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Within first-order phase transitions, pre-transitional effects are nonperturbative, large-amplitude thermal fluctuations which can promote phase mixing before the critical temperature is reached from above. We investigate if these effects are relevant during heavy-ion collisions at RHIC and LHC energies. In contrast with the cosmological quark-hadron transition, we find that the rapid cooling typical of collider experiments and the fact that the quark-gluon plasma is chemically unsaturated suppress the role of pre-transitional effects at current collider energies.

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It is possible to model the gross general features of a phase transition from a quark-gluon plasma (QGP) to a hadronic phase through a phenomenological potential with a scalar order parameter [1]. Assuming the transition to be discontinuous, or first-order, as suggested by some recent lattice QCD simulations [2], the QGP is cooled to a temperature \( T_1 \), where a second minimum appears, indicating the presence of a hadronic phase. With further cooling, the two phases become degenerate at the critical temperature \( T_c \), with a free energy barrier which depends on physical parameters characterizing the system, such as the surface tension \( \sigma \) and the correlation length \( \xi \). This general behavior models both the cosmological quark-hadron phase transition and the production of a QGP during heavy-ion collision experiments, as those under way at the RHIC and planned for the LHC. In the latter case, the plasma generated by the collision expands and cools, relaxing back to the hadronic phase. Recent interest has been sparked by the possibility that this relaxation process is characterized by the formation of disoriented chiral condensates, which are coherent pion condensates similar to the domains typical of quenched ferromagnetic phase transitions [3,4]. An analysis of the nonequilibrium properties of this relaxation process may reveal novel experimental signatures detectable by the RHIC and LHC experiments. Here, we would like to contribute to this analysis by investigating the possible role of pre-transitional phenomena in collider experiments.

Recent work on the dynamics of weak first-order phase transitions have shown that, in certain cases, it is possible to have nonperturbative, large-amplitude fluctuations before the critical temperature is reached, which promote phase mixing [5]. Studies performed in the context of the cosmological electroweak phase transition [6] and quark-hadron phase transition [7] have indicated that, for a range of physical parameters controlling the transition, these effects are present. It is thus natural to consider if similar effects are present during heavy-ion collisions [8]. Whenever pre-transitional phenomena are relevant, one should expect modifications from the usual homogeneous nucleation scenario, which is based on the assumption that critical bubbles of the hadronic phase appear within a homogeneous background of the QG phase. The dynamics of weakly first-order transitions will be sensitive to the amount of phase mixing at \( T_c \): for large phase mixing, above the so-called percolation threshold, the transition may proceed through percolation of the hadronic phase, while for small amounts of phase mixing, by the nucleation of critical bubbles in the (inhomogeneous) background of isolated hadron domains, which grow as \( T \) drops below \( T_c \). An ideal quark gluon plasma in one dimension expands according to the Bjorken scaling, where \( T^3 t \) is constant [9]. Assuming the initial temperature of the plasma produced at RHIC and LHC energies to be 2 to 3 times \( T_c \), scaling implies that the time \( (\Delta t) \) taken by the plasma to cool from \( T_1 \) to \( T_c \) is of order a few fm/c, which could be comparable with the time scale of the subcritical hadronic fluctuations. On the other hand, the expansion rate of the early universe in the range \( T_1 \leq T \leq T_c \) is slow enough [10,11] (\( \Delta t \) could be of the order of a few \( \mu \) secs), that nonperturbative thermal fluctuations may achieve equilibrium. Another difference is that collisions at RHIC and LHC energies will lead to the formation of a highly (chemically) unsaturated plasma, i.e., the initial gluon and quark contents of the plasma remain much below their equilibrium values [12–14]. A chemically unsaturated plasma will cool at an even faster rate than what is predicted from Bjorken scaling [15,16]. The cooling rate will also be accelerated further if expansion in three dimensions is considered. Therefore, we will show that although the equilibrium density distribution of subcritical hadron bubbles is significant - particularly when the transition is weak - unlike the situation in cosmology, they do not contribute strongly to phase mixing. For the range of parameters we investigated, of relevance for RHIC and LHC energies, the plasma cools so rapidly that the subcritical bubbles do not have time to reach their equilibrium distribution.
and promote substantial phase mixing: the phase transition will proceed through the nucleation of critical size hadron bubbles in a (nearly) homogeneous background of the unsaturated quark-gluon plasma.

