Cepheid Mass-Luminosity Relations from the Magellanic Clouds

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Abstract. The OGLE data base is used to obtain periods, effective temperatures and luminosities for fundamental and overtone Magellanic Cloud Cepheids. Masses are then computed for these stars with our linear code with turbulent convection assuming an average composition of ($X=0.716$, $Z=0.010$) for the LMC and of ($X=0.726$, $Z=0.004$) for the SMC. The average $M-L$ relation for the fundamental Cepheids matches closely that for the first overtone Cepheids, this for each Magellanic Cloud. Neither the SMC nor the LMC average Log $M$–Log $L$ relations are straight lines, but have a noticeable curvature.

Our analysis clearly excludes the short distances for both clouds that were adopted by OGLE on the basis of the red clumps.

The current evolutionary tracks systematically predict smaller luminosities than observed, especially at high luminosity. Moreover, the evolutionary tracks of the low mass stars are not in agreement with the observations as they do not extend sufficiently blueward and do not penetrate deep enough into the instability strip, or not at all.

Key words: stars : oscillations – stars: Cepheids – Stars: Evolution, Magellanic Clouds, distance moduli

1. Mass Luminosity Relation

In the last few years high quality data on large numbers of Cepheid variables in the Small and Large Magellanic Clouds have been made available by the EROS and OGLE microlensing projects (Beaulieu et al. 1995, Afonso et al. 1999, Udalski et al. 1999). In particular the OGLE Project has provided standard colors in addition to periods and magnitudes for the largest samples published to date. In this paper we examine some of the constraints that the MC Cepheids impose on stellar evolution and stellar pulsation theories.

We use the full catalogue of publicly available of LMC, SMC single mode Cepheids and SMC double mode Cepheids produced by OGLE in BVI (Udalski et al., 1999abc, with zero point corrections as suggested in April 2000 on the OGLE website, U99 hereafter). The single mode Cepheid catalogues contain 1435 LMC and 2167 SMC stars. We keep objects classified as fundamental mode pulsators or first overtone pulsators, with reliable photometry in both V and I. We exclude stars whose magnitudes are most likely to be strongly contaminated by companions or blending in V or I. The remaining stars form our working sample of OGLE Cepheids. It consists of 670 LMC fundamentals, 426 LMC overtones, 1197 SMC fundamentals and 677 SMC overtones, as well as 24 F/O1 SMC double-modes and 71 O1/O2 SMC double-modes.

The OGLE data base provides intensity averaged magnitudes and colors. With the help of distance moduli these can then be transformed to luminosities and effective temperatures. The Magellanic Clouds (MC) are thought to be relatively uniform in composition, and with the help of observed average compositional information theoretical modelling can then provide the mass of each star.

The distance of the Large Magellanic Cloud remains at the center of the current debates about the distance scale ladder. Whereas a conservative distance modulus of 18.5 $\pm$ 0.1 is widely adopted, extreme values of the distance modulus range from 18.08 to 18.70. The size of this spread reflects largely the limits of the current understanding of newer distance indicators, on the one hand, the red clump stars, which give a very small statistical, but a not well defined and possibly large systematic error (Udalski 2000 and references therein, Cole 1998, Girardi 1998, Stanev et al., 2000, Romaniello et al. , 2000), and, on the other hand, the problem of tackling in the proper way the Hipparcos parallaxes (Feast & Catchpole 1997, Luri et al. 1998, Groenewegen & Oudmaijer, 2000 and references therein).

The method adopted by U99 to derive reddening relies on the controversial red clump distance indicator. It makes the assumption that the $I$ luminosity has a weak metallicity dependence. This assumption which has been questioned on both theoretical and observational grounds, leads to the determination of a short distance to the LMC of $18.22 \pm 0.05$ and $18.73 \pm 0.05$ to the SMC. Moreover it gives a mean reddening for the LMC of 0.147, and of 0.092 for the SMC. We note that these values are different from what is usually given as mean properties for the clouds (especially for the LMC, see Walker 1999 and ref-
Fig. 1. Mass–Luminosity relations for SMC and LMC as derived from the OGLE data adopting choice (A) for distance modulus and reden- ninings in the left panel and our preferred choice (B) in the right panel; fundamental Cepheids are shown as dots and overtones as open circles. Notice the systematic shift of the $M$–$L$ between the choice (A) and the choice (B).

