Dynamical Interpretation
of Chemical Freeze-Out in Heavy Ion Collisions

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Abstract

It is demonstrated that there exists a direct correlation between chemical freeze-out point and the softest point of the equation of state where the pressure divided by the energy density, $p(\varepsilon)/\varepsilon$, has a minimum. A dynamical model is given as an example where the passage of the softest point coincides with the condition for chemical freeze-out, namely an average energy per hadron $\approx 1$ GeV. The sensitivity of the result to the equation of state used is discussed.
Over the last few years the question of chemical equilibrium in heavy ion collisions has attracted much attention [1, 2, 3, 4]. Assuming thermal and chemical equilibrium within a statistical model, it has now been shown that it is indeed possible to describe the hadronic abundances produced at beam energies ranging from 1 to 200 AGeV. The observation was made that the parameters of the chemical freeze-out curve obtained at CERN/SPS, BNL/AGS and GSI/SIS all lie on a unique freeze-out curve in the temperature $T$–chemical potential $\mu_B$ plane [5]. Recently, a surprisingly simple interpretation of this curve has been proposed: the hadronic composition at the final state is determined solely by the average energy per hadron, $\langle E_{\text{had}} \rangle / \langle N_{\text{had}} \rangle$, of approximately 1 GeV in the rest frame of the system under consideration [6, 7]. Using this finding $\langle E_{\text{had}} \rangle / \langle N_{\text{had}} \rangle \approx 1$ GeV as a heuristic definition for the chemical freeze-out point, in this letter we study physical conditions which are realized around this point in evolution of the system. We show that the chemical freeze-out point is intimately related to the softest point of equation of state defined by the minimum of the pressure-to-energy density ratio, $p(\varepsilon)/\varepsilon$, as a function of $\varepsilon$ [8].

Our considerations are essentially based on the recently proposed Mixed Phase (MP) model [9, 10] which is consistent with the available QCD lattice data [11]. The underlying assumption of the MP model is that unbound quarks and gluons may coexist with hadrons forming an homogeneous quark/gluon–hadron phase [9, 10]. Since the mean distance between hadrons and quarks/gluons in this mixed phase may be of the same order as that between hadrons, their interaction with unbound quarks/gluons plays an important role defining the order of the phase transition. Recently the importance of quark/gluon–hadron interactions was also discussed in the context of dilepton production [12].

Within the MP model [9, 10] the effective Hamiltonian is written in the quasiparticle approximation with the density-dependent mean–field interaction. Under quite general requirements of confinement for color charges, the mean–field potential of quarks and gluons is approximated by the following form:

$$U_q(\rho) = U_g(\rho) = \frac{A}{\rho^\gamma}$$

with the total density of quarks and gluons

$$\rho = \rho_q + \rho_g + \sum_j n_j \rho_j$$

where $\rho_q$ and $\rho_g$ are the densities of unbound quarks and gluons outside hadrons, while $\rho_j$ is the density and $n_j$ is the number of valence quarks inside the hadron of type $j$. The
presence of the total density $\rho$ in (1) describes the interaction between all components of the mixed phase. The approximation (1) recovers two important limiting cases of the QCD interaction. Namely, if $\rho \to 0$ the interaction potential goes to infinity, *i.e.* an infinite energy is required to create an isolated quark or gluon which ensures the confinement of color objects. In the other extreme of high energy density corresponding to $\rho \to \infty$, we obtain the asymptotic freedom regime.

The use of the density-dependent potential (1) for quarks and the hadronic potential described by a modified non-linear mean–field model [13] requires certain constraints to satisfy thermodynamic consistency [9, 10]. For the chosen form of the Hamiltonian these conditions show that $U_g(\rho)$ and $U_q(\rho)$ should be independent of the temperature. From these requirements one also obtains an expression for the quark–hadron potential [9]. In the case when the quark-gluon component is neglected, the MP model is reduced to the interacting hadron gas model with a non-linear mean–field interaction [13].

A detailed study of the pure gluonic medium with $SU(3)$ color symmetry and a first order phase transition allows to fix the values of $\gamma = 0.62$ and $A^{1/(3\gamma+1)} = 250 \text{ MeV}$. These values are then generalized to the $SU(3)$ system with dynamical quarks. For two light flavors at zero baryon density, $n_B = 0$, the MP model is consistent with the results from lattice QCD [11] with the deconfinement temperature $T_{\text{dec}} = 153 \text{ MeV}$ and the crossover type of phase transition. The model is then proposed to be extended to baryon-rich systems in a parameter–free way [9].

