How Frustrated Strings Would Pull the Black Holes from the Centers of Galaxies

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Abstract

Recently Pen and Spergel (1997) have shown that a universe whose energy density is dominated by a frustrated network of non-Abelian TeV-scale cosmic strings could account for a broad class of cosmological observations. In this paper we consider the effects of such a string network on the massive black holes widely believed to inhabit the centers of many galaxies. As these black holes traverse the universe together with their host galaxies, they would intersect a large number of string segments. We argue that such segments would become stuck to the black hole, and be stretched by the hole's motion. Stretching the strings would cause significant deceleration of the black holes. Although the black holes would probably not be removed from the galaxies completely, they would be noticeably displaced from the galactic center of mass – by at least 5kpc. This displacement seems to be in contradiction to the observational evidence.

1 Introduction

Recently, Pen and Spergel (1997) proposed that the energy density of the universe is dominated by the contribution from a frustrated network of non-Abelian cosmic strings. Because the network is frustrated, the string energy density scales as only the inverse square of the scale factor. This results in a non-decelerating (though also non-accelerating) universe, and so an alternative cosmology intermediate between dark matter dominated Open CDM and cosmological-constant dominated Λ CDM. [2] presents all the basic features of the frustrated string dominated models. This model is also discussed in [3].

The basis of Pen and Spergel’s proposal is that in some non-Abelian field theories, there are cosmic strings which do not intercommute, i.e. pass through each other, effectively. Consequently, the string network can become frustrated. It then ceases to effectively relax to its minimum energy configuration, it “freezes out.” The vacuum energy density of the strings can then come to dominate the energy density of the universe.
Pen and Spergel refined this scenario with numerical simulations of the evolution of a network of non-Abelian cosmic strings. These simulations showed that for $\Omega_0 \sim 0.4-0.6$ and $H_0 \sim 60-70$ km/s/Mpc their model is consistent with the current observations, including CMB fluctuations, the shape of the galaxy power spectrum, and limits on the age of the galaxies. It was also consistent with observations of high redshift supernova and gravitational lensing statistics.

If the universe today is string dominated, the strings must have formed at an energy scale near the electroweak scale ($\sim 1$ TeV), implying an energy per unit length of approximately $5 \times 10^{-5}$ g/cm. The characteristic comoving separation between strings is approximately $10^{-3}$ of the bubble size during the phase transition, corresponding to $L \simeq 0.1$ A.U. today. (An A.U. is the distance between the earth and the sun.)

This density of strings is surprising – there would be many strings inside our Solar System. However, because the mass per unit length of each string is so low, they would be very difficult to detect. For example the gravitational bending angle due to the string is only $(M_W/M_{Pl})^2 \sim 10^{-32}$ radians, where $M_W$ is the electroweak symmetry breaking scale and $M_{Pl}$ is the Planck scale. Even if the string can catalyze baryon decay, its cross section is so small that the string could pass through the sun, the earth, or even a person, completely undetected. The probability that a string would pass through a specially designed detector in a reasonable time is extremely low.

There is an exception to the innocuousness of the electro-weak scale strings. If a black hole encounters a string at small enough impact parameter that part of the string enters the event horizon, then, as we will show, the black hole will capture the string. If the black hole horizon is large compared to the string cross-section, then the string will remain threaded through the black hole horizon. The exterior ends are unlikely to reconnect for an enormously long time.

Let us assume that such a network of “frustrated strings” really exists in our universe. We will show by simple arguments that such a dense string network would severely impact the motion of black holes in the centers of galaxies.

