Large Extra Dimensions, Sterile neutrinos and Solar Neutrino Data

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Abstract

A desirable feature of models with large extra dimensions is that they lead to naturally light sterile neutrinos in the process of giving small mass to the known neutrinos. We show that extending the brane physics to make the neutrinos of Majorana type can not only accommodate all known indications of neutrino oscillations (e.g. solar, atmospheric and LSND), but also provide a novel way to understand the solar neutrino data by a combination of vacuum and MSW oscillations involving the bulk sterile neutrinos.

I. INTRODUCTION

The four-neutrino scheme in which the solar $\nu_e$ deficit is explained by the $\nu_e \rightarrow \nu_s$ (where $\nu_s$ is a sterile neutrino not having the weak interactions), the atmospheric $\nu_\mu/\nu_e$ anomaly is attributed to $\nu_\mu \rightarrow \nu_\tau$, and the heavier $\nu_\mu$ and $\nu_\tau$ share the role of hot dark matter was originally proposed [1] in order to explain those three phenomena. Later the LSND experiment [2], which observed $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, provided a measure of the mass difference between the nearly degenerate $\nu_e - \nu_s$ and $\nu_\mu - \nu_\tau$ pairs and required the three mass differences that were already present in that neutrino scheme. Exactly this same pattern of neutrino masses and mixings appears necessary to allow production of heavy elements ($A \sim 100$) by type II supernovae [3]. While qualitatively this neutrino scheme seems to explain all existing neutrino phenomena [4], solar neutrino observations are now sufficiently constraining that the small-angle MSW $\nu_e \rightarrow \nu_s$ explanation appears to be in some difficulty, and seemingly

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one must go to some length [5] in order to try to rescue it. Although providing better fits to the solar data, even active-active transitions in a three-neutrino scheme do not provide a quantitatively good explanation of those data [6]. In this letter, we point out that there appears to be a way to achieve a good fit and rescue the apparently needed four-neutrino scheme if large extra dimensions exist. This is motivated by the latest developments in string theories, which have made plausible the interesting possibility that such large extra space dimensions (of order of millimeters) [7] can exist in conjunction with a low fundamental string scale, $M_s \approx \text{few TeVs}$. This is in contrast with earlier perturbative string models where $M_s$ was much closer to the Planck scale, and sizes of all extra dimensions were miniscule.

The low $M_s$, large extra dimension models present an alternative to the conventional high scale SUSY GUT theories in solving the gauge hierarchy problem, and also for the first time lead to observable collider and other low energy signals of string theories. They have therefore evoked a great deal of attention in recent days. A major challenge to these theories, however, comes from the neutrino sector, since there are no large scales here that can help in the implementation of the conventional seesaw mechanism [8] for small neutrino masses. A way out of this difficulty is to include singlet neutrinos in the bulk, in combination with the assumption of an effective global $B - L$ symmetry in the theory below the string scale [9]. The large size of the bulk is then responsible for the suppression of the neutrino masses to the desired level. Furthermore, the low Kaluza-Klein (“KK”) excitations have a mass $\sim R^{-1} \sim 10^{-3}$ eV, which not only provide a natural way to understand the lightness of the sterile neutrino [10], but also provide extra sterile neutrino states in the mass range that can influence the shape of the solar neutrino spectrum. In this letter, we show that, indeed, the effect of the low KK states is such that they provide a novel way to fit the solar neutrino spectrum by a combination of vacuum and small-angle MSW solutions to the solar neutrino data. Thus should the small-angle MSW resolution of the solar neutrino data be definitively ruled out and the LSND observations be confirmed, the mechanism proposed in the paper could not only keep the sterile neutrino solution viable, but also may provide an indirect signal for large extra dimensions.