In his recent review, Walker (1999) noted that the median reden ninings are $E(B-V) \sim 0.1$ for the LMC and $E(B-V) \sim 0.08$ for the SMC. The galactic foreground reden ninings are known to be low to the line of sight of the clouds, viz. 0.06 and 0.04, respectively. The estimation of differential redening inside the clouds based on earlier studies is quite uncertain. In particular, in the LMC heavily reddened stars ($E(B-V) = 0.30$) can be found all over, but the typical range is 0–0.15.

For the derivation of the stellar parameters from the OGLE data we need values for the distance modulus and for the reden nin correction. Hereafter, we consider two alternate choices for these quantities. Choice (A) adopts both the distance moduli and the reden ning from red clump stars as suggested by U99. Choice (B) adopts instead the Cepheid distance modulus to the LMC of $18.55 \pm 0.10$ (Laney & Stobie, 1994). Based on Cepheids the relative distance of the centroid of the SMC to the LMC is fairly well known to be $0.42 \pm 0.05$ (Laney & Stobie 1994), thus leading to 18.97 for the SMC. We use the mean reden ninings of $E(B-V) = 0.1$ and $E(B-V) = 0.08$ for LMC and SMC respectively. The foreground reden ning is estimated to be of 0.06 and 0.04 respectively. Together with this we assume that the reden ning can be represented by a truncated Gaussian distribution with a dispersion $\sigma_{E(B-V)} = 0.06$ in the LMC and $\sigma_{E(B-V)} = 0.05$ in the SMC.

The alternate choices (A) of adopting $E(B-V)$ given by OGLE or (B) of adopting mean values from earlier studies not based on clump stars will be seen to lead to significant systematics shifts in magnitude and color.

The differences, (B) – (A), in distance moduli and in mean reden ninings are $\delta \mu = 0.33\text{mag}$, $\delta(E(B-V)) \sim 0.047$ for the LMC, and $\delta \mu = 0.24\text{mag}$, $\delta(E(B-V)) \sim 0.012$ for the SMC.

We follow Kovács (2000) in the conversion from magnitudes to bolometric, and from colors to effective temperatures using the stellar atmospheric models of Castelli et al. (1997).

$$
\log T_{\text{eff}} = 3.9224 + 0.0046 \log g + 0.0012[M/H]
$$

(1)

$$
-0.2470(V-I_c -(R_V - R_I)E(B-V))
$$

(2)

$$
2.5 \log L = \mu_{MC} - V + R_V \ E(B-V) + BC + 4.75
$$

(3)

$$
BC = 0.0411 + 2.0727 \Delta T - 0.0274 \log g
$$

+0.0482 [M/H] - 8.0634 \Delta T^2
$$

$$
\log g = 2.62 - 1.21 \log P_0
$$

(4)

(L in solar units), where $\Delta T = \log T_{\text{eff}} - 3.772$. The transfor mation to absolute luminosities is then made with the adopted distance moduli $\mu_{MC}$ to the LMC or the SMC for the choice (A) or the choice (B).

This leads to (B) – (A) differences of $\delta \log L \approx 0.2$ and $\delta T_{\text{eff}} = 0.014$ for LMC and to $\delta \log L \approx 0.11$ and $\delta T_{\text{eff}} = 0.0036$ for the SMC.

From the observational data we therefore obtain a period, a luminosity and a $T_{\text{eff}}$ for each fundamental and for each over-
In Fig. 1 we show the Log$L – LogM$ diagram obtained from our Cepheid model calculations that use the observational constraints A in the left panel, and B in the right panel. The fundamental Cepheids are shown as dots and overtones as open circles.

Four features stand out immediately. First, the observations clearly indicate a mass-luminosity relation which appears curved and which will be seen to be much steeper than that of the evolutionary calculations. Second, the average $M–L$ for the fundamental Cepheids agrees with that of the overtones. Third, there is a huge scatter whose nature needs to be discussed, because if the Cepheids formed a homogeneous group, they should all fall on a very tight $M–L$ line. Fourth there is a
large systematic shift of the $M$–$L$ between constraint (A) and (B). Constraint (A) will be seen not to be in agreement with evolutionary calculations.

