Top mixing in effective theories *
F. del Aguila and J. Santiago a

a Departamento de Física Teórica y del Cosmos, Universidad de Granada. E-18071 Granada, Spain

We review how top mixing with light quarks constrains new physics beyond the Standard Model using the effective Lagrangian approach.

1. Introduction

The top quark is expected to be the main probe to new physics beyond the Standard Model (SM) in forthcoming experiments. On one hand its couplings are not measured with high precision. They are known at the tens per cent level [1]. On the other hand, for example, the Large Hadron Collider (LHC) at CERN will be a top factory with more than $10^6$ single and $10^7$ pair produced top quarks per year. This will stand for a precision in the determination of the renormalizable $V_tq$ couplings, $V = W, Z$, of $\sim 1\%$ [1], more than one order of magnitude better than present accuracy. If the new amplitudes scale with the masses involved in the process, the effective top coupling with the charm quark would be $\frac{m_t}{m_c} \cdot \frac{m_c}{m_s} \sim 400$ times larger than the bottom coupling to the strange quark, and then a probe all these times more efficient. Hence it is important to know which new physics can show up at the top and how to discriminate among the different SM extensions.

Our knowledge of the top properties and the precision to be reached in the near future have been revised in detail in Ref. [1] recently. In the following we present the effective Lagrangian description of top mixing and justify why extra vector-like quarks can induce its largest values [2]. We then evaluate the new couplings for the simplest case of an extra quark isosinglet of charge $\frac{2}{3}$, $T_{L,R}$, and comment on the experimental limits.

2. Effective Lagrangian description of top mixing

In order to describe new physics beyond the SM in a model-independent way we must use an effective Lagrangian [3]. Assuming the validity of the SM at the electroweak scale and then that only the SM fields are light, the effects of any SM gauge extension, including the possibility of extra dimensions at the TeV scale, are parametrized by the most general effective Lagrangian involving the SM fields and preserving the SM symmetries. Imposing the almost exact baryon and lepton number conservation such a Lagrangian has been written down up to dimension 6 terms [4],

$$L_{\text{eff}} = L_4 + \frac{1}{\Lambda^2} L_6 + \ldots$$

(1)

The lowest order part $L_4$ containing all renormalizable terms is fixed by the SM. There is no dimension 5 operator allowed by the symmetries; whereas without taking into account flavour indices there are 81 independent dimension 6 operators contributing to $L_6$, 40 of them involving quarks of charge $\frac{2}{3}$.

$L_{\text{eff}}$ should give a good quantitative description of the $1\%$ SM deviations, which is the size of the top mixing $V_tq$ experimentally testable [1]. However this Lagrangian compares with experiment only after electroweak Spontaneous Symmetry Breaking (SSB). Thus $L_6$ gives contributions to dimension 4 operators $O(\frac{v^2}{\Lambda^2})$, with $v$ the elec-
There are also operators of dimension 5 \( t \) roweak vacuum expectation value \( \sim \) and the initial dimension 6 operators \( \mathcal{O}(\frac{1}{\Lambda^2}) \) and the unbroken \( SU(3) \) and the initial dimension 6 operators \( \mathcal{O}(\frac{1}{\Lambda^2}) \). Experimental strategies must be designed to disentangle the different contributions. The obvious way is to start with the simplest vertices. Hence we will concentrate on the corrected gauge couplings of dimension 4 describing the top mixing with light quarks [1]

\[
\mathcal{L}_{4tq}^{Vtq} = -g_4 \bar{t} \gamma^\mu T^a t G_{\mu a} + \frac{2}{3} i \bar{t} \gamma^\mu t A_\mu + \frac{g}{\sqrt{2}} \sum_{q=d,s,b} \bar{t} \gamma^\mu (v_{tq}^W - a_{tq}^W \gamma_5) q W_\mu^+ + \text{h.c.} \\
- \frac{g}{2 \cos \theta_W} \bar{t} \gamma^\mu (v_{tq}^Z - a_{tq}^Z \gamma_5) t Z_\mu \\
- \frac{g}{2 \cos \theta_W} \sum_{q=u,c} \bar{t} \gamma^\mu (v_{tq}^Z - a_{tq}^Z \gamma_5) q Z_\mu + \text{h.c.}
\]

