Light quark masses for dynamical, non-perturbatively O(a) improved Wilson fermions

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We present results for the light quark masses in lattice QCD with two degenerate flavours of dynamical fermions. We used configurations generated by the UKQCD and QCDSF collaborations at six different combinations of $\beta$ and $\kappa$.

1. INTRODUCTION

The calculation of the light quark masses is one of the major aims of lattice gauge theories. In recent years these fundamental parameters have been determined with a rather high accuracy using quenched QCD. At least for the strange quark mass, systematic errors are believed to be under control, except for quenching effects. However, for current simulations of QCD with dynamical fermions this is not yet the case. The main sources of uncertainty are the lack of non-perturbative results for the renormalization constants and the missing possibility to do a reliable continuum extrapolation.

For the calculations presented here we used dynamical configurations with $N_f = 2$ flavours of degenerate quarks for six different combinations of $\beta$ and $\kappa$. The coefficient for the improvement term of the Wilson fermion action, $c_{\text{SW}}(\beta)$, was taken from \cite{1}. The lattice spacing $a$ varies between 0.104 and 0.089 fm. For the lightest sea quarks the ratio $m_{PS}/m_V$ was 0.6. (For further details, see \cite{2}.)

We define the renormalized quark mass in the $\overline{\text{MS}}$-scheme at scale $\mu$ in the following way:

$$\tilde{m}_q^{\overline{\text{MS}}} (\mu) = \frac{(1 + b_A a m_q) Z_{\overline{\text{MS}}} Z_A}{(1 + b_P a m_q) Z_{\overline{\text{MS}}}^{P}\mu} \tilde{m}_q,$$  \hspace{1cm} (1)

where

$$a m_q = \frac{1}{2} \left( \frac{1}{\kappa} - \frac{1}{\kappa_c} \right)$$  \hspace{1cm} (2)

and

$$a \tilde{m}_q = \frac{1}{2} \frac{\langle \partial_t A_4(t) P(0) \rangle}{\langle P(t) P(0) \rangle}$$  \hspace{1cm} (3)

are the bare quark masses from the conserved vector current (CVC) and the partially conserved conserved axial vector current (PCAC), respectively. In order to eliminate $O(a)$ effects introduced by the axial vector current $A_\mu$, one has to replace $A_4$ in the last equation by

$$A_4 \rightarrow A_4 + c_A a \partial_t P,$$  \hspace{1cm} (4)

where $P$ is the pseudoscalar density. We therefore have to compute the following ratios on the lattice:

$$a \tilde{m}_q = a \tilde{m}_q^{(1)} + c_A a \tilde{m}_q^{(2)}$$  \hspace{1cm} (5)

$$= \frac{\langle \partial_t A_4(t) P(0) \rangle}{2 \langle P(t) P(0) \rangle} + c_A \frac{\langle a \partial_t^2 A_4(t) P(0) \rangle}{2 \langle P(t) P(0) \rangle}.$$

Since both ratios can be measured rather precisely, we are, for the rest of this talk, left with basically two problems:

- Determination of the renormalization constants $Z_{\text{O}}$, the coefficients $b_{\text{O}}$, which parametrize the quark mass dependence of the renormalization, and the improvement coefficient $c_A$.

- Extrapolation of the lattice results to the physical quark masses.

In this talk we use $r_0 = 0.5$ fm and take the lattice results for $r_0/a$ from \cite{2} to set the scale.
2. IMPROVEMENT AND RENORMALIZATION

Until now, none of the renormalization constants and improvement coefficient, except for $Z_A [3]$, have been calculated non-perturbatively for $N_f = 2$ improved fermions. We therefore will use perturbative results and compare these with available non-perturbative numbers, e.g. quenched results, in order to have some estimate of the possible errors.

Ordinary perturbation theory on the lattice is known to suffer from problems because of large tadpole diagrams and the expansion in a non-physical (bare) coupling constant. Usually it is better to apply so-called tadpole improved perturbation theory, which refers to a strategy to sum tadpole diagrams and to use a physical coupling constant in the perturbative expansion.