2. Discussion

2.1. Choice (A) or choice (B) for distances and reddening?

With the observational constraints (A), there is a systematic shift to higher luminosities and higher temperature for the observed stars. In the mean, we have systematic shifts of $\delta \log L \sim 0.2$ and $\delta \log L \sim 0.1$, and of $\delta \log T_{\text{eff}} \sim 0.014$ and $\delta \log T_{\text{eff}} \sim 0.0036$ for LMC and SMC, respectively. Once the periods are computed, it leads to $\delta \log M \sim -0.15$ and $\delta \log M \sim -0.1$ respectively. The $M$–$L$ relations derived with such assumptions (A) are not in agreement with evolutionary calculations as discussed in §4.

The LMC is even more strongly affected than the SMC. Taken at face, choice (A) would indicate that evolutionary calculations are quite off the beat. The sensitivity of $M$–$L$ relations to metallicity would have been largely underestimated too. On the other hand, with the conservative distances and reddening to the clouds, the situation is much more satisfactory. Our analysis suggests that the use of the red clump stars as done in U99 is not a satisfactory way of estimating the distance modulus of $18.59 \pm 0.04 \pm 0.08$ mag, in agreement with our choice (B). In the following, we will from now on ignore the choice (A) for reddening and distances, and concentrate on the choice (B).

2.2. Computational Uncertainties

First we examine the computational uncertainties. We expect these to be small because we compute only the linear periods of the fundamental and first overtone. In contrast to the linear growth-rates the periods are very insensitive to the convective parameters ($\alpha$’s in Yecko et al. 1996). The comparison of purely radiative models with our turbulent convective ones gives an idea of the uncertainty. We find that the period shifts are systematic but small, of the order of the size of the dots in Fig. 1. The fact that they are systematic indicates that they cannot contribute to the scatter of Fig. 1. The models have been computed with a mesh of 200 points. Models run with a cruder mesh distribution give essentially the same $M$–$L$ picture. We can safely use linear periods, because nonlinear hydrodynamic modelling shows that the differences are systematic and at most of the order of 0.1% which has no appreciable effect on the $M$–$L$ picture.

None of the computational uncertainties can account for the scatter in the $M$–$L$ relation, and we have to look in the observational data.
2.3. Scatter in the M–L Relations

In the upper panel of figure 2 we plot the residuals of the period-luminosity (P–L) relation in V and I for both the LMC and the SMC fundamental Cepheids OGLE data. These diagrams illustrate the structure of the Cepheid P–L relation (see fig 5 and 6 from Sasselov et al. 1997). The dispersion is mainly along the reddening vector in the LMC, whereas in the SMC the cloud of points it is not, because depth effects are another source of scatter. We recall that there is a near degeneracy between lines of constant period and reddening, therefore one cannot just minimize the dispersion along reddening vector in this plane to correct for the reddening. It would lead to an over-correction. When we use the reddening derived by U99 we note but a marginal improvement of the residuals as shown in the lower panel of Fig. 1. One concludes that the differential reddening within the clouds on a star per star basis persists as a major source of dispersion that is not compensated for by the reddening maps from red clump stars.

In order to see whether the size of error that is inherent in the observations is responsible for the scatter in our M–L relation we have made the following test. First we construct a sequence of fundamental Cepheid models with a specific M–L relation, \( \log L = 0.79 + 3.56 \log M \), and with a range of \( T_{\text{eff}} \) that spans the corresponding instability strip. We transform these \( L \) and \( T_{\text{eff}} \) to I and V magnitudes. These data are then maculated with Gaussian noise of 0.02 in the I and V magnitudes and with a Gaussian noise in the reddening with \( \sigma_{E(B-V)} = 0.06 \). Using these surrogate stars as input we then proceed to compute the surrogate stellar masses the same way as we handled the OGLE data. Figure 4 show the resulting M–L relation. It is seen to exhibit the same type of scatter as the OGLE-derived M–L relations. The reason for the scatter is thus seen to originate in the extreme sensitivity of the masses to small errors in the colors and magnitudes.

It is tempting to use the observational deviations in Fig. 2 to tighten the derived M–L data. The question is whether we can use the deviations parallel to the reddening line to estimate (and correct) for reddening and observational noise. We find that because of the finite width of the IS the spread in \( T_{\text{eff}} \) has an effect parallel to the reddening, so we cannot decouple just the reddening error from it. The spread in mass (for a given \( L \)) has an effect not parallel (in a right angle close to 45 degrees) to the reddening, so it has a component perpendicular to the reddening line. Because of this projection angle the perpendicular direction alone cannot be used to estimate the observational errors in I or V. In summary, unfortunately, it is therefore not possible to use the residuals of Figure 2 either to correct for the observational reddening errors on a star by star basis.

