Using a chiral Lagrangian we show that strongly interacting models of electroweak symmetry breaking are not in conflict with precision data. Such models, like Technicolor, need not lead to a heavy Higgs-like signal. Furthermore, the allowed values for the low-energy constants in the effective Lagrangian, derived from bounds on the oblique correction parameters $S, T, U$, are not unnatural. Finally, we point out that there are some problems with gauge invariance, if one tries to relate the oblique parameters to the low-energy constants in the ordinary chiral Lagrangian for QCD of Gasser and Leutwyler. In particular, $S$ cannot be identified with $r_G^{L_2}$.

1 The electroweak chiral Lagrangian

If the electroweak (EW) symmetry is dynamically broken by some strongly interacting underlying theory, similarly to chiral symmetry breaking in QCD, and if there is a mass gap between the observed particles in the Standard Model (SM) and the scale of this underlying theory, one can construct an effective field theory in analogy to chiral perturbation theory. In the bosonic sector, the corresponding EW chiral Lagrangian is of the form $\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots$, where $\mathcal{L}_k$ is of order $p^k$. The Lagrangian $\mathcal{L}_2$ describes a gauged non-linear sigma model. $\mathcal{L}_4 = \sum_i a_i \mathcal{O}_i$, where the $SU(2)_L \times U(1)_Y$ gauge-invariant operators $\mathcal{O}_i$ are built from the light fields, i.e. the photon and the $W$- and $Z$-bosons. The low-energy constants (LEC’s) $a_i$ parametrize different underlying theories.

2 Are strongly interacting models ruled out by precision data ?

a) Light Higgs: A heavy Higgs boson is excluded since all recent SM fits of precision data point to a low Higgs mass, $M_H < 170$ GeV at 95% C.L. However, it is not true that all strongly interacting theories lead to a heavy Higgs-like signal. As shown in Table 1, the pattern of LEC’s $a_i$ for a heavy Higgs boson differs from the one in a simple Technicolor model, estimated using VMD.
Table 1: Non-vanishing, renormalized LEC’s $a_i(\mu = M_Z)$ for the SM with $M_H = 1$ TeV and for a two-flavor, QCD-like Technicolor (TC) model with a Technirho mass of 2 TeV.

<table>
<thead>
<tr>
<th></th>
<th>$10^3 \times a_0$</th>
<th>$10^3 \times a_1$</th>
<th>$10^3 \times a_2$</th>
<th>$10^3 \times a_3$</th>
<th>$10^3 \times a_4$</th>
<th>$10^3 \times a_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>-11.8</td>
<td>-2.6</td>
<td>-0.8</td>
<td>0.8</td>
<td>1.6</td>
<td>8.9</td>
</tr>
<tr>
<td>TC</td>
<td>-14.7</td>
<td>-8.9</td>
<td>-5.4</td>
<td>5.4</td>
<td>5.2</td>
<td>-0.3</td>
</tr>
</tbody>
</table>

\(b) S\)-parameter: The oblique correction parameters \(S, T, U\) describe effects of new physics beyond the SM that enter the self-energies of EW gauge bosons. The PDG quotes the value \(S = -0.16 \pm 0.11\) for \(M_H = 300\) GeV, whereas estimates for Technicolor models lead to \(S \approx O(1)\). The parameters \(S, T, U\) can be related to the LEC’s \(a_i\), e.g., \(S = 16\pi \left[-a_1(\mu) + \text{EWChL loops}(\mu)\right]\).

Interpreting the PDG values as bounds on the deviations from the SM, \(\Delta S \equiv S - S_{SM}(M_H)\), one obtains with the values for the \(a_i\) for a heavy Higgs boson:

\[
\Delta S = 16\pi \left[-a_1(\mu) - \frac{1}{12} \left(\frac{1}{6} + \ln\left(M_T^2/\mu^2\right)\right)\right] \Rightarrow 10^3 \times a_1(M_Z) = 1.8 \pm 2.2 ,
\]

\[
\Delta T = \frac{8\pi c}{c_W^2} \left[a_0(\mu) + \frac{3}{8} \left(\frac{1}{6} + \ln\left(M_T^2/\mu^2\right)\right)\right] \Rightarrow 10^3 \times a_0(M_Z) = -6.4 \pm 4.3 ,
\]

\[
\Delta U = 16\pi a_8 \Rightarrow 10^3 \times a_8(M_Z) = 2.4 \pm 3.0 .
\]

Although this rules out QCD-like Technicolor models, mainly from \(a_1\), the size of the \(a_i\) is not unnatural for strongly interacting theories, see also Refs. 6,7.

3 \(S \neq l^{GL}_5\) or the issue of gauge invariance

There are some subtle problems with gauge invariance, if one tries to relate \(S\) to \(a_1\) and \(a_1\) to \(l^{GL}_5\) in the ordinary chiral Lagrangian. There is a qualitative difference to ChPT because the gauge fields in the EW chiral Lagrangian are dynamical. Using the equations of motion for the gauge fields one can in fact remove the operators corresponding to \(a_1\) and \(a_8\) from the basis. Therefore, one cannot simply map estimates for the LEC’s \(l^{GL}_i\) from QCD or from models into the LEC’s \(a_i\) without performing a complete matching calculation.

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References