A particular consequence of the MP model is that for $n_B = 0$ the softest point [8] of the equation of state is located at comparatively low values of the energy density: $\varepsilon_{SP} \approx 0.45 \text{ GeV}/\text{fm}^3$ which is in a good agreement with recent lattice estimates [11]. This value of $\varepsilon_{SP}$ is close to the energy density inside a nucleon. Thus, reaching this value signals that we are dealing with a single 'big' hadron consisting of deconfined matter. For baryonic matter the softest point is gradually washed out at $n_B \gtrsim 0.4 \ n_0$. As shown in [9, 10], this thermodynamic behavior differs drastically from both the interacting hadron gas model which has no softest point and the two–phase approach, based on the bag model, having by construction a first order phase transition and the softest point at $\varepsilon_{SP} > 1 \text{ GeV}/\text{fm}^3$ independent of $n_B$ [8]. These differences should manifest themselves in the expansion dynamics.

In Fig.1 we show the evolution trajectories for central $Au + Au$ collisions in the $T - \mu_B$ plane together with the freeze-out parameters obtained from hadronic abundances. Using
the quark-gluon string model [15], the initial state was estimated as a state of hot and dense nuclear matter inside a cylinder in the center-of-mass frame with radius \( R = 4 \, \text{fm} \) and length \( L = 2R/\gamma_{\text{c.m.}} \). The calculated temporal behavior of energy and baryon densities is close to that in RQMD or UrQMD transport models and the procedure of defining the initial state of a fireball is described in [9, 10]. The subsequent isentropic expansion of this fireball is treated within the Frankfurt expansion model [16] based on a scaled hydrodynamical prescription assuming \( t \sim V \). As seen in Fig.1, the turning points of these trajectories, \( i.e. \) the points where \( \partial T/\partial \mu_B \) changes the sign, are in a good agreement with the extracted chemical freeze-out parameters and clearly correlate with the smooth curve for the ideal gas model with \( \langle E_{\text{had}} \rangle / \langle N_{\text{had}} \rangle = 1 \, \text{GeV} \) [6] defined above as a condition of chemical freeze-out. Existence of the turning point is related to two limiting equilibrium regimes for temperature behavior of the chemical potential: ultrarelativistic regime dominated by light particles (quarks, pions) when \( \mu_B \sim T^2 \) and non-relativistic one defined mainly by nucleons and deltas with \( \mu_B \sim T^{-1} \).

Footnote: Other approach have been recently applied in [14] where conditions for thermodynamic equilibrium in heavy ion collisions were studied within the transport UrQMD model and the calculation results at a fixed time moment were approximated by the ideal hadron gas equation of state to extract the time dependence of \( T \) and \( \mu_B \). It is of interest that our results for dynamical trajectories below the turning point practically coincide with that from [14] for equilibrium part of \( T - \mu_B \) trajectory. However, at higher temperature, where according to [14] the system is in a non-equilibrium state, their results differ from ours and exhibit no turning point.

. Uncertainty in the initial \( \varepsilon \) and \( n_B \) in 10-20% shifts slightly the dynamical trajectory but the turning point nevertheless stays on the \( \langle E_{\text{had}} \rangle / \langle N_{\text{had}} \rangle = 1 \, \text{GeV} \) curve. Notice that there is a finite region in \( T \) where \( \partial \mu_B/\partial T \approx 0 \), \( i.e. \) the chemical potential in the expanding system is kept constant around this turning point.

The observed correlation is further elucidated in Fig.2. The quantity \( p/\varepsilon \) is closely related to the square of the velocity of sound and characterizes the expansion speed.

Footnote: In simplified hydrodynamic models was shown that, for example, the transverse expansion of a cylindrical source is governed by the pressure-to-enthalpy ratio, \( p/(p + \)
by $p/\varepsilon$ rather than by the sound velocity. 

so the system lives for an appreciable amount of time around the softest point which facilitates to reach the chemical equilibrium. This statement is evidenced directly in the right-hand side of Fig.2 where the time evolution of $p/\varepsilon$ is shown: The system spends about 1/3 of the total expansion time in a state near the softest point. During this time the baryon chemical potential $\mu_B$ remains practically constant.

In Fig.2 (left-hand side) the position of the softest point correlates with the average energy per hadron being about 1 GeV for all beam energies except for 2 AGeV case. One should note that the quantity $\varepsilon/\rho_{\text{had}}$, where $\varepsilon$ is the total energy density, coincides with $\langle E_{\text{had}} \rangle / \langle N_{\text{had}} \rangle$ considered in [6] only in the case when there are no unbound quarks/gluons in the system. In the MP model, all components are interacting with each other and therefore the quantity $\langle E_{\text{had}} \rangle$ is not well defined. However the admixture of unbound quarks at the softest point $\varepsilon_{SP}$ amounts to about 13% and 8% at beam energies $E_{\text{lab}} = 150$ and 10 AGeV, correspondingly. Thus thermodynamical quantities and in particular the ratios of hadron abundance in the MP and resonance gas model are very close to each other at the freeze-out point.