3. Beat Cepheids

OGLE have also published data on SMC beat Cepheids. Because the knowledge of a (precise) second period adds an additional piece of information, these stars should be even more constraining than the single-mode Cepheids for extracting an M–L relation. In fact Kovács (2000) has used the two observed periods and \( T_{\text{eff}} \) and a radiative linear Cepheid models to infer luminosities and thus the distance modulus to the SMC.

In order to check the self consistency of the observational data and pulsation models we can make the following test on the SMC beat Cepheids. We take three of the four observed \( P_{\text{k}} \) and \( P_{k+1} \) \((P_0\) for the F/O1 and \( P_1\) for the O1/O2 beat Cepheids). From these three parameters (ignoring \( P_{k+1} \) for the time being) we calculate the mass and then the second period \( P_{k+1}\)\((\text{calc})\). Then we compare this calculated period to the observed one \((P_{k+1}\)\((\text{obs})\)). On the \( \epsilon = P_{k+1}\)\((\text{calc})/P_{k+1}\)\((\text{obs})\) vs. \( P_k \) diagram, with the choice (B) of distance modulus and E(B-V) we observe the following facts:

- For all but one of the F/O1 stars \( \epsilon < 1 \), and the data are along an almost horizontal line. To get a self-consistent solution for these stars, we have to increase the luminosity relative to our parameter choice (B) by \( \delta \log(L) = 0.12 \). Decreasing the metal content \((Z)\) to 0.001 also shifts the \( \epsilon \) values to the right direction, but by itself it does not solve the discrepancy.

- For the O1/O2 beat Cepheids the \( \epsilon \) values are along a line with a slope of \( \approx 0.03 \). There is only a limited range around \( P_1 \approx 1.0 \) days, where self consistent solution exist for the stellar parameters.

- The scatter on the \( \epsilon \) vs. \( P_k \) plots are consistent with the observational noise in \( E(B-V) \), I and V. With the help of surrogate data with the same noise as described in Sect. 2.3,
we found that these error sources do not introduce systematic trends (like the slope of $\epsilon$).

For the second set of tests we allowed systematic shifts in $\log L$. For the O1/O2 Cepheids the slope of $\epsilon$ strongly depends on $\delta \log L$. With $\delta \log L = -0.05$ to $-0.10$, the slope is removed but the scatter of the points is increased, and $\epsilon < 1$ for all of the stars. Consistent solutions exist again only if the metallicity ($Z$) is decreased to 0.001. In the case of F/O1 stars the distance modulus has a less significant effect on the slope of $\epsilon$. The best agreement was found with $\delta \log L = 0.10$ which is just opposite to the value we found for the O1/O2 Cepheids.

We have checked whether this discrepancy can be removed by allowing a wider range of the initial assumptions on the input parameters. For our first set of tests the distance modulus was fixed, and we have allowed a wide range in reddening ($-0.1 < \Delta E(B-V) < 0.1$) as well as various changes in the composition and metallicity mixtures with the customized OPAL library. All these changes in the input data result in some vertical shifts in the $\epsilon$ vs. $P_{k}$ diagram, but not enough to get consistent solutions for the F/O1 stars. The metallicity would need to be decreased to $Z = 0.001$ to get the mean value of $\epsilon$ to be 1. We also note that there is no significant difference in $\epsilon$ between the radiative and convective models.

Our conclusion agrees with the work of Buchler, Kolláth, Beaulieu & Goupil (1996), but is in apparent disagreement with Kovács (2000). The reason for this apparent disagreement is that Kovács did not really construct models with the observational parameters, but simply minimized what he called $\sigma$, viz. the deviation from observed to model periods, and in fact this sigma is not zero for his ‘solutions’. Furthermore in those cases where a solution can be found, the mass is determined with a very large uncertainty by the two period constraint, as already pointed out by Buchler et al. (1996).