II. BASIC IDEAS OF THE MODEL AND THE MIXING OF BULK MODES

The model we will present consists of the standard model in the brane and one bulk neutrino. The bulk may be five, six or higher dimensional; we will assume that only one of those extra dimensions is large. The bulk neutrino will be assumed to couple to $\nu_e^1$. Let us begin our discussion by focusing on the $\nu_e$ in the brane and one bulk neutrino. Obviously, the fields that could propagate in the extra dimensions are chosen to be gauge singlets. Let us denote bulk neutrino by $\nu_B(x^\mu, y)$. It has a five-dimensional kinetic energy term and a coupling to the lepton field $L$. The effective 4-dimensional Lagrangian for this system can

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1In general, the bulk neutrino will couple to all three species of brane neutrinos; however, when the $\nu_{\mu,\tau}$ masses are much bigger than the mass parameters in the $\nu_e$ and $\nu_B$ sector, one can decouple the $\nu_{\mu,\tau}$ and restrict oneself to only the $\nu_e, \nu_B$ sector to analyze the effect of the bulk neutrino, as we do in the paper.
be written as:

\[ \mathcal{L} = \kappa \bar{L} H \nu_{BR}(x, y = 0) + \int dy \ \bar{\nu}_{BL}(x, y) \partial_5 \nu_{BR}(x, y) + h.c., \]  

(1)

where from the five-dimensional kinetic energy, we have only kept the 5th component that contributes to the mass terms of the KK modes in the brane; \( H \) denotes the Higgs doublet, and \( \kappa = \frac{h M^*}{M_{Pl}} \) the suppressed Yukawa coupling. It is worth pointing out that this suppression is independent of the number and radius hierarchy of the extra dimensions, provided that \( \nu_B \) propagates in the whole bulk. For simplicity, we will assume that there is only one extra dimension with radius of compactification as large as a millimeter, and the rest with much smaller compactification radii. The smaller dimensions will only contribute to the relationship between the Planck and the string scale, but its KK excitations will be very heavy and decouple from the neutrino spectrum. Thus, all the analysis could be done as in five dimensions.

To address the issue of naturalness of the sterile neutrino mass, let us note that the direct Dirac or Majorana mass terms for the bulk neutrino can be forbidden by an appropriate choice of geometry and dimension of the bulk in which the \( \nu_B \) resides. For instance, in 5 dimensions the \( Z_2 \) orbifold symmetry under which \( y \rightarrow -y \) combined with \( B - L \) symmetry guarantee this, and in 6-dimensions choosing \( \nu_B \) as a four-component chiral spinor is sufficient for the purpose.

The second point we wish to note is that we will include new physics in the brane that will generate a Majorana mass matrix for the three standard model neutrinos as follows:

\[ \mathcal{M} = \begin{pmatrix} \delta_{ee} & \delta_{e\mu} & \delta_{e\tau} \\ \delta_{e\mu} & \delta_{\mu\mu} & m_0 \\ \delta_{e\tau} & m_0 & \delta_{\tau\tau} \end{pmatrix}. \]  

(2)

The origin of this pattern of brane neutrino masses will be discussed later in this paper. We assume that \( m_0 \gg \delta_{ij} \), and as a result the \( \nu_{\mu,\tau} \) in effect decouple from the \( \nu_{e,s} \) and do not affect the mixing between the bulk neutrino modes and the \( \nu_e \). If we choose \( m_0 \gg \delta_{\mu\tau} \), then we have maximal mixing in the \( \mu - \tau \) sector as needed to understand the atmospheric neutrino data. Furthermore, if we choose \( m_0 \sim eV \), then this provides an explanation of the LSND observations. We will therefore not discuss the \( \nu_\mu \) and \( \nu_\tau \) anymore but rather concentrate in the rest of the Letter on the solar neutrino data.

Concerning the effect of the bulk modes, note that the first term in Eq. (1) will be responsible for the neutrino mass once the Higgs field develops its vacuum expectation value. The induced Dirac mass parameter will be given by \( m = \kappa v \), for which \( M^* = 1 \) TeV is about \( h \cdot 10^{-5} \) eV. Obviously this value depends only linearly on the fundamental scale. Larger values for \( M^* \) will increase \( m \) proportionately. After introducing the expansion of the bulk field in terms of the KK modes, the Dirac mass terms in (1) could be written as

\[ (\nu_e \nu_{0B} \nu_{B,-} \nu_{B,+}) \begin{pmatrix} \delta_{ee} & m \sqrt{2m} & 0 \\ m & 0 & 0 \\ \sqrt{2m} & 0 & 0 \\ 0 & 0 & \partial_5 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_{0B} \\ \nu_{B,-} \\ \nu_{B,+} \end{pmatrix}, \]  

(3)

where our notation is as follows: \( \nu_B \) represents the KK excitations, the off-diagonal term \( \sqrt{2m} \) is actually an infinite row vector of the form \( \sqrt{2m}(1,1,\cdots) \). The operator \( \partial_5 \) stands for the diagonal and infinite KK mass matrix whose \( n \)-th entry is given by \( n/R \).
Using this short-hand notation makes it easier to calculate the exact eigenvalues and the eigenstates of this mass matrix \[10\]. Simple algebra yields the characteristic equation

\[ m_n = \delta_{ee} + \pi m^2 R \cot(\pi m_n R), \]

where \( m_n \) is the mass eigenvalue \([9,11]\).

