Chiral Symmetry Breaking for Domain Wall Fermions in Quenched Lattice QCD

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The domain wall fermion formulation exhibits full chiral symmetry for finite lattice spacing except for the effects of mixing between the domain walls. Close to the continuum limit these symmetry breaking effects should be described by a single residual mass. We determine this mass from the conservation law obeyed by the conserved axial current in quenched simulations with \( \beta = 5.7 \) and 6.0 and domain wall separations varying between 12 and 48 on \( 8^3 \times 32 \) and \( 16^3 \times 32 \) lattices. Using the resulting values for the residual mass we perform two complete and independent calculations of the pion decay constant. Good agreement is found between these two methods and with experiment.

1. INTRODUCTION

An important feature of both domain wall and overlap fermion formulations is that the chiral limit is disentangled from the continuum limit. For the case of the domain wall fermions, the chiral symmetry breaking is solely controlled by the amount of coupling between the domain walls[1]. To exploit the chiral properties of the domain wall fermions, it is essential to understand these chiral symmetry breaking effects.

We reported earlier[2,3] that for quenched simulations, although the quantity \( m^2_\pi(m_f = 0) \) obtained from the pseudoscalar density correlator decreased as the separation between the domain walls \( L_s \) was increased, it did not appear to vanish in the \( L_s \to \infty \) limit. Further investigation shows that the chiral limit of the pion mass is distorted by many factors. Besides the order \( a^2 \) effects and finite volume effects common to both dynamical and quenched simulations using domain wall fermions, the presence of topological near-zero modes in the quenched approximation significantly affects the determination of the pion mass[4,5]. These zero-mode effects can be suppressed by going to a larger volume. Other non-linear behaviors caused by the absence of the fermion determinant in quenched simulations, such as “quenched chiral logs”, may also play an important role in the behavior of the pion mass as the chiral limit is approached. These factors make it difficult to determine the degree of chiral symmetry breaking from examination of the pion mass in the quenched chiral limit. The technique described in detail below quantifies the chiral symmetry breaking effects more precisely by a single residual mass measured from the extra term in the divergence of the axial current[6].

2. RESIDUAL CHIRAL SYMMETRY BREAKING

Close to the continuum, the effect of mixing between the domain walls produces chiral symmetry breaking terms with a coefficient \( m_{\text{res}} \), the residual mass, in the low energy effective Lagrangian. This results in an effective quark mass of \( m_{\text{eff}} = m_f + m_{\text{res}} \) entering all low momentum Green’s functions.

We can measure \( m_{\text{res}} \) starting from the divergence of the axial current, given by[6]

\[
\Delta \mu \cdot A_\mu^a(x) = 2m_f J_5^a(x) + 2J_{5q}^a(x).
\]

Compared with the corresponding continuum expression, the additional term from \( J_{5q}^a \) referred as the “mid-point” contribution, should be equiva-
lent to $m_{\text{res}}J_q^a$ up to $O(a^2)$. Therefore when $t$ is large enough such that low energy physics dominates, the ratio

$$R(t) = \frac{\langle \sum_{\vec{x}} J_{q_5}(\vec{x}, t) \pi^a(0) \rangle}{\langle \sum_{\vec{x}} J_{q_3}(\vec{x}, t) \pi^a(0) \rangle}$$

should be equal to $m_{\text{res}}$. Fig. 1 shows that the plateau for $R(t)$ at $\beta = 6.0$, $L_s = 16$ starts from $t = 2$, and at $\beta = 5.7$, $L_s = 48$ it starts from $t = 4$.

Figure 1. $R(t)$ versus $t$ at $\beta = 6.0$, $L_s = 16$ (circles) and $\beta = 5.7$, $L_s = 48$ (diamonds).

Being part of the mass term of the low energy effective Lagrangian close to the continuum, $m_{\text{res}}$ should provide a universal description of the residual chiral symmetry breaking effects in all long distance observables. This is demonstrated in Figure 2 where $m_{\text{res}}$ shows little $m_f$ or volume dependence at $\beta = 5.70$, $L_s = 48$.

Figure 2. The residual mass $m_{\text{res}}$ versus $m_f$ at $\beta = 5.70$, $L_s = 48$. The open circles are obtained from 106 configurations with volume $16^3 \times 32$, while the closed diamonds are obtained from 335 configurations with volume $8^3 \times 32$.