Moreover, although this does not directly affect the absence of solutions, we remark that Kovács adopted reddening from red clump stars following U99. In the mean, these reddening are $\sim 0.01$ larger than the mean reddening towards the SMC, so it will marginally affect his temperature scale compared to ours. However the distance he derives is not in agreement with the distance adopted by U99 to the SMC. Then he adopts the differential distance from LMC to SMC given by U99, and derives the distance to the LMC. We do not find this procedure satisfactory either since there is an internal contradiction in using reddenings from red clump stars, red clump star differential distance between the clouds, but ignoring the discrepancy between these distances and his Cepheid distance.

We note that the same trouble arises when we use the 3 observational data, $(P_{k}, P_{k+1}, T_{\text{eff}})$ and compute $L$ and $M$. For many stars in the SMC sample there is no solution, i.e., no mass and luminosity can be found that satisfies these three observational constraints! The same difficulty appears when, instead, one tries to satisfy the 3 observational constraints $(P_{k}, P_{k+1}, L)$ to compute a $T_{\text{eff}}$ and $M$.

In the few cases where there are solutions based on three pieces of observational data, they are generally not compatible with the fourth one, i.e., if the periods and $T_{\text{eff}}$ are given, the calculated luminosity and mass are not fully acceptable. Why there are no solutions for the observed beat Cepheids in the SMC remains an unsolved puzzle that the introduction of turbulent convection in the linear codes did not resolve.

4. Comparison to Evolutionary Tracks

It is of course of great interest to confront the predictions of evolution calculations with the $M$–$L$ values which we have extracted from the OGLE data. Recently a number of such calculations, all performed with the OPAL opacities, have become available: Alibert et al. (1999), Girardi et al. (2000), Bono et al. (2000). We present a confrontation of these calculations with our OGLE-derived stellar parameters in Fig. 3. For both the SMC and the LMC we show a theoretical HR diagram
with superposed evolutionary tracks, and a mass-luminosity diagram with the $M-L$ relations from these authors. We show the evolutionary tracks of Girardi (with $X=0.756$, $Z=0.004$ and $X=0.742$, $Z=0.008$), respectively, which are the closest to our chosen compositions. The Girardi et al. $M-L$ relations for the 2nd/3rd crossing (taken as the points of slowest evolution at the blue edge) are shown as solid lines, those of Alibert et al. as long dashes and those of Bono et al. with ($Y=0.226$, $Z=0.004$ and $Y=0.216$, $Z=0.004$) as short dashes.

We note that none of the evolutionary calculations is fully in agreement with our OGLE-derived LMC and SMC $M-L$ data. At fixed mass, the computed stars are not luminous enough. The results of Girardi et al. are closest to our derived $M-L$ relations and seem also to have the right curvature (Alibert et al. and Bono et al. used straight line $M-L$ fits). Indeed, if the $M-L$ of Girardi et al are shifted by -0.09 in $\log M$ for SMC (respectively by -0.06 in $\log M$ for LMC) metallicities, a reasonable agreement is achieved at low and high luminosities.

We have not shown the $M-L$ relations for the faster, first crossing to avoid cluttering the figures. It can be seen from the left-hand subfigures that the luminosities are about 0.2 lower for the same mass on these crossings.

The density of stars is definitely lower at the low luminosity end. A natural explanation would be that the low luminosity stars are first crossers. The Girardi et al. tracks for the LMC are compatible with this interpretation, but it would be useful to do the statistics on the basis of the evolution speed along the tracks. However, the Girardi low $L$ tracks do not loop sufficiently far for the SMC. The problem is slightly worse for both the LMC and SMC tracks of Alibert et al. (1999).

We conclude that the evolutionary calculations are not in total agreement with constraints from stellar pulsation theory and observations. They exhibit a known failure for low mass SMC Cepheids, viz. the blue loops do not penetrate the instability strip. But we stress that it is also important, when comparing constraints from stellar evolutionary calculations with observations, not to remain in the $\log L - \log T_{\text{eff}}$ and $P-L$ planes, because discrepancies can show up in other quantities such as the masses.

A comparison with the predictions of the recent stellar evolution calculations show a discrepancy both in the theoretical HR diagrams where the low mass tracks do not extend sufficiently blueward to penetrate the instability strip. They also show a perhaps more serious discrepancy at the level of the $M-L$ relations.

Short distances to the clouds based on the red clump method as adopted by OGLE are not in agreement with the current understanding of stellar evolution and stellar pulsation calculations.

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