The equation for eigenstates is

\[ \tilde{\nu}_n = \frac{1}{N_n} \left[ \nu_e + \frac{m}{m_n} \nu_{0B} + \sqrt{2 \pi m_n} \nu_{B,-} + \sqrt{2 \pi m} \nu_{B,+} \right], \]

where the sum over the KK modes in the last term is implicit. \( N_n \) is the normalization factor given by

\[ N_n^2 = 1 + m^2 \pi^2 R^2 + \left( \frac{m_n - \delta_{ee}}{m} \right)^2. \]

Thus, the weak eigenstate is actually a coherent superposition of an infinite number of massive modes. Therefore, even for this single flavour case, the time evolution of the mass eigenstates involves in principle all mass eigenstates and is very different from the simple oscillatory behaviour familiar from the conventional two or three neutrino case. The survival probability depends strongly on \( m_R \), reflecting the universal coupling of all the KK components of \( \nu_B \) with \( \nu_L \) in (1).

Note that in the limit of \( \delta_{ee} = 0 \), the \( \nu_e \) and \( \nu_{B,0} \) are two, two-component spinors that form a Dirac fermion with mass \( m \). We will be interested in the small \( m_R \) region of the parameter space. In that case, the KK modes come in pairs of mass \( m_n = \pm k_n / R \), with \( k_n \) a positive integer. It is easy to see that the coupling of the \( \nu_e \) to the KK modes is approximately \( mR \). Once we include the effect of \( \delta_{ee} \neq 0 \), they become Majorana fermions with masses given by: \( m_1 \approx +\delta_{ee}/2 + m \) and \( m_2 \approx +\delta_{ee}/2 - m \), and they are maximally mixed; i.e., the two mass eigenstates are \( \nu_{1,2} \approx \nu_e \pm \nu_{B,0} \). Thus as the \( \nu_e \) produced in a weak interaction process evolves, it oscillates to the \( \nu_{B,0} \) state with an oscillation length \( L \approx E / (2 m \delta_{ee}) \). Later on in this paper we will find that for natural values of \( m, \delta_{ee} \) in this model, \( L \) is of order of the Sun-Earth distance so that our model leads to vacuum oscillation (“VO”) of the solar neutrinos.

Furthermore since the \( \nu_e \) also mixes with the KK modes of the bulk neutrinos with a mass difference square of order \( 10^{-5} \, \text{eV}^2 \), this brings in the MSW resonance transition of \( \nu_e \) to \( \nu_{B,KK} \) modes at higher energies. In our discussion of the solar neutrino puzzle, we will take into account both these effects.

### III. FITTING SOLAR NEUTRINO DATA BY A COMBINATION OF VACUUM AND MSW OSCILLATIONS

In order to clarify the role of the KK excitations in the fit to the solar neutrinos, let us briefly recall the general constraints on fits to solar neutrino data using vacuum oscillation. The simplest way to reconcile the rates for the three classes of experiments at different energies is to “kill” the \( \text{Be}^7 \) neutrinos, reduce the \( \text{B}^8 \) neutrinos by half and leave the \( \text{pp} \) neutrinos alone. One might at first think that to achieve this in the VO case, one may put
a node of the survival probability function $P_{ee}$ around 0.86 MeV. However, for an arbitrary node number, the oscillatory behaviour of $P_{ee}$ before and after 0.86 MeV cannot in general satisfy the other two requirements mentioned above. For instance, if one uses the first node to “kill Be$^7$”, then for B$^8$ neutrino energies the $P_{ee}$ is close to one and not half as would be desirable; on the other hand, if one uses one of the higher nodes (higher $\Delta m^2$), then pp neutrinos get reduced. Therefore, the general strategy employed is to not to kill Be$^7$ altogether but reduce it and reduce the B$^8$ neutrinos by more than 50%. The water data then requires an additional contribution. In the case of active VO, this additional contribution is provided by the neutral current cross section which amounts to about 16% of the charged current one. What this implies is that in a pure two-neutrino oscillation picture, VO will not work for active to sterile oscillation.

