Challenges in Hyperon Decays

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Abstract

We give an personal overview of some of the unsolved problems related to hyperon decays. We cover nonleptonic decays, radiative decays and magnetic moments. Some of the theoretical issues are also touched upon.

1 Introduction

While the frontier of high energy physics marches on to higher and higher energies, there are many unsolved puzzles left over in low energy physics. Hyperon decays account for many of them. These puzzles are generally not considered serious enough to deserve major attention. Our inability to calculate accurately with strong interaction is usually the reason invoked to explain why these puzzles remains so and why they may not be directly relevant to our understanding of the fundamental physics.

Nevertheless, one should be reminded that, even if their solutions do not require changes in the fundamental physics, it is still utmost important to understand the dynamics of low energy hadronic physics through the thicket of strong interaction. In addition, one should keep in mind that the surprises can potentially occur that alter our understanding at the fundamental level. One example is the potential new source of CP


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that can be revealed itself in hyperon decays. There are of course many other corners that
the new physics may have already revealed itself in the data, and yet unrecognized by us
through our inability to quantify strong interaction. One example that stands out is the
$\Delta I = 1/2$ rule that had been repeatedly used to motivate new physics in the literature in
the past.

Since hyperon decays is a vast and hard subject, it is beyond my ability to review
most of them in detail [1]. My goal is only to recount some of them with some personal
perspective so as to help motivate more future experiments on this subject.

2 Problems and tools

The various tools or models was invented to understand hadronic dynamics without fac-
ing the strong coupling issues head on. One can start by taking advantage of only the
symmetry property which presumably is respected by the strong dynamics except the
potential spontaneous symmetry breaking by vacuum condensates. One first identifies, or
assumes, the low energy degrees of freedom and their associate symmetry, and proceeds
to write down the most general low energy effective theory describing these degrees of
freedom consistent with the required symmetry. This is the approaches taken by chiral
perturbation theory (ChPT)[2] and QCD sum rule[3]. These low energy effective theories
typically involve infinite numbers of terms, or couplings. To make such theories useful
some expansion parameters and the associated cut-off scheme is necessary to derive pre-
dictions. For example in the simplest ChPT, the octet of Goldstone bosons associated
with chiral symmetry breaking are identified as the low energy degrees of freedom and
using its energy and momentum as expansion parameter an infinite series of Lagrangian
can be written down according to the chiral symmetry. The higher dimensional operators
are suppressed by powers of $p/\Lambda_\chi$ where $\Lambda_\chi$ is the chiral symmetry breaking scale. Similar
expansion series is employed in QCD sum rule approach.

Unfortunately, there is no guarantee that either the expansion is convergent, or, the
degrees of freedom included is sufficient. This is especially the case when the energy
involved in the hadronic process is not limited to very small $p/\Lambda_\chi$ and can sometimes
be quite close to where the high resonance occurs. In such case it is often impossible
to justify the approximation or to account for the data without extra inputs. However, there is so far no reliable principles about how such extra inputs should be invoked or deployed. As a results, the theory has degenerated into models that are mostly invoked to account for particular special subsets of phenomenology. For example, it was well known that the lower order chiral perturbation theory cannot account for the data on hyperon radiative decays [4, 5] even after the heavy baryon chiral perturbation theory formalism is employed [6, 7, 8]. In an attempt to resolve this puzzle, some additional resonances are added in the analysis recently [9]. While the resulting ”model” can account for the data (in fact it can account for the data in more than one way), however it is not clear how much fundamental understanding is actually gained in the process. With these general comments on the theoretical difficulties, we can go over the specific challenges.

The challenges facing the hyperon decays can be summarized in the following categories:
1) Nonleptonic decays;
2) CP violation;
3) Semi-leptonic decays;
4) Radiative decays;
5) Magnetic moments.

I shall leave the topic of CP violation to German Valencia and topic of semi-leptonic decays to Earl C. Swallow in this workshop.

### 3 Nonleptonic Decays

The nonleptonic decays, $B \to B'\pi$, includes:
- $\Lambda \to p\pi^-$; $\Lambda \to n\pi^0$
- $\Sigma^+ \to n\pi^+$; $\Sigma^+ \to p\pi^0$; $\Sigma^- \to n\pi^-$
- $\Xi^0 \to \Lambda\pi^0$; $\Xi^- \to \Lambda\pi^-$

The amplitude can be written as

$$M = \bar{u}_{B'}(p')(A + B\gamma_5)u_B(p)$$

where amplitudes $A$’s are parity violating, S-wave while $B$’s are parity conserving and P-wave.
3.1 \( \Delta I = 1/2 \) Rule

Each amplitude can be further decomposed into two isospin channels. For example, \( A \) amplitude for \( \Lambda \to p\pi^- \) can be decomposed into \( A(\Lambda \to p\pi^-) = \sqrt{2}A^{1/2}_{\Lambda} - A^{3/2}_{\Lambda} \) and similarly for \( B \) amplitudes. Experimentally [10], the ratio of \( A^{3/2}/A^{1/2} \) for each S-wave, \( A \), or P-wave, \( B \), amplitude is found to be small and of the order of that in the nonleptonic kaon decays. For example,

\[
\begin{pmatrix}
S \\
P
\end{pmatrix} =
\begin{pmatrix}
\Lambda \\
\Sigma \\
\Xi
\end{pmatrix}
\begin{pmatrix}
0.014 & 0.006 \\
-0.017 & -0.047 \\
0.034 & 0.023
\end{pmatrix}
\]

The fact that these ratios, as well as the corresponding one in kaon nonleptonic decays are all of the order of 0.02 in absolute value indicates that \( \Delta I = 1/2 \) rule may reflect something general about the strangeness changing nonleptonic decays. However, so far we have not been able to understand this selection rule in general.