To study the dynamics of a first order phase transition, we use a generic form of the potential in terms of a real scalar order parameter \( \phi \) given by \( V(\phi) = a(T)\phi^2 - b T\phi^3 + c \phi^4 \) [1,7]. The parameters \( a, b \) and \( c \) are determined from physical quantities, such as the surface tension \( (\sigma) \) and the correlation length \( (\xi) \) of the fluctuations, and also from the requirement that the second minimum of the above potential should be equal to the pressure difference between the two phases [7].

The bag equation of state is used to calculate the pressure in the two phases. The potential \( V(\phi) \) has a minimum at \( \phi = 0 \) and a metastable second minimum at \( \phi = (3bT + \sqrt{9b^2T^2 - 32ac})/8c \) below \( T \leq T_1 \). In the thin wall approximation [17], \( b, c \) and \( T_1 \) can be written as [7],

\[
b = \frac{1}{\sqrt{6\sigma \xi^3 T_c^2}}; \quad c = \frac{1}{12\xi^3 \sigma}; \quad T_1 = \left[ \frac{BT_c^4}{B - \frac{3\pi}{16\sigma} V_b} \right]^\dagger,
\]

where \( B \) is the bag constant and \( V_b(\phi_m) = [3\sigma/16\xi(T_c)] \) is the height of the degenerate barrier at \( T = T_c \) or at \( a(T_c) = b^2T_c^2/4c \). A wide spectrum of first-order phase transitions, ranging from very weak to strong, can be studied by either changing \( \sigma \) or \( \xi \) or both. For example, for a fixed value of \( \xi \), the strength of the transition is controlled by \( \sigma \), becoming very weak first order or second order when \( \sigma \to 0 \).

We follow Ref. [5] to obtain the equilibrium number density of subcritical bubbles. Let \( n(R, t) \) be the number density of bubbles with a radius between \( R \) and \( R + dR \) at time \( t \) that satisfies the Boltzmann equation

\[
\frac{\partial n}{\partial t} = +|v|\frac{\partial n}{\partial R} + (1 - \Gamma_0 - \Gamma_+) \tag{2}
\]

The first term on the right-hand side is the shrinking term with velocity \( v = \partial R/\partial t \). The term \( \Gamma_0 \) is the rate per unit volume for the thermal nucleation of a bubble of radius \( R \) of phase \( \phi = \phi_+ \) (hadron phase) within the phase \( \phi = 0 \) (QGP phase). Similarly, \( \Gamma_+ \) is the corresponding rate of the phase \( \phi = 0 \) within the phase \( \phi = \phi_+ \). The factor \( \gamma_+ \) is defined as the volume fraction in the hadron phase. Assuming \( \Gamma_0 \approx \Gamma_+(= \Gamma) \) for a degenerate potential at \( T = T_c \), we write for the rate \( \Gamma = AT^4 \exp[-F(\phi_+)/T] \) where \( A \) is a constant of order unity. Using the Gaussian ansatz for subcritical configurations \( \phi(\mathbf{r}) = \phi_+ \exp(-r^2/R^2) \), the free energy functional \( F(\phi) = 4\pi \int r^2dr[1/2(|\partial \phi|/\partial r)^2 + V(\phi, T)] \) can be written as [5] \( F(\phi_+) = \alpha R + \beta R^3 \) where \( \alpha = 3\sqrt{2\pi^3/2}\phi_+^2/8 \) and \( \beta = \pi/2\phi_+^2[\sqrt{2\alpha}/4 - \sqrt{3bT}\phi_+/9 + c\phi_+^2/8] \). The equilibrium number density \( n_0 \) of subcritical bubbles is found by solving Eq. (2) with \( \partial n/\partial t = 0 \) and imposing the physical boundary condition \( n(R \to \infty) = 0 \).

