Descent Relation of Tachyon Condensation
from Boundary String Field Theory

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Abstract

We analyze how lower-dimensional bosonic D-branes further decay, using the boundary string field theory. Especially we find that the effective tachyon potential of the lower-dimensional D-brane has the same profile as that of D25-brane.

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1 Introduction

Since the discovery of D-brane [1], our understanding of string theory has been deepened. Among other things it is now possible to discuss the open string tachyon [2, 3, 4, 5]. It was conjectured that bosonic D-brane of any dimension can decay into the open string vacuum or lower-dimensional D-brane. Moreover the vacuum energy of the bosonic D-brane is considered to correspond to the tension of the D-brane.

Old days calculation [6] in the open string field theory [7] has been renewed to discuss the tachyon condensation [8, 9, 10, 11, 12, 13, 14, 15]. Since in these cases all scalar quantities may acquire the vacuum expectation value, we can only analyze the tachyon condensation by truncating the infinite levels of string excitations. Some attempts for the exact manipulation are found in [16, 17, 18, 19].

However, recently this difficulty is overcome [20, 21, 22]. Using another formulation called boundary string field theory (BSFT) [23, 24, 25, 26, 27] we have only to consider the tachyon field in discussing the tachyon condensation. This is because the general property of the renormalization group flow ensures that the quadratic modes of the tachyon field decouple from the other modes. Exact analysis was performed in this formulation. Both the situations that D25-brane decays into the open string vacuum and that D25-brane decays into lower-dimensional brane are analyzed, and Sen’s conjectures are confirmed exactly. As a result, the derivative truncated tachyon potential is found to agree with the toy model proposed in [28, 29]. Several related works are also found in [30, 31, 32, 33, 34, 35].

In this paper we use this formulation to discuss the descent relation of tachyon condensation. We analyze, after D25-brane decays into lower-dimensional brane, how it further decays. Especially we find that the effective tachyon potential of lower-dimensional branes has the same profiles as that of D25-brane.

In the next section, we review some results of BSFT for later necessity. After the review we proceed in Sec. 3 to analyze the descent relation using BSFT. Since the analysis is unusual from the field theoretical viewpoint, we restate the results using the field theoretical analysis in Sec. 4. We conclude in the final section.

2 Boundary String Field Theory

In this section, we shall review some results of BSFT [23, 24, 25, 26, 27] which is necessary for later analysis. The BSFT is constructed by identifying several contents in the Batalin-Vilkovisky (BV) formalism with those in string field theory. Hence we shall first shortly recall the BV formalism. The BV formalism is defined as follows. Assume that a supermanifold $\mathcal{M}$
with local coordinates \( \lambda^i \) and fermionic closed 2-form \( \omega_{ij} \) is given. If the action \( S \) satisfies the master equation \( \{ S, S \} = 0 \) with the antibracket defined as \( \{ A, B \} = A \partial_i \omega^{ij} \partial_j B \), the action \( S \) will necessarily have gauge symmetry. The existence of the action \( S \) is equivalent to the existence of a vector field \( V^i \) satisfying the nilpotency condition \( V^2 = 0 \) and the local integrability condition \( d(i_V \omega) = 0 \). The two quantities \( S \) and \( V^i \) are related via \( i_V \omega = dS \).

To construct BSFT we have to identify the contents in the BV formalism with those in string field theory. We first identify the supermanifold \( \mathcal{M} \) as the space of open string fields \( \{ \mathcal{O}_i \} \) and the fermionic 2-form \( \omega_{ij} \) as the two point function of the string fields \( \omega_{ij} = \langle \mathcal{O}_i \mathcal{O}_j \rangle \). Here the string field \( \mathcal{O} \) has ghost number one and is related to the boundary perturbation \( S_{\partial \Sigma} = \int d\theta \mathcal{V} \) by the relation \( \mathcal{O} = c \mathcal{V} \). Besides if the vector \( V^i \) is identified as the BRST current vector, the derivative of the action is given as

\[
d S = \frac{1}{2} \int d\theta d\theta' \langle d(c \mathcal{V})(\theta) \{ Q_{\text{BRST}}, c \mathcal{V} \}(\theta') \rangle_{\mathcal{V}}. \tag{2.1}
\]