There is convincing evidence that almost all galaxies contain very massive compact objects, probably black holes, at their centers. Analysis of data such as galactic rotation curves, luminosity and mass-to-light ratio indicates the existence of a central extreme mass concentration in many galaxies, including our Milky Way[1], M87[4], and others[5] - [6]. The most convincing case for a black hole per se is from our own Galaxy[1], for which the stellar rotation curves about Sag A* have been measured to approximately $0.01 - 0.03$ pc, and preclude the presence of a stable cluster of stars, white dwarfs, neutron stars, or even stellar-mass black holes. Other than a massive ($2.6 \times 10^6 M_{\odot}$) black hole the only possible explanations are a super-dense cluster of either very light (< $0.02 M_{\odot}$) black holes or weakly interacting dark matter. If anything, each of those seems more far-fetched than a single massive black hole.

The range in mass of these black holes is $10^6 - \text{few} \times 10^9 M_{\odot}$, implying a range in Schwarzschild radii of $10^9 - \text{few} \times 10^{12}$ m. Especially for the more massive black holes, this is larger than the inter-string separation. Thus as the black hole moved through the universe it would encounter many strands of cosmic string. These strands would be stretched by the motion of the black hole through space. As the strings are stretched, they reduce the kinetic energy of the black hole, slowing it. Although each string has a
minimal effect on the black hole, even integrated over the lifetime of the universe, the
cumulative effect of the large number of strings that are being stretched, can be very
significant. Typically we find that, despite the dynamical response of the host galaxy,
a typical galactic core black hole would be observably shifted from the center of the
galaxy.

As we argued briefly above, the motions of ordinary stars are not likely to be affected
– the strings pass right through even a neutron star. Stellar mass black holes will be
affected, albeit to a lesser extent than galactic black holes. This is because the cross-
sectional area of a black hole, and hence the number of strings it is expected to capture,
is proportional to the square of the black hole mass. The total black hole deceleration
due to string drag is therefore proportional to the first power of the black hole mass.

2 String Network and a Black Hole

Consider a super-massive black hole traversing the universe and encountering a string.
(We choose to view things in the rest frame of the string, because the string network is
frustrated and nearly static in the frame of the CMB.) Since the area of the black hole
is large compared to the string cross section, after the black hole hits the string, the
string sticks to the black hole [8]. The portion of the string which enters the horizon
cannot be pulled back out. The response of the rest of the string is limited by causality,
i.e. only the portion of the string within the sound horizon $c_s t$ of the initial point of
string-black hole impact can respond to the encounter of the string with the black hole.
Here $c_s$ is the sound velocity in the string (typically $c_s \simeq c$), and $t$ is the time elapsed
since the black-hole encountered the string. With the string fixed beyond $c_s t$ and the
middle pinned to the black hole, the string begins to stretch. The notion of the string
length in the background space of the black hole is different from the flat space one.
However, by minimizing the length of the string from some point outside the horizon to
the horizon itself, one can infer that, for a string velocity perpendicular to the string,
after the equator of the black hole passes the string, the string prefers energetically
to enter the black hole horizon at normal incident, and the string thereafter stretches
rather than recombining behind the black hole.

An isolated string has a constant energy per unit length, and so the string config-
uration which minimizes the length is also the configuration which minimizes the total
energy. Actually, there is no just one string, but rather a string network. If the strings
were connected in an isolated two-dimensional surface, then the minimum energy con-
figuration would be the surface of minimum area. Since the frustrated non-Abelian
strings are connected in a highly three-dimensional network, the minimum energy con-
figuration is the configuration which minimizes the volume of the deformed network.
By axial symmetry, the volume of the network is the volume of rotation of a curve $f(y)$
about the axis between the original point of contact of the black hole with the network
($y = a$), and the black hole’s position at some time $t$ later ($y = b$). The volume of
rotation of a curve $f(y)$ about the $y$ axis for $a < y < b$ is:

$$V = \pi \int_a^b [f(y')]^2 dy'$$  (1)
Our variational equation is:

$$\delta \int_{0}^{y} [f(y')]^2 dy' = 0 \quad (2)$$

with \( f(0) = L \) and \( f(y) = c_s t \). The solution is a curve which is zero everywhere except at the end points — the configuration shown in Fig. (1). Note that string network can be treated as a continuous medium only on scales bigger than the characteristic distance, \( L \), between the the string segments. Thus, the width of the tube in Fig. (1) is approximately \( L \).