The MP equation of state plays a decisive role for the regularity considered here, describing both the order of the phase transition and the deconfinement temperature. The two-phase (bag) model exhibits a first order phase transition with $T_{\text{dec}} = 160$ MeV and has a \textit{spatially separated} Gibbs mixed phase but the corresponding trajectories in the $T - \mu_B$ plane are quite different from those in the MP model as shown in [10]. The exit point from the Gibbs mixed phase at $E_{\text{lab}} = 150$ AGeV is close to the corresponding freeze-out point in Fig.1. However it does not lead to the observed correlation with the softest point position in the whole energy range considered. The interacting hadron gas model has no softest point effect as was demonstrated in [9, 10] but nevertheless it exhibits the turning point in $T - \mu_B$ plane. These facts are seen also from Fig.1 and Fig.2 where for $E_{\text{lab}} = 2$ AGeV quark-hadron interactions are negligible ($\approx 1\%$). Indeed, at this energy instead of a minimum one has a monotonic fall-off specific for hadronic models with only a small irregularity in $p/\varepsilon$ near the point $\varepsilon/\rho_{\text{had}} \sim 1$ GeV.

\textit{Footnote:} Note that at the SIS energies the chemical freeze-out point practically coincides with the thermal freeze-out [19].
At higher energies the dynamical trajectories are very different for hadronic and mixed phase models but the positions of turning points in the $T - \mu_B$ plane turn out to be still rather close to each other and both lying on the curve corresponding to $\varepsilon/\rho_{\text{had}} = 1 \text{ GeV}$. Therefore, one could expect similar results for the ratios of hadronic abundance in these two models. However the model predictions differ essentially as long as evolution of the system is concerned.

It is noteworthy that similarly to the results presented in Fig.2, the softest point of the equation of state correlates with an average energy per quark, $\varepsilon/\rho \approx 350 \text{ MeV}$ which is close to the constituent quark mass. So, at higher values of $\varepsilon/\rho$ we are dealing with a strongly-interacting mixture of highly-excited hadrons and unbound massive quarks/gluons forming (in accordance with Landau’s idea [20]) an ‘amorphous’ fluid suitable for hydrodynamic treatment. Below the softest point the interaction decreases, the relative fraction of unbound quarks/gluons is getting negligible, higher hadronic resonances decay into baryons and light mesons. At this stage the application of hydrodynamics becomes questionable.

In summary, we have shown in this paper an example of an equation of state where there exists a direct correlation between chemical freeze-out point and the softest point of the equation of state. This description correlates with the observation that chemical freeze-out occurs when the average energy per hadron, $\langle E_{\text{had}} \rangle / \langle N_{\text{had}} \rangle$, drops below the value of 1 GeV as found in [6]. In the examples considered such a clear correlation is observed for the equation of state of the mixed phase model, where the energy per hadron seems to be a relevant observable defining chemical freeze-out conditions.

We thank B. Friman, Yu. Ivanov, E. Kolomeitsev and W. Nörenberg for useful discussions. E.G.N., K.R. and V.D.T. gratefully acknowledge the hospitality at the Theory Group of GSI, where some part of this work has been done. J.C. gratefully acknowledges the hospitality of the Physics Department of the University of Bielefeld. This work was supported in part by DFG under the program of scientific-technological collaboration (project 436 RUS 113/558/0) and by Russian Found of Fundamental Investigations (project 00-02-04012). K.R also acknowledges the partial support of the State Committee for Scientific Research (KBN).
References


Figure 1: The compiled chemical freeze-out parameters (from [6, 7]) obtained from the observed hadronic abundances and dynamical trajectories calculated for central Au + Au collisions at different beam energies $E_{lab}$ with the mixed phase equation of state. The smooth dashed curve is calculated in the ideal hadron gas model for $\langle E_{had} \rangle / \langle N_{had} \rangle = 1 \text{ GeV}$ [6].
Figure 2: The ratio of pressure to energy density, \( \frac{p}{\epsilon} \), versus the average energy per hadron, \( \frac{\epsilon}{\rho_{\text{had}}} \), (left-hand side) and its time evolution for different systems (right-hand side). The curve for \( p\bar{p} \) collisions at \( \sqrt{S} = 40 \, \text{GeV} \) is calculated for isentropic expansion of a sphere with \( R = 1 \, \text{fm} \). Other cases are calculated for central \( Au + Au \) collisions at the given beam energy and under the same conditions as in Fig. 1.