![Energy dependence of the $\nu_e$ survival probability](image)

FIG. 1. Energy dependence of the $\nu_e$ survival probability when $R = 58 \mu m$, $m = 0.32 \times 10^{-4}$ eV, and $\delta_{ee} = 0.81 \times 10^{-7}$ eV. The distribution of neutrino origins within the Sun depends on the nuclear reaction producing them. The dot-dashed part of the curve assumes the distribution of the pp reaction, the solid part assumes the distribution of $^{15}O$ neutrinos, and the dashed part assumes the distribution of $^8B$ neutrinos.

In our model, we expect both vacuum oscillations and MSW oscillations to be important, since the lowest mass pair of neutrinos are split by a very small mass difference, whereas the KK states have to be separated by at least $10^{-2}$ eV because of the limits from gravity experiments. Therefore, we can use the first node of $P_{ee}$ to suppress the Be$^7$ completely. Going up in energy toward B$^8$ neutrinos, the VO contribution to survival probability, which normally would have risen to very near one, is suppressed by the small-angle MSW transitions to the different KK excitations of the bulk sterile neutrinos. This fact is clear from Fig. 1. Most dips in Fig. 1 are a consequence of the MSW resonances of successive KK states. This provides a new way to fit the solar neutrino data in models with large extra dimensions and is the main observation of our paper. Coming to the details, we note that as we move to higher energies, more and more modes satisfy the resonance condition, but
their contribution to survival probability decreases; however, the net effect is to increase the 
P_{ee}, a feature that is obvious from the figure.

To proceed with the calculation, we call \( \vec{\alpha} \) the state vector of neutrinos in a basis of vacuum eigenstates, with \( U \vec{\alpha} \) the same vector in the basis used in Eq. 3. Then in the presence of matter, with \( \rho_e = \sqrt{2}G_F(n_e - 0.5n_n) \), and \( n_e \) and \( n_n \) the electron and neutron densities, \( \vec{\alpha} \) evolves according to

\[
i \frac{d}{dt} \vec{\alpha} = \rho_e(t) \left( \sqrt{2} \vec{\alpha} \right)
\]

for \( c_r(t) = e^{-itm^2_e/2E U^*_1r} \).

The equation for the survival probability can be written as:

\[
|a_e|^2 \approx \sum_{k=1}^{\infty} |U_{1k}\alpha_k(t)|^2 + 2Re \left( \sum_{j=1}^{\infty} U^*_{12j-1}U_{12j}\alpha_{2j-1}(t)\alpha_{2j}(t)e^{i\frac{m^2_{2j-1} - m^2_{2j}}{2E} t_d} \right).
\]

The eigenvectors and eigenvalues can be found in the presence of matter, replacing \( M^2 \) with \( M^2 + H \), where \( M \) is the mass matrix of Eq. 3, \( H = 2E\rho_e \) when acting on \( \nu_e \), and \( H \) is zero on sterile neutrinos. Define

\[
w = \frac{E\rho_e}{m\delta_{ee}} + \sqrt{1 + \left( \frac{E\rho_e}{m\delta_{ee}} \right)^2}; \tag{9}
\]

\( w = 1 \) in vacuum. The characteristic Eq. 4 becomes

\[
m_n = w\delta_{ee} + \pi m^2 R \cot(\pi m_n R), \tag{10}
\]

the eigenvectors are found to be

\[
\tilde{\nu}_n = \frac{1}{N_n} \left[ \nu_e + \frac{m}{wm_n} \nu_B - \frac{\sqrt{2}mm_n}{w(m_n^2 - \partial_5^2)} \nu'_B,+ + \frac{\sqrt{2}m\delta_5}{m_n^2 - \partial_5^2} \nu'_B,- \right], \tag{11}
\]

and \( N_n \) is given by

\[
N_n^2 = 1 + \frac{1 + \frac{1}{w^2}}{2} \left( \pi^2 m^2 R^2 + \frac{(m_n - w\delta_{ee})^2}{m^2} \right) - \frac{1 - \frac{1}{w^2}}{2} \frac{(m_n - w\delta_{ee})}{m} \tag{12}
\]