![Figure 1: Actual shape of the deformed string medium](image)

It should be noted that the response of a particular frustrated string network configuration might be intermediate between the volume-minimization and area-minimization results quoted above (or even the length minimization result). However, we will adopt the most conservative possibility — the minimization of volume — to obtain limits on the string network.

### 3 String Recombination and Snapping

All the effects which can result in a black hole shedding the strings have to be taken into account. For example, if two ends of (not necessarily the same) string on the black hole horizon come within a distance characterized by the string thickness \( \sim \frac{1}{T_{\text{eV}}} \) apart, then they can recombine and detach from the black hole.

In traveling a distance \( y \) through the universe, the black hole accumulates \( N(y) = \frac{y L^2}{2 g L_{g}} \) strings. (\( g \) is a geometrical factor having to do with the connectivity of the network.) The ends of these strings will accumulate on a circle of radius \( l = \frac{R_{\text{Sch}} L}{y} \).

Recombination will happen when \( \frac{2g}{N(y)} < \frac{1}{T_{\text{eV}}} \), i.e. when the black hole has traveled

$$y_{\text{rec}} \approx \sqrt{\frac{2g L^4 T_{\text{eV}}}{R_{\text{Sch}}}} \quad (3)$$

For a \( 10^9 \) Solar masses black hole, and \( g = 3 \) we get \( y_{\text{rec}} \approx 2 \times 10^7 \) pc. A black hole traveling at \( (1-2) \times 10^{-3}c \) would take \( \mathcal{O}(10^{11}) \) years to travel this distance.
Figure 2: String recombination

One thing which could affect above calculation is string oscillations. Collision of a string with a black hole would excite oscillatory modes of the string. These oscillations would enhance the probability for string segments to collide with each other. However, the initial amplitude of these oscillations are relatively small because the black hole is moving very slowly compared to the sound speed in the strings. Also, these string oscillations are rapidly damped by several effects. First, the energy of string oscillations are rapidly propagated away from the black hole into the bulk network. Second, the links nearest the black hole are being stretched, effectively “red-shifting” away the oscillations. Also, for most non-Abelian string networks, most string-string collisions do not result in reconnections which result in a string loop falling into the black hole. Indeed, because generic reconnections would be with string links that were captured by the black hole long before, they would cause the network to become more tangled, rather than less, and hence not result in any decrease in the force applied by the stretching string network on the black hole. Correct and full treatment of these oscillations requires a concrete model of the strings and probably a detailed numerical evolution of the frustrated string network in the black hole wake.

There is also possibility that the strings might snap. The snapped ends could terminate on monopoles or mini black holes and the black hole could tear the network rather than get caught. However, one can easily show that the probability for this to happen is negligible. If the underlying field theory contains monopoles, then the probability of monopole-antimonopole nucleation per unit length per unit time is \( \sim M_W^2 e^{-\pi m_m^2/u^2} \) where \( m_m \) is the mass of the monopole and \( u^2 \) is the string tension or the energy per unit length of the string [10]. From \( m_m \sim 4\pi M_{m}/\alpha \), where \( \alpha \) is the fine structure constant and \( M_m \) is the energy scale at which the monopoles were formed, we see that the process is suppressed by a huge exponential factor even if \( M_m \sim M_W \). Even integrated over the lifetime of the universe and over the causal length corresponding to this time, the probability is still negligibly small due to the non-perturbative nature of this effect. The snapped string ends could also terminate on mini black holes. For a mini black hole of the Planck mass the probability of pair nucleation per unit length per unit time is \( \sim M_W^2 e^{-\pi M_{Pl}^2/u^2} \) which is even more suppressed than the monopole-
antimonopole creation. Finally, in the case when the underlying field theory does not support monopoles, the presence of the black hole changes the topology of the space-time and allows monopole solutions which wind around the black hole or the black hole itself could carry magnetic charge and be a monopole itself. Due to their enormous mass, the probability of nucleating of such monopoles is enormously small.