The first two terms are fixed by the unbroken gauge symmetry \( SU(3)_C \times U(1)_Q \). The charged currents are modified \( v_{tq}^W, a_{tq}^W = \frac{1}{\sqrt{2}} (1 + \mathcal{O}(\frac{\Lambda^2}{\Lambda^2})) \), where \( V_{tq} \) is the Cabibbo-Kobayashi-Maskawa (CKM) matrix [5], receiving corrections of order \( \frac{x_t^2}{\Lambda^2} \). The neutral currents are similarly modified \( v_{tq}^Z, a_{tq}^Z = \mathcal{O}(\frac{x_t^2}{\Lambda^2}) \) and \( v_{tq}^Z, a_{tq}^Z = \mathcal{O}(\frac{x_t^2}{\Lambda^2}) \).

These non-standard contributions come from the \( \mathcal{L}_6 \) operators in Table 1. We follow the notation in Ref. [4] where \( q \) stands for left-handed (LH) doublets and \( u \) and \( d \) for the right-handed (RH) singlets, \( \phi \) is the scalar doublet, \( W^I \mu, B_{\mu \nu} \) are the usual \( SU(2)_L \) and \( U(1)_Y \) gauge field strengths and \( D_\mu \) is the covariant derivative. Other \( \mathcal{L}_6 \) operators involving the top quark do not contribute to \( \mathcal{L}_{4tq}^{Vtq} \) at this order. As a matter of fact \( \mathcal{O}_{d\phi} \) does not contribute to this order either. It redefines the down quark mass eigenstates, but this redefinition can be reabsorbed in the CKM matrix. On the other hand the operators \( \mathcal{O}_{WB,\phi,\phi} \) give flavour independent corrections. They redefine the gauge couplings which are known to fulfill the SM relations to an accuracy better than 1% [6]. Hence, in practice we are only interested in \( \mathcal{O}_{\phi q}^{(1,3)}, \mathcal{O}_{u\phi,\phi u,\phi \phi} \).

\[
\begin{align*}
\mathcal{O}_{\phi q}^{(1)} &= (\phi^I I_D \phi)(\bar{q} \gamma^\mu q) \\
\mathcal{O}_{\phi u} &= (\phi^I I_D \phi)(\bar{u} \gamma^\mu u) \\
\mathcal{O}_{\phi W} &= \frac{1}{2} (\phi^I I_D \phi) W_{\mu \nu}^I W_{\mu \nu}^I \\
\mathcal{O}_{\phi B} &= \frac{1}{2} (\phi^I I_D \phi) B_{\mu \nu} B_{\mu \nu} \\
\mathcal{O}_{WB} &= (\phi^I I_D \phi) W_{\mu \nu} L_{\mu \nu} B_{\mu \nu} \\
\mathcal{O}_{\phi}^{(1)} &= (\phi^I I_D \phi)(D_\mu \phi D^\mu \phi) \\
\mathcal{O}_{\phi}^{(3)} &= (\phi^I I_D \phi)(D_\mu \phi D^\mu \phi)
\end{align*}
\]

Table 1

Dimension 6 operators contributing to renormalizable \( Vtq \) couplings after SSB.
Table 2
Top quark flavour changing branching ratios for the SM, the two Higgs (2H) model, supersymmetric models (SUSY) without R and with R parity breaking and the SM extensions with exotic vector-like quarks [1]. The branching ratio is defined as $B = \frac{\Gamma}{\Gamma_{\text{total}}}$.

<table>
<thead>
<tr>
<th></th>
<th>SM</th>
<th>2H</th>
<th>SUSY (R,R)</th>
<th>Exotic quarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(t\rightarrow qZ)$</td>
<td>$\sim 10^{-4}$</td>
<td>$\sim 10^{-3}$</td>
<td>$\sim 10^{-3}, 10^{-4}$</td>
<td>$\sim 10^{-2}$</td>
</tr>
</tbody>
</table>

It is worth before, however, to note that large top mixing can be induced by new vector-like quarks and only by them. In Ref. [7] it was shown that large $L^6$ coefficients mean operators generated by tree level integration of heavy modes and this implies for $L^V_{tq}$ heavy fermions or heavy gauge bosons. As the mixing of the SM gauge bosons is known to be typically small [6], we are left with heavy vector-like quarks as the only possibility [2].