This model was tested using a program that evolved from one supplied by W. Haxton [12] in 1987. The program was updated to use the solar model of BP98 [13] and modified to do all neutrino transport within the sun numerically. For example, no adiabatic approximation was used. It was also necessary to generalize beyond the two-neutrino model. Up to 20 neutrinos were allowed, but no more than 12 contribute for the solutions we considered. Finally, there were changes to account for the fact that oscillation is into sterile neutrinos.

For comparison with experimental results, tables of detector sensitivity for the Chlorine and Gallium experiments were taken from Bahcall’s web site [13].

The Super-Kamiokande detector sensitivity was modeled as follows. For an electron neutrino of energy \( E_\nu \), the differential cross section was given by ’t Hooft’s [14]. His result
was adjusted to include those radiative corrections that can be accounted for by modifications of $g_A$ and $g_V$ as described by the Particle Data Group [15].

The response of Super-K to electrons is given in Table 6 of the NIM paper describing their calibration [17]. That table gives the percent resolution in the signal from Cherenkov light, averaged over the detector for various total electron energies. To within the number of digits accuracy given in the table, the fractional resolution is $(0.443 + 0.0038 E)/\sqrt{E}$, where $E$ is the total electron energy (kinetic energy plus rest mass). In the approximation that electrons of the relevant energies lose energy at a rate proportional to $1/\beta^2$, the amount of Cherenkov light is proportional to $(E - E_{th})^2/E$, where $E_{th} \approx 0.755 \text{ MeV}$ is the threshold for Cherenkov light production in water, with index of refraction $n = 1.33$. Knowledge of the fractional resolution in signal, combined with knowledge of the relation between amount of Cherenkov light and the true electron energy, gives an energy resolution of

$$
\sigma_E = \frac{E(E - E_{th})(0.443 + 0.0038 E)}{(E + E_{th})\sqrt{E}}.
$$

(13)

The response of Super-K to neutrinos of energy $E_\nu$ is given by smearing 't Hooft's electron energy distribution with a Gaussian resolution function of standard deviation $\sigma_E$, with the integration restricted to measured energy within an energy bin, or within the range of the total flux measurement (5.5-20 MeV).

Calculations of electron neutrino survival probability, averaged over the response of detectors, were compared with measurements.

The Chlorine result, from Homestake [18], is $2.56 \pm 0.16(\text{stat}) \pm 0.16(\text{syst}) = 2.56 \pm 0.23(\text{tot})$ SNU. This result, compared with $7.7_{-1.0}^{+1.1}$ SNU from BP98 [13], gives a $\nu_e$ survival probability of $0.332 \pm 0.030$, neglecting the theoretical uncertainty, but including experimental systematic errors. With the BP98 uncertainty folded in, the survival probability becomes $0.332 \pm 0.056$.

Gallium results were given at Neutrino 2000 [19] for SAGE by Y. Gavrin and for GALLEX and GNO by E. Bellotti. Combining statistical and systematic errors, and combining the three experiments, gives a detection rate of $74.7_{-5.0}^{+5.1}$ SNU. This is to be compared with $129_{-6}^{+8}$ SNU from BP98. Neglecting the theoretical uncertainty of BP98, the Gallium $\nu_e$ survival probability is $0.579 \pm 0.039$. With the theoretical uncertainty of BP98 this would be $0.579 \pm 0.050$.

From Suzuki’s talk at Neutrino 2000 [19], the $5.5 - 20 \text{ MeV}$ 1117 day Super-K experimental result was $\text{Data}/BP98 = 0.465 \pm 0.015$ with systematic and statistical errors added in quadrature, and with no theoretical uncertainty from BP98. The BP98 theoretical uncertainty in the $^8\text{B}$ flux is $^{+19}_{-14}$%, which is much larger than the uncertainty in the experimental measurement.