4 Deviation from the Free-Motion Path due to Strings

The change in energy of one string due to the motion of the black hole a distance $\Delta y$ past their initial point of encounter is $\Delta E \approx 2\Delta y u^2$, where $u^2$ is energy per unit length of the string. The total change in energy due to all the strings that the black hole encounters is

$$\Delta E_{\text{tot}} = \frac{\pi R_{\text{Sch}}^2 u^2}{L^3 g} \int_0^y 2(y - y_0)dy_0 = \frac{\pi R_{\text{Sch}}^2 u^2 y^2}{L^3 g}$$  \hspace{1cm} (4)

Imposing energy conservation we find:

$$\frac{1}{2} M_{\text{bh}} \dot{y}_0^2 = \frac{1}{2} M_{\text{bh}} \dot{y}^2 + \frac{\pi R_{\text{Sch}}^2 u^2 y^2}{L^3 g}$$  \hspace{1cm} (5)

where $\dot{y}_0$ is the initial velocity of the black hole.

Let $\Delta = \dot{y}_0 t - y$ measure the deviation of the black hole from its free path. Suppose that $\Delta$ is small, i.e. $\Delta \ll \Delta_{\text{max}} \equiv \dot{y}_0 t$. Linearizing in $\Delta$ we find:

$$\dot{\Delta} = \frac{\pi R_{\text{Sch}}^2 u^2 \dot{y}_0}{g L^3 M_{\text{bh}}} t^2,$$  \hspace{1cm} (6)

the solution of which is

$$\Delta = \frac{\pi R_{\text{Sch}}^2 u^2 \dot{y}_0}{3g L^3 M_{\text{bh}}} t^3.$$  \hspace{1cm} (7)

(Not that equation (5) can be solved exactly giving

$$y = \dot{y}_0 \sqrt{\frac{g L^3 M_{\text{bh}}}{2 \pi R_{\text{Sch}}^2 u^2}} \sin \left( \sqrt{\frac{2\pi R_{\text{Sch}}^2 u^2}{g L^3 M_{\text{bh}}} t} \right)$$  \hspace{1cm} (8)

and

$$\Delta = \dot{y}_0 \left[ t - \sqrt{\frac{g L^3 M_{\text{bh}}}{2 \pi R_{\text{Sch}}^2 u^2}} \sin \left( \sqrt{\frac{2\pi R_{\text{Sch}}^2 u^2}{g L^3 M_{\text{bh}}} t} \right) \right]$$  \hspace{1cm} (9)

This is a monotonically increasing function which for a small argument expansion in sin gives the result (7).)

Taking $u^2 = (1 TeV)^2$, $\dot{y}_0 = 300\text{km/s}$, $g = 3$, $L = 0.1$ A.U., $M_{\text{bh}} = 10^9 M_{\odot}$ and the elapsed time since the black hole began moving to be only $t = 3 \cdot 10^9$ years, we get:

$$\Delta \sim 3 \cdot 10^6 \text{pc}$$  \hspace{1cm} (10)
Obviously, $\Delta$ is bigger than $\dot{y}_0 t \sim 3 \cdot 10^5 \text{pc}$ which means that our assumption $\Delta \ll \Delta_{\text{max}} \equiv \dot{y}_0 t$ would break down. However, result (10) has ignored the gravitational pull of the galaxy in which the black hole is embedded, which will counteract the deceleration due to the string.