The choice of the coupling constant to be used is not unique. We will use the boosted coupling constant $g^* = g/\Lambda^{\overline{\text{MS}}}$ for bare quantities and the coupling constant in the $\overline{\text{MS}}$-scheme ($g_{\overline{\text{MS}}}^2 = 4\pi\alpha_{\overline{\text{MS}}}(1/a)$) for the computation of renormalized quantities. $\alpha_{\overline{\text{MS}}}(1/a)$ can be calculated from the 4-loop expansion of the $\beta$-function [4], using $\Lambda^{\overline{\text{MS}}} = 250$ MeV.

Figure 1. Ratio $R_1(a)$ defined in Eq. (6) as a function of the lattice spacing using $c_A = 0$ (◦) and the tadpole-improved result (●).

The improvement coefficient $c_A$ is known from 1-loop perturbation theory [5]. In the quenched approximation $c_A$ has also been computed non-perturbatively [6]. To parametrize the difference between the non-perturbative and the tadpole-improved results at the strange quark mass, it is convenient to define the ratio

$$R_1(a) = \frac{\bar{m}_q^{(1)} + c_A \bar{m}_q^{(2)}}{\bar{m}_q^{(1)} + c_A^{(\text{NP})} \bar{m}_q^{(2)}} \bigg|_{m_0 = \sqrt{2} m_K}.$$  \hspace{1cm} (6)

In Fig. 1 we show $R_1(a)$ using our quenched results [7]. In the continuum limit we expect $R_1(0) = 1$. Although $R_1(a)$ approaches this limit very quickly, the results for the bare quark mass show discretization effects of up to 15% for $a \approx 0.1$ fm.

As a next example we compare the tadpole-improved result for $\kappa_{\text{val}}$ to $\kappa_{\text{val}}^{(\text{NP})}$ [8]

$$\kappa_{\text{val}} = \frac{1}{8} \left[ 1 + g^2 (0.025 - 0.029 c_{\text{SW}} u_0^3 - 0.012 (c_{\text{SW}} u_0^3)^2) \right] u_0^{-1}, \hspace{1cm} (7)$$

with the non-perturbative results. For each $(\beta, \kappa_{\text{sea}})$ we calculated $\bar{m}_q$ for three different values of $\kappa_{\text{val}}$. From a linear extrapolation in $1/\kappa_{\text{val}}$ to $\bar{m}_q^{(\text{val})} = 0$ we determine $\kappa_{\text{val}}$. In Fig. 2 we compare our non-perturbative results with the results from Eq. (7). In this case we find the relative difference between both results to be less than 0.1%.

The renormalization constants are known to 1-loop tadpole-improved perturbation theory [9], $Z_{\overline{\text{MS}}}^P(\mu)$ is scale dependent and we will use the...
renormalization group equations, to scale our results to a convenient scale, i.e.
\[
\tilde{m}_q(2\text{ GeV}) = \frac{\Delta Z^\text{MS}(1/a)}{\Delta Z^\text{MS}(2\text{ GeV})} \tilde{m}_q(1/a),
\]  
\(8\)
where \(\Delta Z(\mu)\) [7] is the factor which translates a scale and scheme dependent value of the quark mass to its renormalization group invariant value.