It is possible to integrate this derivative as

\[
S(\lambda) = Z(\lambda) \left( 1 + \beta^i(\lambda) \frac{\partial}{\partial \lambda^i} \log Z(\lambda) \right), \tag{2.2}
\]

with some \( \beta(\lambda) \) functions and the partition function \( Z(\lambda) \) given as

\[
Z(\lambda) = \int \mathcal{D}X \exp \left( -S_{\Sigma}(X) - S_{\partial \Sigma}(X, \lambda) \right). \tag{2.3}
\]

In the special case of the tachyon field \( T \) in the quadratic profile,

\[
T = a + \sum_i \frac{u_i}{2\alpha'} X_i^2, \tag{2.4}
\]

the BSFT action is given as

\[
S(a, u_i) = e^{-a} \prod_i Z_1(u_i) \left( 1 + a + \sum_i u_i - \sum_i u_i \frac{\partial}{\partial u_i} \log Z_1(u_i) \right), \tag{2.5}
\]

where \( Z_1(u) \) is defined as

\[
Z_1(u) \equiv \sqrt{ue^u} \Gamma(u). \tag{2.6}
\]

If we expand this action with respect to the coordinate \( u \) up to the next to leading order,

\[
S(a, u_i) = \left( 14 + a + \sum_i u_i + O(u^2) \right) e^{-a} \prod_i \frac{1}{\sqrt{u_i}} \tag{2.7}
\]

we can rewrite the action in the field theoretical form:

\[
S = T_{25} \int d^{26}x \left( \alpha' e^{-T} (\tilde{\partial}_a T)^2 + e^{-T}(T + 1) \right). \tag{2.8}
\]
\[ S = 4e \cdot T_{25} \int d^{26}x \left( \alpha'(\partial_x \phi)^2 - \frac{1}{4} \phi^2 \log \phi^2 \right), \quad (2.9) \]

with the D25-brane tension \( T_{25} \) defined as \( 1/(2\pi \alpha')^{13} \) in this case. (See [21, 22].) Here we have normalized the action in the usual way via the field redefinition \( \log \phi^2 = -(T + 1) \). The coordinates \( x^i \) of the target space correspond to the zero modes of the worldsheet fields \( X^i \).

The exact BSFT action (2.5) is analyzed carefully in [21]. If we set all \( u_i \) zero except only one direction \( u_1 = u \), we can depict the profile of the action as in Fig. 1. Here three stationary points are found and they correspond to the perturbative vacuum of D25-brane \((a = 0, u = 0)\), D24-brane \((a = \infty, u = \infty)\) and the open string vacuum where open strings condense \((a = 0, u = \infty)\), respectively. When D25-brane decays into D24-brane we have to take both \( a \) and \( u \) to infinity under the relation

\[ a = -u + u \frac{\partial}{\partial u} \log Z_1(u), \quad (2.10) \]

which is the stationary condition of \( a \).

Figure 1: The profile of the BSFT action \( S(a, u) \). The D24-brane is expressed as \( a, u \to \infty \) along the ridge (2.10).

### 3 Descent Relation from Boundary String Field Theory

Here we shall analyze how lower-dimensional D-brane further decays. For simplicity we shall study the effective tachyon potential and further decays of D24-brane. Therefore we keep only
two directions $T = a + (uX^2 + vY^2)/2\alpha'$ for consideration: one ($u_1 = u, X_1 = X$) is used when D25-brane decays into D24-brane and the other ($u_2 = v, X_2 = Y$) is kept to discuss further decays. Hence the previous action $S(a, u_i)$ is rewritten as $S(a, u, v)$. The generalizations to other decays are straightforward.

First note that after D25-brane decays into D24-brane both the coordinates $a$ and $u$ go to infinity. However, if we would like to discuss further decay, it is necessary to identify the coordinate that plays the role of the tachyon zero mode of D24-brane, just like the coordinate $a$ in the case of D25-brane. There must be some choices. But as we shall discuss later, it should be natural to consider the section of a large constant $u = u_*$ and use again the coordinate $a$ for the tachyon zero mode.

More precisely, we shall evaluate $S(a_*, a, u_*, v)$ in the limit $a_*, u_* \to \infty$. The double limit is taken under the relation (2.10) for $(a_*, u_*)$ since we consider the decay of D24-brane here. The calculation is straightforward if we use the asymptotic forms around $u \sim \infty$:

$$
\log Z_1(u) \sim u \log u - u + \gamma u + \log \sqrt{2\pi},
\frac{d}{du} \log Z_1(u) \sim u \log u + \gamma u.
$$

(3.1)

After a little algebra we find the following result:

$$
\lim_{a_*, u_* \to \infty} S(a_*, a, u_*, v) = \sqrt{2\pi} e^{-a} Z_1(v) \left( 1 + a + v - v \frac{d}{dv} \log Z_1(v) \right).
$$

(3.2)

Amazingly, this has exactly the same form as the tachyon potential of D25-brane except the factor $\sqrt{2\pi}$. Hence, here we have found heuristically that the profile of the tachyon potential of D24-brane is the same as that of D25-brane. This result makes it possible to discuss further decays of D24-brane in the same way as that of D25-brane.

