B_0 \gtrsim B_{cr}$ occurs exclusively when only one mode of splitting is operative, it may be possible to resolve by observation the question of whether one or several modes of splitting operate in high fields. The splitting and pair production cutoffs are expected around 100 MeV in long period pulsars (slightly higher if the emission region is above the surface). If only the $\perp \rightarrow \parallel$ mode operates, the splitting cutoff will occur at an energy around a factor of 2–3 lower than the pair production cutoff. GLAST (depending on sensitivity below 100 MeV) or a future medium energy $\gamma$-ray detector would then see 100% $\parallel$ polarization of the highest energy photons, i.e. those between $\varepsilon_{\text{esc}} \parallel$ and $\varepsilon_{\text{esc}} \parallel - e^- e^+$. On the other hand, the absence of this signature would be consistent with, but not necessarily imply, at least two active splitting modes. These distinctive signatures establish a strong case for performing gamma-ray polarimetry experiments, an objective that may not be that far in the future for medium-energy Compton gamma-ray telescopes.

5. CONCLUSION

We have presented a detailed study of the comparative attenuation of photons by magnetic pair production and photon splitting, of pair cascades and of conditions for pair suppression in very highly magnetized pulsars. While ground-state pair creation and positronium formation can act at $B_0 \gtrsim 0.1 B_{cr}$ to reduce the number of free pairs, only photon splitting has the potential capability of dramatically inhibiting the creation of pairs, bound or free. Our quantitative study of cascade pair yields at different field strengths confirms the location of the approximate radio quiescence boundary in the $P - \dot{P}$ diagram proposed by Baring & Harding (1998), assuming that copious pair production is a requirement for pulsar radio emission. However, it also shows that the existence of such a boundary requires the rate of photon splitting to be non-zero for both photon polarization modes. This is not believed to be true in the low-field, weakly dispersive limit, where kinematic selection rules prohibit splitting of photons of $\parallel$ polarization. Whether photon splitting can significantly suppress pairs in highly magnetized radio pulsars depends on presently unexplored physics. It therefore has become critically important to study the photon splitting process in the high-field, highly dispersive regime. Understanding the behavior of splitting in this regime will not only resolve the radio quiescence question, but is crucial to modeling acceleration and radiation in high-field pulsars and magnetars.

The circumstantial evidence, that the most highly magnetized pulsars (AXPs and SGRs) tend to be radio quiet, argues for some pair suppression at high field strengths. The radio pulsar and magnetar populations are no longer cleanly separated in $P - \dot{P}$ space, which indicates that another dimension of phase space is necessary to divide these two source populations. Delineation of these two source groups is complicated by the likelihood of alternate spin-down mechanisms and evolution issues for magnetars. More systematic surveys of the overlap region of $P - \dot{P}$ space at X-ray and $\gamma$-ray energies, correlated with radio pulsar surveys are needed. We have given some guidelines for such high-energy searches based on the expected physics of particle acceleration and photon attenuation, in anticipation of the next generation of medium and high-energy gamma-ray experiments such as GLAST. Spectral and polarization observations may be crucial in identifying signatures of photon splitting and even in resolving the issue of which splitting modes are operating in ultra-magnetized sources, an attractive possibility for both astrophysicists and physicists.

We are grateful to Peter Gonthier and Marty Knecht for help with the splitting cascade code development. We also thank Alex Muslimov, Jim Cordes, Eric Gotthelf, David Thompson, Peter Gonthier and Matthew Bailes for discussions. This work was supported by the NASA Astrophysics Theory Program.