The best fits were with $R \approx 58 \mu\text{m}$, $mR$ around 0.0094, and $\delta_{ee}$ corresponding to $\delta m^2 \sim 0.52 \times 10^{-11} \text{ eV}^2$. These parameters give average survival probabilities for Chlorine, Gallium, and water of 0.382, 0.533, 0.454, respectively, and give a $\nu_e$ survival probability whose energy dependence is shown in Fig. 1. For theories with two-neutrino oscillations, the coupling between $\nu_e$ and the higher mass neutrino eigenstate is often characterized by \(\sin^2 2\theta\). For our case, the coupling between $\nu_e$ and the first K excitation would have $\sin^2 2\theta$ replaced by $4/(mR)^2 = 0.00035$.

Vacuum oscillations between the lowest two mass eigenstates nearly eliminate electron neutrinos with energies of .63MeV/(2n+1) for $n = 1, 2, \ldots$. This accounts for Fig. 1
showing nearly zero $\nu_e$ survival near 0.63 MeV, which helps eliminate the $^7$Be contribution at 0.862 MeV, and for the dip at the lowest neutrino energy. Table 1 of the BP98 paper [13] gives predictions for rates in Chlorine and Gallium. Since the Chlorine measurement is sensitive almost entirely to the $^7$Be and $^8$B contributions, and Super-Kamiokande, which is sensitive only to $^8$B, measures higher $\nu_e$ survival probabilities than Homestake, suppression of the $^7$Be contribution is needed to make Homestake and Super-Kamiokande compatible. But $^7$Be is also a large contributor to the Gallium measurement, whose largest contribution from the pp spectrum between 0.24 MeV and 0.42 MeV gets some suppression from the dip at low energies. These effects make the Gallium prediction low compared with experiment, increasing $\delta_{ee}$ moves the low energy dip to the right, which exacerbates the Gallium problem by moving the lowest energy dip into Gallium’s most sensitive pp energy range. Decreasing $\delta_{ee}$ helps Gallium by moving the lowest energy dip to the left, but hurts the Chlorine fit by moving the higher energy vacuum oscillation dip further to the left of the $^7$Be peak.

MSW resonances start causing the third and fourth eigenstates to be significantly occupied above $\sim 0.95$ MeV, the fifth and sixth eigenstates above $\sim 3.9$ MeV, and the 7’th and 8’th above $\sim 8.7$ MeV. Fig. 1 shows dips in survival probability just above these energy thresholds. The typical values of the survival probability within the $^8$B region ($\sim 6$ to $\sim 14$ MeV) are quite sensitive to the value of $mR$. As can be seen from Eq. 6, higher $mR$ increases $1/N \approx m/m_n \approx mR/n$ for various $n$, and thereby increases $\nu_e$ coupling to higher mass eigenstates, strengthens MSW resonances, and lowers $\nu_e$ survival probability.

After folding in the Super-Kamiokande detection sensitivity, the expected energy dependence of the $\nu_e$ survival probability compared with preliminary data from a plot shown by Suzuki at Neutrino 2000 [19] is shown in Fig. 2. The uncertainties are statistical only. The parameters used in making Fig. 2 were chosen to provide a good fit to the total rates only; they were not adjusted to fit the shape of this spectrum.

[FIG. 2. Super-Kamiokande energy spectrum: measured (error bars) and predicted (curve) for the same parameters as in Fig. 1, which do not involve a fit to this spectrum. Data are preliminary results shown at Neutrino 2000 by Y. Suzuki, based on 1117 days.]
The seasonal effect was computed for a few points on the earth’s orbit. If \( r \) is the distance between the earth and the sun,
\[
\frac{r_0}{r} = 1 + \epsilon \cos(\theta - \theta_0),
\]
(14)
where \( r_0 \) is one astronomical unit, \( \epsilon = 0.0167 \) is the orbital eccentricity, and \( \theta - \theta_0 \approx 2\pi(t-t_0) \), with \( t \) in years and \( t_0 = January \ 2, \ 4h \ 52m \). Table I shows very small seasonal variation, in accord with observations.

**TABLE I.** Predicted seasonal variations in \( \nu_e \) fluxes, excluding the \( 1/r^2 \) variation. The model assumed the same parameters as were used for Fig. 1.