Using \( \gamma_0 \approx 4\pi R^3 n_0/3 \), we get a coupled equation for \( \gamma_0 \), which can be solved to get

\[
\gamma_0 = \frac{I}{1 + 2T}; \quad I = \int_R^{\infty} \frac{4\pi}{3\sigma} R^3 \Gamma(R', \phi_+) dR' \tag{3}
\]

We will consider the statistically dominant fluctuations with \( R \approx \xi \) and estimate \( \gamma_0 \) integrating Eq. (3) from \( \xi \) to \( \infty \). Neglecting the shrinking term in Eq. (2), the time-dependent solution of \( n(\xi, t) \) can be written as [5] \( n(\xi, t) = n_0(\xi)[1 - \exp(-q(\xi)t)] \) where \( q(\xi) = [(8\pi \xi^3/3)\Gamma] \) and \( n_0(\xi) = \Gamma(\xi)/q(\xi) \). Alternatively, in terms of \( \gamma_+ \), the above solution has the form

\[
\gamma_+(\xi, t) = \gamma_0(\xi)[1 - \exp(-q_0t)] \tag{4}
\]

where \( q_0 = (4\pi \xi^3/3)\Gamma/\gamma_0 \). The relaxation time \( \tau = q_0^{-1} \) depends on two factors \( \gamma_0 \) and \( \Gamma \) out of which only the \( \gamma_0 \) is affected by shrinking (if included). Since we know the complete solution of \( \gamma_0 \) that includes shrinking [Eq. (3)], Eq. (4) can also be used to estimate its time dependence. Note that the presence of a shrinking term in Eq. (2) results in a reduction of \( \gamma_0 \) and also in a faster relaxation process.

**FIG. 1.** (a) \( \gamma_0 \) versus \( \sigma \) at \( T = T_c \) for a few typical values of \( A \). \( \xi \) is fixed at 0.5 fm and \( T_c \) at 160 MeV. (b) \( \Delta t \) as a function of \( \sigma \) for various initial conditions as shown in table 1.

First we consider the slow evolution of the medium as in the case of early universe [10,11] so that the equilibrium scenario is applicable. Fig. 1(a) shows the plot of \( \gamma_0 \) as a function of \( \sigma \) at \( T = T_c \) for a few typical values of the prefactor \( A \). We have fixed \( \xi \) at 0.5 fm, \( T_c \) at 160 MeV and \( v = 1/\sqrt{3} \). As expected, the equilibrium hadronic fraction increases with decreasing \( \sigma \) and becomes as large as 0.5 for weak transitions. Recent lattice QCD predictions [2] suggest that the quark-hadron...
weaker the transition. In the standard scenario, we can see Eq. (1), \( \Delta \lambda \) of the deviation of the gluon (quark) density from the formation time rate. Assuming ideal scaling, we can estimate the time \( \Delta t \) taken by the plasma to cool from \( T_1 \) to \( T_c \) as

\[
\Delta t = \frac{T_0}{T_c} t_0 \left[ 1 - \frac{T_c}{T_1} \right], \tag{5}
\]

where \( \nu = 3 \) in (1+1) dimensions. Since \( T_1 \) depends on \( \sigma \) [see Eq. (1)], \( \Delta t \) will also depend on \( \sigma \), being smaller the weaker the transition. In the standard scenario, we can assume the initial temperature \( T_0 \approx 320 \text{ MeV} \) and the formation time \( t_0 \approx 1 \text{ fm} \). However, several perturbative-inspired QCD models [13,14] suggest a very different collision scenario at RHIC and LHC energies, which lead to the formation of unsaturated plasma with high gluon content. Such a plasma will attain thermal equilibrium in a short time \( t_0 \approx 0.3 - 0.7 \text{ fm} \), but will remain far from chemical equilibrium. Since the initial plasma is gluon rich, more quark and anti-quark pairs will be needed in order to achieve chemical equilibrium. The dynamical evolution of the plasma undergoing chemical equilibration was studied initially by Biro et. al. [15] and subsequently by many others [16] by solving the hydrodynamical equations along with a set of rate equations governing chemical equilibrations. It was found that a chemically unsaturated plasma cools faster than what is predicted by Bjorken scaling, since additional energy is consumed during chemical equilibration. Following Ref. [16], we have studied chemical equilibration and dynamical evolution of the QGP with different initial conditions, as listed in table I. We consider two dominant reaction channels \( q \bar{q} \leftrightarrow qg \) and \( gg \leftrightarrow ggg \) that contribute to the chemical equilibrium. The fugacity \( \lambda_{qg}(\leq 1) \) gives the measure of the deviation of the gluon (quark) density from the equilibrium value; chemical equilibrium is achieved when \( \lambda_i's \to 1 \). For the present purpose, we skip the details, and parameterize the cooling rate in terms of \( \nu \) in the range \( T_1 \leq T \leq T_c \). In table I, \( \nu \) has been listed for two sets of initial conditions obtained using HIJING [13] and Self Screened Parton Cascade Model(SSPM) [14] models at RHIC and LHC energies (Note that \( \nu < 3 \) implies a faster cooling). Fig. 1(b) shows the plot of \( \Delta t \) as a function of \( \sigma \) as obtained from Eq. (5) for different \( \nu \) values. The time \( \Delta t \) depends on the initial values of the temperature \( T_0 \), formation time \( t_0 \) and also on the cooling rate \( \nu \). However, except for the SSPM initial conditions at LHC energies, values of \( \Delta t \) obtained with other initial conditions have nearly similar values.