We shall make a few comments here.

First our result here seems natural. D-brane of any dimension can decay into lower-dimensional D-brane or the open string vacuum. Hence it is natural to expect that D-brane of any dimension should all have the same structure in the tachyon mode.

Secondly, as we have mentioned before our analysis, there are some choices to identify the tachyon zero mode of D24-brane. For example, though here we have fixed the coordinate $u$ to be a large constant $u_*$, we can alternatively fix the coordinate $a = a_*$ and try to regard the coordinate $u$ as the tachyon zero mode. This time we have to evaluate the action $S(a_*, u_* - u, v)$ in the limit $a_*, u_* \to \infty$. In this case it is not the coordinate $u$ but

$$
a_{\text{eff}} \equiv u_* \log u_* - (u_* - u) \log(u_* - u) + (\gamma - 1)u
$$

(3.3)

that plays the role of the tachyon zero mode. However, we shall discuss in the next section that it is more natural to identify the coordinate $a$ as the tachyon zero mode as in (3.2).
4 Field Theoretical Analysis

In the previous section we have analyzed how the lower-dimensional D-brane further decays using the exact BSFT action. We have found that the profile of the tachyon potential is unchanged under the descent relation. However, what we have done is somewhat unusual from the field theoretical viewpoint. In the present section we would like to complement the previous analysis using the field theoretical action (2.9) \[28, 29\] obtained by truncating the higher derivatives. In this method we can also obtain the same results. Besides, it would shed some light on our previous question: what coordinate plays the role of the tachyon zero mode of the lower-dimensional D-brane.

The derivative truncated action (2.9) is written as
\[
S = 4e \cdot T_{25} \int d^{25}y dx \left( \alpha'(\partial_\vec{y} \phi)^2 + \alpha'(\partial_x \phi)^2 - \frac{1}{4} \phi^2 \log \phi^2 \right).
\] (4.1)

As in the previous section here we also consider the effective action of the tachyon on D24-brane for simplicity. The coordinate $x$ denotes the transverse direction of the D24-brane and $\vec{y}$ the longitudinal direction. The D24-brane is expressed as a classical lump solution $\bar{\phi}(x)$. To see the fluctuation on the D24-brane we have to expand the tachyon field into infinite modes around this solution as
\[
\phi(x, \vec{y}) = \bar{\phi}(x) + \tilde{\phi}(x, \vec{y}),
\]
\[
\tilde{\phi}(x, \vec{y}) = \sum_n \xi_n(\vec{y}) \psi_n(x).
\] (4.2)

If we want to obtain the effective action for the lowest mode of the D24-brane, we have to put the expansion (4.2) with the classical solution $\bar{\phi}(x)$ and the fluctuating modes $\tilde{\phi}(x, \vec{y})$ into the original derivative truncated action (4.1) and integrate over the coordinate $x$. As shown in \[29\], when we calculate the effective action for the lowest mode, the effect of the higher excited modes does not come in. It was also found that both the profiles of the lump solution $\bar{\phi}(x)$ and the lowest mode of D24-brane $\psi_0(x)$ are the gaussian function:
\[
\bar{\phi}(x) = \psi_0(x) = \exp\left(-\frac{1}{8\alpha' x^2}\right).
\] (4.3)

After a straightforward calculation we find the derivative truncated action (4.1) becomes
\[
S = 4e \cdot T_{25} \sqrt{4\pi \alpha'} \int d^{25}y \left( \alpha' \left( \partial_\vec{y} \left(1 + \xi_0(\vec{y})\right) \right)^2 - \frac{1}{4} \left(1 + \xi_0(\vec{y})\right)^2 \log \frac{(1 + \xi_0(\vec{y})^2)}{e} \right).
\] (4.4)

If we rewrite the lowest mode $\xi_0(\vec{y})$ into a new field $\phi_{eff}(\vec{y})$ as
\[
1 + \xi_0(\vec{y}) \equiv \sqrt{e} \phi_{eff}(\vec{y})
\] (4.5)
we find that the derivative truncated action is unchanged except that the dimension of the field theory is lowered by one and the tension becomes $T_{25}2\pi \sqrt{\alpha'} \cdot \left( e / \sqrt{\pi} \right)$ which is $T_{24}$ in the interpretation of [29, 21].