<table>
<thead>
<tr>
<th>( \theta - \theta_0 ) (eqn 14)</th>
<th>Chlorine</th>
<th>Gallium</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (January 2)</td>
<td>0.3848</td>
<td>0.5356</td>
<td>0.4538</td>
</tr>
<tr>
<td>( \pm \pi/2 )</td>
<td>0.3824</td>
<td>0.5333</td>
<td>0.4536</td>
</tr>
<tr>
<td>( \pi ) (July 4)</td>
<td>0.3800</td>
<td>0.5294</td>
<td>0.4534</td>
</tr>
</tbody>
</table>

**IV. NEW PHYSICS IN THE BRANE AND THE NEUTRINO MASS MATRIX**

There appears to be a rather simple symmetry reason that can explain the neutrino mass pattern in the brane. Consider an \( L_e + L_\mu - L_\tau \) symmetry [20]. We assign \( L_e = 1 \) to the bulk neutrino. The neutrino mass pattern allowed in the basis \( (\nu_e, \nu_\mu, \nu_\tau) \) corresponds to the mass matrix in Eq. (2) with all \( \delta \)’s (except \( \delta_{e\tau} \)) set to zero. The various \( \delta \) entries will arise after we turn on the terms in the Lagrangian that break this symmetry. The model we will choose for this purpose has the new nonstandard model fields: singlet charged Higgs fields \( \eta^+, h^{++} \) which are blind with respect to lepton number. We further include \( SU(2)_L \) triplet fields, \( \Delta_{e,\mu,\tau} \), with \( Y = 2 \) which carry two negative units of lepton numbers \( L_{e,\mu,\tau} \), respectively. We then write the Lagrangian involving these fields consisting of two parts: one \( \mathcal{L}_0 \) which is invariant under \( (L_e + L_\mu - L_\tau) \) number and another \( \mathcal{L}_1 \) that breaks it:

\[
\mathcal{L}_0 = \eta(f_{e\tau}L_\mu L_\tau + f_{\tau\tau}L_e L_\tau) + h^{++}(f'_{e\tau}e_R^\tau R^\tau + f'_{\mu\tau}\mu_R^\tau R^\tau) + \lambda_e L_e L_e \Delta_e + \lambda_\mu L_\mu L_\mu \Delta_\mu + \lambda_\tau L_\tau L_\tau + M h^{++} \eta^2 + h.c.
\]
(15)
and

\[
\mathcal{L}_1 = h^{++}(\sum_{i=e,\mu,\tau} M_{ii} \Delta_i^2 + M_{0\mu} \Delta_\mu \Delta_\tau + M_{0\tau} \Delta_e \Delta_\tau) + h.c.
\]
(16)

With these couplings, the neutrino Majorana masses arise from two-loop effects (similar to the mechanism of ref. [21]), and we have the \( L_e + L_\mu - L_\tau \) violating entries \( \delta_{ij} \sim c \ m^2_{ij} \). This leads to the prediction that if \( \delta_{ee} \sim 10^{-8} \text{ eV} \) then \( \delta_{\tau\tau} \sim 10^{-2} \), as would be required to understand the atmospheric neutrino data.

Finally we wish to comment that models with bulk sterile neutrinos lead to new contributions to supernova energy loss, as well as to the energy density in the early universe,
which has important implications for the evolution of the universe. Currently these issues are under discussion [22,23], and if these models [24] are preferred by neutrino oscillation data, possible questions that can arise from cosmological considerations must be addressed. Note, however, that this model is less sensitive to these issues than other fits because for VO $\Delta m^2$ is an order of magnitude smaller, for MSW $\sin^2 2\theta$ is more than an order of magnitude smaller, and there is only a single KK tower starting from a very small mass [22].

In conclusion, we have found a new fit to the solar neutrino data in models with large extra dimensions using a combination of vacuum and MSW transitions between the $\nu_e$ and a sterile neutrino and its KK excitations. If the LSND results are confirmed and the small-angle MSW $\nu_e - \nu_x$ mechanism is ruled out by the energy distribution in water experiments, then one will have to invoke this mechanism to understand all neutrino data. The idea may be tested earlier, however, since the extra dimension size needed is 0.06 mm, which is within the range of some gravity experiments, and it is likely that the better energy resolution of the SNO detector can detect predicted dips in the spectrum. On the other hand, the small value found for the neutrino mass of $3 \times 10^{-5}$ eV is undetectable even by any conceivable double beta decay experiment, and the seasonal effect (shown in Table I) will be difficult to observe.

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