Remark on the Electron Self-Energy Calculation

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Abstract

A quantum-electrodynamical description of the self-energy in which the interaction is propagated with the velocity of light instead of instantaneously gives rise to a modification of the photon propagator identical with the convergence factor. We discuss a physical meaning of the convergence procedure used in the electron self-energy calculation.

In 1934, W. Pauli asked to calculate the self-energy of an electron according to the positron theory of Dirac. It was a modern repetition of an old problem of electrodynamics. In 1939, V. F. Weisskopf [1] showed that the corresponding calculation in the positron theory resulted in a logarithmic divergence of the electron self-energy. In 1949, R. P. Feynman [2] put forward an intelligible method of using diagrams in attacking this problem. It provides an intuitive way of arriving at the correct result with the aid of a cut-off procedure. But the cut-off procedure is not unique and is adopted only to define the mathematics [3]. Its physical implication remains basically unsatisfactory. I should like to point out a problem in the Feynman diagram and its connection with the convergence factor used in the electron self-energy calculation.

Looking at the Feynman diagram for the self-energy, one can see that it describes a static interaction phenomenon. The diagram represents the self-interaction due to the emission and reabsorption of a virtual transverse photon. Tacitly it assumes the instantaneous interaction of the electron with the Coulomb field created by the electron itself. We know, however, in classical electrodynamics that the Coulomb potential does not act instantaneously, but is delayed by the time of propagation. The description of the self-interaction as the reaction back on the electron of its own radiation fields is a classical notion in that it means a complete neglect of the velocity of propagation. The Feynman graph illustrating the self-energy is unnatural. To be a physical description, it exhibits a causal behavior associated with the self-interaction.

What causal effect can be assumed in the Feynman diagram without violating the established thoughts? Perhaps, the vacuum polarization effect is an illustration. The simplest and most natural assumption will be to incorporate an electron-positron pair as existing part of time into the propagation of a virtual photon.

If the photon virtually disintegrates into an electron-positron pair for a certain fraction of the time, the electron loop gives an additional $e^2$ correction to the photon
propagator through which an electron interacts with itself. The modification of the photon propagator is then the replacement

\[
\frac{1}{q^2} \rightarrow \frac{1}{q^2} \left[-\Pi_{\mu\nu}\right] \frac{1}{q^2},
\]

where the \(i\epsilon\) prescription is implicit. The polarization tensor \(\Pi_{\mu\nu}\) is written as the sum of a constant term \(\Pi(0)\) for \(q^2 = 0\) and terms proportional to \((q_\mu q_\nu - \delta_{\mu\nu} q^2)\). The leading term \(\Pi(0)\) is a positive, real constant that depends quadratically on the cut-off \(\Lambda\). In the limit \(q^2 \rightarrow 0\), the \(q^2\) term is absorbed into the renormalization constant. At least formally, we see that (1) reads approximately

\[
\frac{1}{q^2} \rightarrow \frac{1}{q^2} - \frac{1}{q^2 - \Pi(0)},
\]

where we have used the operator relation \((A - B)^{-1} = A^{-1} + A^{-1}BA^{-1} + \cdots\).

We recall that the convergence factor introduced by Feynman is

\[
\frac{1}{q^2} \rightarrow \frac{1}{q^2 \left(q^2 - \Lambda^2\right)}, \quad \text{that is,} \quad \frac{1}{q^2} - \frac{1}{q^2 - \Lambda^2},
\]

where the \(i\epsilon\) prescription is again implicit. It becomes evident that the vacuum polarization effect gives rise to a modification of the photon propagator corresponding to the convergence factor considered in connection with the divergent self-energy integral. The formal agreement draws our attention to the causal effect which has been assumed as necessity. Indeed, there is more.

Weisskopf’s calculation is based upon the positron theory, in which the presence of an electron causes a static polarization in the distribution of the vacuum electrons. In his calculation, Weisskopf shows a physical significance of the static polarization induced in the vacuum due to the presence of an electron itself. However, Feynman’s diagram is not a physical description of self-energy according to the positron theory. It is no more than a quantum-theoretical translation of the classical notion of self-energy. To be consistent with the positron theory, the self-energy diagram must be modified to include the effect of the static polarizability of the vacuum electrons. We have already assumed the vacuum polarization effect as a causal effect. It is at once evident that the assumption brings the diagram into accord with the positron theory. The necessity of the vacuum polarization effect may now be regarded as justified, in which the cut-off \(\Lambda^2\) finds a physical interpretation in terms of \(\Pi(0)\). As Feynman explains, the convergence procedure (3) can be looked upon as the result of superposition of the effects of quanta of various masses. However, it should be noted that the minus sign in front of the term for the propagation of a photon of mass \(\Lambda\) has not been explained so far from this point of view.

There have been many arguments that explain why the quadratically divergent constant \(\Pi(0)\) must be discarded [4]. They essentially say that the formalism must be
set up in such a way that the observed photon mass is strictly zero. Even though any “honest” calculation gives $\Pi(0) \neq 0$, the way we compute the vacuum polarization is consistent with assigning a null value to $\Pi(0)$ which leads to a nonvanishing photon mass. But when viewed from the present point, one may say that the convergence procedure (3) amounts to the use of $\Pi(0)$ in the propagation of a virtual photon. Whenever the photon propagator is supplied with the convergence factor, it amounts to taking account of the closed loop contribution to the photon propagator.

The following conclusion must be drawn from the qualitative argument of the positron theory: The self-energy of an electron is a result of the static polarization induced in the vacuum due to the presence of the electron. The cut-offs appearing in the mass and charge corrections must therefore be equivalent. Although the respective corrections are cut-off dependent, their ratio would contain no dependence upon the cut-off.

References


Note added in proof: Likewise for the closed electron loop, Feynman suggests taking the difference of the result for electrons of mass $m$ and mass $(m^2 + \lambda^2)^{1/2}$. If the closed loop, integrated over some finite range of $p$, is called $-\Pi_{\mu\nu}(m^2)$ and the loop over the same range of $p$, but with $m$ replaced by $(m^2 + \lambda^2)^{1/2}$ is $-\Pi_{\mu\nu}(m^2 + \lambda^2)$, the modification

$$\Pi_{\mu\nu} \rightarrow \int_0^\Lambda [\Pi_{\mu\nu}(m^2) - \Pi_{\mu\nu}(m^2 + \lambda^2)] d\lambda$$

is then characteristic for the problem of polarization of the vacuum even though such a procedure has no meaning in terms of physically realizable particles. In a certain sense the closed electron loop is an artificial device for computational purpose of the vacuum polarization effect. In form and content the convergence procedure corresponds to the calculation in terms of a virtual electron-positron pair of a modification $q^{-2} \rightarrow q^{-2} \Pi_{\mu\nu} q^{-2}$. Again, we can note a connection between the convergence procedure and a causal behavior associated the induced current with an external field.