Here we have repeated the analysis of the effective action of D24-brane using the field theoretical method. We have found that the result is perfectly consistent with our expectation and the result of the previous section.

Let us return back to the question raised in the previous section: why we have identified the coordinate $a$ as the tachyon zero mode. To answer this question from the field theoretical analysis, first let us see how the manipulation in this section relates to the manipulation in the previous section.

In the analysis of BSFT in the previous section, we have first assumed the quadratic profile for the tachyon field (2.4) and obtained directly the integrated expression (2.5). After obtaining the BSFT action we have restricted ourselves in the section of a large constant $u = u_*$ and considered $S(a_* + a, u_*, v)$.

On the other hand, in obtaining the effective tachyon action using the field theoretical analysis in the present section, we have rewritten the field $\phi(x, \vec{y})$ as $\exp(-x^2/8\alpha') \times (1+\xi_0(\vec{y}))$ and integrated over the $x$ direction. In other words, since both the classical lump solution $\bar{\phi}(x)$ and the lowest fluctuating mode $\psi_0(x)$ have the same quadratic exponent with fixed coefficient $1/8\alpha'$, it is possible to factorize the field $\phi$ as the product of the classical solution $\bar{\phi}$ in the direction $x$ and the fluctuating mode $\phi_{\text{eff}}$ in the direction $\vec{y}$ as

$$\phi = \bar{\phi}[a_*, u_*, 0] \times \sqrt{e^{\phi_{\text{eff}}}[a, 0, v]}.$$  \hspace{1cm} (4.6)

Here we have defined the notation $\phi[a, u, v]$ (and their cousins $\bar{\phi}$ and $\phi_{\text{eff}}$) as

$$\phi[a, u, v] \equiv \exp\left\{-\frac{1}{2}\left(a + \frac{u}{2\alpha'} x^2 + \frac{v}{2\alpha'} y^2 + 1\right)\right\},$$  \hspace{1cm} (4.7)

using the relation $\log \phi^2 = -(T+1)$ and set $a_* = -1$ and $u_* = 1/2$ for the derivative truncated action (4.1). Note that the right-hand-side of (4.6) is equal to $\phi[a_* + a, u_*, v]$. Hence, what we have done in this section is exactly to consider the action $S(a_* + a, u_*, v)$, which corresponds to see the integrated form (2.7) in the section of $u_* = 1/2$ and shift the origin of $a$ by $a_* = -1$. Here the coordinate $a$ corresponds to the tachyon zero mode on D24-brane.

However, the exact BSFT action (2.5) is written not as the field theory but as the integrated form. Hence, though we know that the classical solution should be quadratic, it is difficult in principle to discuss the fluctuating modes around the classical solution and see whether the higher fluctuating modes decouple. Moreover, if we want to relate the two manipulations, the classical lump solution $\bar{\phi}(x)$ and the lowest fluctuating mode $\psi_0(x)$ should have the same gaussian profiles. Here even in the exact case, we have optimistically assumed these properties and analyzed $S(a_* + a, u_*, v)$ as in the derivative truncated case.
5 Conclusion

In this paper we have analyzed the descent relation of the tachyon condensation. Especially we have found that the profile of the tachyon potential of lower-dimensional D-brane is the same as that of D25-brane. Namely, the tachyon potential has kind of self-similarity.

In analyzing the tachyon mode of the lower-dimensional D-brane using BSFT, we have fixed the coordinate \( u = u_\ast \) and identified the coordinate \( a \) as the tachyon zero mode. We have also shown that this identification is natural from the field theoretical viewpoint.

When we discuss the situation that D25-brane decays into the open string vacuum, we set all \( u_i \) zero and consider the coordinate \( a \) becomes infinity. However, if we discuss the decay of D24-brane into the open string vacuum, first we have to consider \( a \to \infty \) with a large constant \( u = u_\ast \) and send \( u_\ast \to \infty \) afterwards. Hence, our open string vacuum in this case seems to be obtained in the limit \( a, u \to \infty \) and different from the original open string vacuum \( (a = \infty, u = 0) \). This is a very delicate problem that relates to how we take the double limit. To clarify the vacuum structure at \( a = \infty \) should be an interesting direction.

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