For example, in kaon decays, the factor of 20 in \( \Delta I = 1/2 \) enhancement is partially accounted for (by a factor of 3 to 4) by the renormalization group (RG) effect of QCD between the weak scale and the low energy kaon scale. However, numerically this enhancement is still insufficient and this actually leads to many proposal of new physics in the literature in order to account for it.

For hyperon, the understanding improves a little bit. The similar RG enhancement effect is still operating. Pati/Woo and Miura/Minamikawa [11] discovered another potential enhancement effect. They observed that, using Fierz transformation, the \( O^{\Delta=3/2} \) operator that can mediate hyperon decays, can be written in such a way that it is symmetric under the exchange of color indices of either \((u, s)\) pair or \((d, s)\) pair. If the baryon are taken naively as consisting of three valence quarks only, then the anti-symmetry of color wavefunction as well as this color property of the \( \Delta I = 3/2 \) operator imply that only the \( \Delta I = 1/2 \) operator can mediate the transition \( \langle B'|H_W|B \rangle \) where \( H_W \) is the weak Hamiltonian, that is, \( \langle B'|O^{\Delta=3/2}|B \rangle >= 0 \). Of course, \( B \to B' \) matrix elements are not the only mechanism for \( B \to B'\pi \) decays. And, baryon wavefunction is not exactly just three quark only. However, it is a good indication that the \( \Delta I = 1/2 \) amplitudes are dynamically favored. The most unsatisfactory aspect of this explanation is probably
that the same mechanism clearly does not apply to kaons. Therefore it does not provide a general understanding of the strangeness changing nonleptonic decays together. Nevertheless, with such qualitative mechanism at hand, one can conclude that, overall, $\Delta I = 1/2$ rule is probably not as serious a problem theoretically as in the case of kaons.

### 3.2 Nonleptonic enhancement

The data clearly shows the typical non-leptonic branching ratios, say, $\Gamma(\Lambda \rightarrow p\pi^-)$ is about $10^3$ larger than the typical semi-leptonic ones, say, $\Gamma(\Lambda \rightarrow p\bar{e}\nu_e)$. Why? We have not been able to account for that based on the fundamental principle. There were various attempts to resolve this difference using quark model plus some QCD inputs many years ago. However, none can be considered compelling. This problem has not received too much recent attention. Nevertheless, I am wondering whether one can derive some better qualitative understanding by learning from the recent works on related issues in the charm and especially the B systems, such as heavy quark perturbation theory [12] or QCD inspired factorization models [13].

### 3.3 S-Wave/P-Wave problem

In chiral models or current algebra approach, S-wave ($A$) is due to a contact term or, in ChPT language, the lowest order term in chiral Lagrangian. It is typically represented as in Fig. 1a. Note that the same vertex does not contribute to the P-wave amplitudes, $B$. To get P-wave amplitudes, one need to invoke the pole diagrams in Fig. 1b and Fig. 1c. The lowest order weak Lagrangian gives

$$L_W^0 = h_D Tr(\bar{B}\{\lambda, B\}) + h_F Tr(\bar{B}[\lambda, B])$$

where $\lambda = \xi^\dagger \xi$, $\xi^2 = \Sigma$ is the usual Goldstone bosons in matrix form. The $L_W^0$ contributes to S-wave amplitudes, $A$, directly, but not to $B$. The two parameters $h_D$ and $h_F$ can be used to fit the S-wave amplitudes quite well at this order. In particular, $A(\Sigma^+ \rightarrow n\pi^+)$ is found to be zero at this order while experimentally, indeed, it is much smaller than others.
Using the fitted $h_D$ and $h_F$, together with strong interaction parameters, one can calculate the pole diagrams and predict the P-wave amplitudes, $B$. The result turns out to be a poor fit of many of the $B$ amplitudes extracted from the data. Actually, alternatively, one can also chose to use $h_D, h_F$ to fit the P-wave amplitudes, and use them to predict S-wave amplitudes. The result is an equally poor fit. The is the famous S-wave/P-wave problem.

There are a few potential ways out of the puzzle. One can see if the inclusion of higher order effective Lagrangian can rectify the situation. However, there are just too many parameters in the next order Lagrangian and there is no unique, convincing fit to the data. Alternatively, one can adopt a model that include new resonances, such as Spin 1/2 Parity-odd resonances or the Spin 1/2 parity-even Roper resonances, as the intermediate states, $B''$ in the pole diagrams[9]. The result can fit both S- and P-wave amplitudes, however the fit is clearly model dependent.