5 Influence of the Galaxy

Because the force on the black hole due to the stretching of the string network is quite small, the displacement of the black hole relative to the center of its host galaxy is slow enough that the galaxy can respond dynamically. We must therefore include the gravitational pull of the galaxy in which the black hole is embedded, which will counteract the deceleration due to the string. Looking from the rest galaxy frame, the displaced black hole will start to drag a part of galaxy with it. This could be an considerable effect – if the galaxy were a rigid body attached to the black hole, then the acceleration of the black hole would be reduced by a factor of $M_{\text{Galaxy}}/M_{\text{bh}} > 10^3$. The galaxy is not a rigid body, nor is it rigidly attached to the black hole, hence we must model the galactic mass density distribution, and the galaxy response, although the details of the model will have little effect on our final conclusions. We adopt the following simple, but conservative, model – we take the galaxy to be spherically symmetric with the mass density excluding the black hole given by

$$
\rho(r) = \rho_0 \begin{cases} 
0 & r < r_c \\
\left(\frac{r_c}{r_0}\right)^{-(3+\gamma)} & r_c < r < r_0 \\
\left(\frac{r_c}{r_0}\right)^{-2} & r > r_0
\end{cases}
$$

(11)

The region $r < r_c \equiv R_{\text{Sch}}$ is occupied by the black hole. The region $r_c < r < r_0$, where $\rho(r) \sim r^{-3}$ or steeper, is the central core of the galaxy. Empirically, $r_0 \simeq 2 - 10 \text{kpc}$. In the third region ($r > r_0$), $\rho(r)$ goes more or less like $r^{-2}$. $\rho_0$, the density at the radius where the velocity dispersion or orbital velocity turns over, can be determined from the condition

$$
\frac{GM(r_0)+M_{\text{bh}}}{r_0^2} = \frac{\dot{y}_0^2}{r_0^3}
$$

where $\dot{y}_0$ is the linear orbital velocity of galaxy at the distance $r_0$ from the center of the galaxy. Accordingly, the galactical mass distribution $M(r) \equiv 4\pi \int_0^r \rho(r) r^2 dr$ is:

$$
M(r) = \begin{cases} 
\frac{(\frac{r_c^3}{r_0^3} - M_{\text{bh}})}{(1-(\frac{r_c}{r_0})^{3+\gamma})} (1 - (\frac{r_c}{r})^{\gamma}) & \gamma > 0, \ r_c < r < r_0, \\
\frac{(\frac{r_c^3}{r_0^3} - M_{\text{bh}})}{\ln\left(\frac{r}{r_c}\right)} \ln\left(\frac{r}{r_c}\right) & \gamma = 0, \ r_c < r < r_0.
\end{cases}
$$

(12)

We then assume that if the black hole has been displaced by the string-force by $\Delta$ from the center of the galaxy, then it carries with it that portion $M(\Delta)$ of the galactic mass within the radius $\Delta$.

The energy conservation equation (5) is now replaced by:

$$
\frac{1}{2} M(\Delta) \dot{y}_0^2 + \frac{1}{2} M_{\text{bh}} \dot{y}_0^2 = \frac{1}{2} M(\Delta) \dot{y}^2 + \frac{1}{2} M_{\text{bh}} \dot{y}^2 + \frac{\pi R_{\text{Sch}}^2 u^2 y^2}{L^3 g}
$$

(13)
Again, we are interested in small $\Delta$, obviously much smaller than $y_0 t$, but we also have much stronger constraints. If $\Delta$ is of the order of $r_0 \sim 2 - 10$ kpc, then it means that the black hole has been displaced $2 - 10$ kpc from the center of the galaxy – clearly inconsistent with the observations of elliptical galaxies with candidate black holes. We therefore take $\Delta < r_0$. Linearizing again in $\Delta$, (13) can be rewritten:

$$\dot{\Delta} = (M(\Delta) + M_{bh})^{-1} \frac{\pi R_{Sch}^2 u^2 y_0}{L^3 g} t^2.$$  \hspace{1cm} (14)

Since $M(\Delta) = 0$ for $\Delta < r_c$, in this region $\Delta$ is given by equation (7). The black hole therefore reaches $\Delta = r_c$ in time

$$t_c = \left( \frac{3gL^3M_{bh}r_c}{\pi R_{Sch}^2 u^2 y_0} \right)^{\frac{1}{3}}$$  \hspace{1cm} (15)