3. A simple example: one extra quark isosinglet $T_{L,R}$

The simplest SM extension with large top mixing results from the addition of only one charge three quark isosinglet $T_{L,R}$ to the SM. In this renormalizable model the mixing of the new vector-like quark with the SM quarks violates the GIM mechanism introducing tree level Flavour Changing Neutral Currents (FCNC). This extension is so simple that it is easier to work in the full theory in this particular case, but it will be instructive to use the effective Lagrangian formalism to guide the analysis of more complicated cases. The complete integration of the general case will be given elsewhere [2].

Let us assume without loss of generality canonical covariant derivative terms, diagonal SM Yukawa couplings $\lambda_i q_i u_i \tilde{q} + h.c.$, mixing Yukawa terms $\lambda^i q_i T_R \tilde{q} + h.c.$ and a heavy mass term $MT_L T_R + h.c.$, where $\lambda_i$ and $M$ are real and $\lambda^i$ are in general complex. Then the integration of $T$ generates only new LH currents, i.e. only $C_{\phi q}^{(1,2)}$ get non-zero contributions. Now $\Lambda = M$. After SSB $C_{\phi q}^{(1,2)}$ give the $L^V_{tq}$ corrections of order $v^2 M^2$ through diagram (a) in Figure 1

$$\mathcal{L}^W_{tq} = -\frac{g}{\sqrt{2}} W^+_{\mu} A_{jk} \bar{u}_L^j \gamma^\mu d_L^k + h.c.,$$

$$\mathcal{L}^Z_{tq} = -\frac{g}{2 \cos \theta_W} Z_\mu (B_{jk} \bar{u}_L^j \gamma^\mu u_L^k - \bar{d}_L^j \gamma^\mu d_L^j - 2 \sin^2 \theta_W J_{EM}^\mu),$$

where

$$A_{jk} = \left( \delta_{jl} + \frac{m^i_j m^i_k}{M^2} \frac{m^2_j}{-\delta_{ij} m^2_j - m^2_j} \right) V_{ik},$$

$$B_{jk} = \delta_{jk} - \frac{m^i_j m^i_k}{M^2},$$

and $m_i = v \sqrt{\frac{\lambda_i}{2}}$ and similarly $m^i_j = v \sqrt{\frac{\lambda^i_j}{2}}$. The star stands for complex conjugation.

![Figure 1](image-url)

Figure 1. $\mathcal{L}_6$ contributions after SSB to the dimension 4 effective Lagrangian $\mathcal{L}^V_{tq}$. 
Figure 2. Quark isosinglet contributions to the dimension 4 effective Lagrangians $\mathcal{L}_4^{Wtq}$, $\mathcal{L}_4^{Ztq}$.

In the full theory the contributions from diagram (a) in Figure 2 correspond to a part of the contribution depicted by diagram (a) in Figure 1. The other part results from diagram (b) in Figure 2. This second diagram gives a $\frac{v^2}{M^2}$ correction to $\bar{u}_L \phi u'_L$, where the arrow in the diagram stands for the derivative. Then using the equations of motion in order to compare with the $\mathcal{L}_u$ contributions in Figure 1, $\phi u'_L$ is replaced by a mass term, which will match with the contribution of diagram (b) in Figure 1, and the remaining gauge terms in the covariant derivative, which will complete the matching with diagram (a) in Figure 1.

When confronted to experiment this simple model allows for a large top mixing, in particular for instance $|B_{tc}| = \frac{m'_t m'_c}{M^2} \leq 0.082$ [8]. This is almost one order of magnitude larger than the expected precision at LHC, which corresponds to the branching ratios in Table 3 [1].

Table 3
Experimental limits expected at LHC for the top quark flavour changing branching ratios.

<table>
<thead>
<tr>
<th>q=u</th>
<th>q=c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Br}(t \rightarrow qZ)$</td>
<td>$10^{-4}$</td>
</tr>
</tbody>
</table>

REFERENCES