The five points can also be fit extremely well to a double-exponential function; this may suggest the existence of multiple decay modes. The exact asymptotic form for $m_{\text{res}}$ as a function of $L_s$ still needs further investigation. However for $L_s = 16$, $m_{\text{res}}$ is already as low as $3.87(16)$ MeV in the MS scheme at 2 GeV, about 1/30 of the strange quark mass. This value is so small that the residual chiral symmetry breaking is not very important for these simulations. Even at the stronger coupling $\beta = 5.7$ ($a^{-1} \sim 1$ GeV), when $L_s$ is increased from 32 to 48, $m_{\text{res}}$ drops from 0.0105(2) to 0.00688(13), which is about 1/14 of the strange quark mass. Good chiral properties can be obtained by simulating at this value of $L_s$. Studies of this issue can also be found in [7,8].

3. CALCULATION OF $f_\pi$

As a test of these measurements and chiral properties, we calculate the pion decay constant using the pseudoscalar density correlator, which
Figure 3. The residual mass $m_{\text{res}}$ versus $L_s$ at $\beta = 6.0$. The dotted line is a simple exponential fit. The long dashed line is an exponential plus constant fit. The solid line is a double-exponential fit.

requires knowledge of the residual mass, and compare the results with those obtained from the axial vector current correlator.

Following the definition of the pion decay constant, $f_\pi$ can be related to the amplitude of the axial current correlator $A_{AA}$:

$$A_{AA} = (\frac{f_\pi}{Z_A})^2 \frac{m_\pi}{2}. \tag{3}$$

In Eq. 3, the renormalization factor $Z_A$ is needed to relate the local axial current to the conserved current in the domain wall fermion formulation. $Z_A$ can be measured by studying the large $t$ behavior of the quantity $Z_A(t)$, defined as the ratio of the coupling between the conserved current and the pion to the coupling between the local current and the pion. Figure 4 shows that nice plateaus for $Z_A(t)$ can be found over the ranges $4 \leq t \leq 14$ and $18 \leq t \leq 28$ for both $\beta = 6.0$, $L_s = 16$ with $16^3 \times 32$ volume and $\beta = 5.7$, $L_s = 48$ with $8^3 \times 32$ volume. The $L_s$ dependence of $Z_A$ at $\beta = 6.0$ is presented in Fig. 5. For $L_s$ ranging from 12 to 48, the change in $Z_A$ is less than 1%. Therefore, the effect of finite $L_s$ on the result for $Z_A$ is negligible.

Another method to calculate $f_\pi$ uses the relation between $f_\pi$ and the amplitude of the pseudoscalar correlator $A_{PP}$:

$$A_{PP} = - (\frac{f_\pi}{m_f + m_{\text{res}}})^2 \frac{m_\pi^2}{8}, \tag{4}$$

which directly depends upon the residual mass $m_{\text{res}}$.

Results for $\beta = 5.7$, $L_s = 48$ with lattice volume $8^3 \times 32$ show a discrepancy in the $m_f$ dependence of $f_\pi$ between these two methods. While the difference appears to become somewhat smaller at the larger volume of $16^3 \times 32$, we attribute the discrepancy between these two methods of calculating $f_\pi$ to $O(a^2)$ errors.

Figure 6 shows the comparison of these two independent calculations at the weaker coupling $\beta = 6.0$. Consistent results are seen at each valence quark mass, and the $f_\pi$ values measured by
extrapolation to $m_f = -m_{\text{res}}$ agree well with the physical value of 130 MeV, where $m_\rho$ is used to set the scale. It is important to note that these good properties disappear if the value for $m_{\text{res}}$ in Eq. 4 is replaced by 0 or by the $x$ intercept of the simple linear fit to $m_\pi^2$. Therefore, the good agreement seen between the two approaches serves as a consistency check of the residual mass analysis and a demonstration of the good chiral properties of the domain wall fermion formulation.

4. CONCLUSIONS

The chiral symmetry breaking effects in the domain wall fermion formulation can be characterized by a residual mass, which enters the effective quark mass in the low energy effective Lagrangian. This quantity can be determined accurately from the additional term in the divergence of axial current. Good chiral properties can be achieved for the quenched domain wall formulation for lattice spacing $a^{-1} \sim 1 - 2$ GeV. The results for $m_{\text{res}}$ are further checked by our determinations of the pion decay constant.

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