To first order we expect that the following relation between \(\tilde{m}_q\) and \(m_q\) holds:
\[
\tilde{m}_q = \frac{Z_P}{Z_S Z_A} m_q + O(m_q^2).
\]  
\(9\)
From the slope of the fits shown in Fig. 2 we get a non-perturbative value for the ratio \(Z_P/Z_S Z_A\). In Fig. 3 we compare these results with the corresponding tadpole-improved numbers. In this case we find the difference to be of the order of 10%. However, one might hope that the results for \(Z_m(\mu) = Z_A/Z_P(\mu)\) from tadpole-improved perturbation theory to be more reliable, since the contributions from the tadpole diagrams cancel. For the quenched approximation this can be checked, since \(Z_m(\mu)\) has been calculated non-perturbatively in the Schrödinger functional (SF) scheme [10]. In order to compare perturbative and non-perturbative results we looked at the ratio
\[
R_2(a) = \frac{\Delta Z^\text{MS}(1/a) Z^\text{SF}(1/a)}{\Delta Z^\text{SF}(1/L) Z^\text{MS}(1/L)},
\]  
\(10\)
where we expect \(R_2(a) = 1 + O(a)\). The results for \(R_2(a)\) are shown in Fig. 4.

In perturbation theory we have \(b_A \approx b_P\). This has recently been confirmed non-perturbatively for quenched QCD [11]. It is therefore expected that these coefficients will have only a small effect on the results.

3. QUARK MASSES

We are now able to combine our results and to calculate \(m_q^\text{MS}(2\text{ GeV})\). In Fig. 5 we show the squared pseudoscalar mass as a function of the renormalized quark mass at \(\kappa^{\text{val}} = \kappa^{\text{sea}}\).

Even for large quark masses the deviation from linearity is very small. Making a linear ansatz we can compute the light and strange quark mass by solving the equations
\[
b_1 2 r_0 \tilde{m}_l := (r_0 m_{\pi}^{\text{phys}})^2, \quad (11)\]
\[
b_1 (r_0 \tilde{m}_l + r_0 \tilde{m}_s) := (r_0 m_{K}^{\text{phys}})^2. \quad (12)\]
Using the experimental values \(m_{\pi}^{\text{phys}} = 137.3\text{ MeV}\) and \(m_{K}^{\text{phys}} = 495.7\text{ MeV}\) yields the results \(m_{\pi}^\text{MS}(2\text{ GeV}) = 3.8(3)\text{ MeV}\) and \(m_{K}^\text{MS}(2\text{ GeV}) = 96(4)\text{ MeV}\).

For a more sophisticated ansatz we make use of partially quenched chiral perturbation theory. To 1-loop order the pseudoscalar mass can be written in the following form [12]:
\[
m_{PS}^2 = b_1(\kappa^{\text{sea}}) 2 \tilde{m}_l^{\text{val}} + b_2(\kappa^{\text{sea}}) (2 \tilde{m}_l^{\text{val}})^2 \quad (13)\]
The coefficients \(b_1\) and \(b_2\) can be calculated from a quadratic fit to our results at constant \((\beta, \kappa^{\text{sea}})\). The results, which show a rather mild dependency on \(\kappa^{\text{sea}}\), are plotted in Fig. 6. To calculate the
Figure 5. Renormalized quark mass as function of the squared pseudoscalar mass in units of $r_0$.

We finally get the following results:

$$\tilde{m}_{l}^{\overline{MS}}(2 \text{ GeV}) = 3.5(2) \text{ MeV},$$

$$\tilde{m}_{s}^{\overline{MS}}(2 \text{ GeV}) = 90(5) \text{ MeV}. \quad (15)$$

In a previous quenched calculation [7] we found $\tilde{m}_{s}^{\overline{MS}}(2 \text{ GeV}) = 105(4) \text{ MeV}$. Since systematic errors, which are currently not under control, might be of the order 10%, we can not conclude that the (small) difference between these results is a quenching effect.

Figure 6. Coefficients $b_1$ and $b_2$ as a function of the squared pseudoscalar mass at $\kappa_{\text{sea}} = \kappa_{\text{val}}$. The lines show a linear fit.

4. CONCLUSIONS

We did a preliminary analysis of the light quark masses using two flavours of degenerate, non-perturbatively improved fermions. We used tadpole-improved perturbation theory, but tried to give some idea of the systematic errors. For the extrapolation to the physical region we used partially quenched chiral perturbation theory. The difference between our final results and earlier published quenched results was found to be small.

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REFERENCES

2. A. Irving, this conference.