Next we proceed to estimate the density of subcritical hadron bubbles built up at time \( t = \Delta t \). Fig. 2 shows \( \gamma(t)/\gamma_0 \) as a function of \( t \) at three different \( \sigma \) values. The equilibration rate of the subcritical hadron bubbles of a given radius depends on the ratio \( \Gamma/\gamma_0 \). Although both \( \Gamma \) and \( \gamma_0 \) are larger for weaker transitions, their ratio decreases with decreasing \( \sigma \). Therefore, as can be seen, equilibration is faster for a stronger transition as compared to the weak one.

![FIG. 2. The ratio \( \gamma(t)/\gamma_0 \) as a function of \( t \) at three typical values of \( \sigma \) for \( A=1 \).](image-url)
large. It may be mentioned here that we have considered expansion only in (1+1) dimensions. Inclusion of transverse expansion, significant at RHIC and LHC energies, will accelerate the cooling rate further, reducing the amount of phase mixing considerably. Since phase mixing at $T = T_c$ is negligible, the plasma will supercool and the phase transition will proceed by the nucleation of critical-size hadron bubbles within a (nearly) homogeneous background of the metastable QGP phase.

We have also studied the effect of other parameters like $T_c$ and $\xi$ on $\gamma$ and found that $\gamma(\Delta t)$ is very insignificant to promote phase mixing. The prefactor $A$ is the only sensitive parameter on which $\gamma(\Delta t)$ depends. While the choice of $A \approx 1$ is quite reasonable [17], we have also varied $A$ from 1 to 10 and did not find significant phase mixing particularly when $\sigma$ is small.

In conclusion, we have studied the effect of phase mixing promoted by thermal subcritical hadron bubbles during a first-order quark-hadron phase transition as predicted to occur during heavy-ion collisions. Although the equilibrium density distribution of these subcritical bubbles can be quite large, their equilibration time-scale is larger than the cooling time-scale for the QGP. As a consequence, for RHIC and LHC energies, they will not build up to a level capable of modifying the predictions from homogeneous nucleation theory: hadronization will proceed through the nucleation of critical bubbles within a nearly homogeneous background of quark-gluon plasma. This situation is to be contrasted with the cosmological quark-hadron transition, where substantial phase mixing may occur, altering the dynamics of the phase transition.

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**TABLE I.** Initial conditions are taken from ref [18] as predicted by SSFM and HIJING calculations. The fugacities $\lambda_i$'s give a measure of the deviation of the gluon or quark densities from the equilibrium values.

<table>
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<th>CODE</th>
<th>ENERGY</th>
<th>$t_0$ (fm)</th>
<th>$T_0$ (GeV)</th>
<th>$\lambda_g$</th>
<th>$\lambda_q$</th>
<th>$\nu$</th>
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<td>0.55</td>
<td>0.05</td>
<td>0.008</td>
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<td>0.82</td>
<td>0.124</td>
<td>0.02</td>
<td>1.8</td>
</tr>
</tbody>
</table>

**FIG. 3.** (a) The fraction $\gamma$ at $t = \Delta t$ as a function of $\sigma$ at $A = 1$ and $v = 0.577$. (b) Same as above at $A=10$, but for different $v$ values with SSFM initial conditions at RHIC energy.
[11] The solution of Einstein's relativistic field equation yields a relation between the age of the universe and the temperature [10], $t = \sqrt{\frac{\alpha}{G}} T^{-2}$ where $\alpha = 9/(164\pi^3)$. Since the Newton's constant $G$ is very small, the above relation would imply 
\[ \Delta t = \sqrt{\frac{\alpha}{G}} (T_c^{-2} - T_1^{-2}) \sim \text{few } \mu \text{sec.} \]