For $t > t_c$ (or equivalently $\Delta > r_c$) we have two cases, $\gamma = 0$ and $\gamma = 1$. Integration from $t_c$ to $t$ gives:

$$\frac{(\frac{v_0^2 r_c}{c^2} - M_{bh})}{ln(\frac{\Delta}{r_c})}(ln(\frac{\Delta}{r_c}) - 1)\Delta + M_{bh} \Delta = \frac{\pi R_{Sch}^2 u^2 y_0}{L^3 g} t^3 + \frac{\frac{v_0^2 r_c}{c^2} - M_{bh}}{ln(\frac{\Delta}{r_c})} r_c$$  \hspace{1cm} (16)

when $\gamma = 0$, and

$$\frac{(\frac{v_0^2 r_c}{c^2} - M_{bh})}{(1 - \frac{r_c}{r_0})} r_c ln \frac{\Delta}{r_c} + \left( \frac{\frac{v_0^2 r_c}{c^2} - M_{bh}}{1 - \frac{r_c}{r_0}} \right) + M_{bh} \Delta = \frac{\pi R_{Sch}^2 u^2 y_0}{L^3 g} t^3 + \frac{\frac{v_0^2 r_c}{c^2} - M_{bh}}{(1 - \frac{r_c}{r_0})} r_c$$  \hspace{1cm} (17)

when $\gamma > 0$. Using $M_{bh} = 10^9 M_{Sun}$, $\Delta_{max} = r_0 \sim 3$ kpc, $v_0 = 300$ km/s, $t = 3 \times 10^9$ years and $(ln(\frac{\Delta}{r_c}))_{max} \sim 17$, we find $\Delta \sim 50$ kpc for both $\gamma = 0$ and $\gamma = 1$.

This result should not, or course, be believed for $\Delta > r_0$, but even $\Delta = r_0$ is enough to rule out the frustrated string network model. The time needed to reach $\Delta = r_0$, is $t_0 \sim 10^9$ years for both $\gamma = 0$ and $\gamma = 1$. We can also estimate the displacement $\Delta$ of the black hole in the last $\Delta t = 250 \cdot 10^6$ years, which is a characteristic dynamical time of the galaxy. For this purpose, we integrate equation (14) from $(t_{now} - \Delta t)$ to $t_{now}$ with the very conservative assumption that $\Delta(t_{now} - \Delta t) = 0$. For both $\gamma = 0$ and $\gamma = 1$, we find $\Delta \sim 5$ kpc.

If we accept that the astronomical observations indicate massive black holes, some of mass greater than $10^9 M_{Sun}$, located within 400 pc of the galactic center (400 pc is an underestimate of the accuracy of the black hole position [9]), then the possibility of a cosmologically dominant frustrated non-Abelian string network is severely complicated.

### 6 Conclusion

The possibility that a frustrated network of non-Abelian strings dominates the energy density of the universe is a fascinating one. Although there are models of such strings which agree remarkably well with a broad class of observations, we have shown that
with the string and network parameters required to play the desired cosmological role, the string network would severely affect the relative motion of central galactic black holes with respect to the rest of the galaxy, if the black hole mass is greater than about $10^8 M_{\text{Sun}}$. The black hole would be noticeably displaced from the center of the galaxy even in the last galactic dynamical time. This does not necessarily rule out a cosmologically important frustrated non-Abelian string network. It could for example be that the superdense objects at the centers of galaxy are not black holes, but some other exotic objects which would interact far more weakly with the string network. One might also be able, by altering the field-theory model which gave rise to the strings, to alter the strings' linear density or network properties and thereby reduce the dynamical effects on the galactic black holes. In any case, these effects must be considered in any ongoing attempts to incorporate cosmic string networks into cosmology.

References