4 Radiative Decays

The radiative decays are:

$\Sigma^+(uus) \rightarrow p(uud)\gamma$  BF $\sim (1.23 \pm 0.05) \times 10^{-3}$ ; **

$\Sigma^0 \rightarrow n\gamma$  BF too small to see due to EM decay $\Sigma^0 \rightarrow \Lambda\gamma$

$\Xi^0 \rightarrow \Sigma^0\gamma$;  BF $\sim (3.5 \pm 0.4) \times 10^{-3}$ ;

$\Xi^0 \rightarrow \Lambda\gamma$;  BF $\sim (1.06 \pm 0.16) \times 10^{-3}$ ;

$\Lambda \rightarrow n\gamma$;  BF $\sim (1.75 \pm 0.15) \times 10^{-3}$ ;

$\Xi^- \rightarrow \Sigma^-\gamma$;  BF $\sim (1.27 \pm 0.23) \times 10^{-4}$ ; **

$\Sigma^0 \rightarrow \Lambda\gamma$  BF $\sim 100\%$ electromagnetic decay .

(Note that ** are charged modes between U-spin doublets.)
Generally, the amplitudes can be rewritten as

$$A = -\frac{e}{M_B + M_{B'}} i \epsilon^{\mu
u} q^\nu \bar{u}(p') \sigma_{\mu\nu} (C + D \gamma_5) u(p)$$

where $C$ is parity conserving magnetic dipole transition (M1), $A$ is parity violating electric dipole transition (E1). The decay rate is $\Gamma \propto |C|^2 + |D|^2$. The asymmetry is $A_\gamma = \frac{2 \Re \{C^* D\}}{|C|^2 + |D|^2}$, that is, one needs both nonzero $C$ and $D$ amplitudes to get nonzero asymmetry. There is an old theorem[14] by Hara regarding the vanishing of $D$ amplitudes.

### 4.1 Hara’s Theorem

**Hara’s Theorem:** Assuming that the amplitudes are not singular in the $SU(3)_F$ limit, parity violating $D$ amplitudes must vanish for decays between states of a $U$-spin doublet in the $SU(3)_F$ limit. The theorem implies that

$$D(\Sigma^+ \to p\gamma) = D(\Xi^- \to \Sigma^- \gamma) = 0$$

and as a result the asymmetries $A_\gamma(\Sigma^+ \to p\gamma) = A_\gamma(\Xi^- \to \Sigma^- \gamma) = 0$

Accepting the assumption of Hara’s theorem, even when $SU(3)$ breaking is taken into account, asymmetry $A_\gamma$ should remain small. Unfortunately, experimentally $A_\gamma(\Sigma^+ \to p\gamma)$, which is the only one measured to be nonzero, was found to be $-0.76 \pm 0.08$, indeed large and negative. The others, given in Table 1., have much larger error.

The hadronic models did not have a great deal of success in explaining the details of the data. All the models of this type (except those that include vector mesons) preserve Hara’s theorem in their formulations. General analysis which include $SU(3)$ breaking[21] actually predicts a small and positive asymmetry for the $\Sigma^+$ decay while the data is negative and relatively large. Models that assume vector-meson dominance[22] can introduce effects that violate Hara’s theorem due to mixing of the vector meson with the photon. In models using quarks, it was pointed out[23] that the diagrams in which a $W$-boson is exchanged between two constituent quarks can give rise to violation of Hara’s theorem. In addition, models which include vector-meson dominance are in better agreement with the data, though the situation is still not satisfactory. Among hadronic models, the observed negative asymmetry parameter for $\Sigma^+$ decay is best accounted for using QCD sum-rules.
Other approaches can be found in Refs. [24, 25, 26]. A detailed overview on both experimental and theoretical aspects of weak radiative decays of hyperons is given in Refs. [18, 4, 5].

Many papers questioned the assumption of Hara Theorem based on general ground[27, 28], including some recent references[28]. The point is that it is possible that the parity violating amplitude, \( D \), becomes singular when one takes the \( SU(3) \) limit. It reflects that how to treat the \( SU(3) \) breaking can have a crucial influence on the outcome of the analysis. As elaborated in the magnetic moment section later, we believe that more careful analyses are still wanting in this aspect in the literature.

One may try to employ ChPT, that was useful in describing low energies hadronic processes involving only mesons, to tackle hyperon. For application to processes involving baryons, it is most consistently formulated in the heavy-baryon formulation [12], in which the \( SU(3) \) invariant baryon mass, \( \hat{m} \), is removed by a field transformation (see Ref. [6] for detail). In this approach an amplitude for a given process is expanded in external pion four-momenta, \( q \), baryon residual four momenta, \( k \), and the quark mass, \( m_s \). If one neglects the up and down quark mass. One can collectively write down \( q, k, \) and \( m_s \) as \( E \) (and adopt the convention that \( k \) and \( m_s \) are of the same order in the chiral expansion). The perturbation theory is reliable only when \( E \) is smaller than the chiral symmetry breaking scale \( \Lambda_\chi \). In the heavy-baryon formulation there is an additional expansion in \( 1/\hat{m} \). However, all these terms can be effectively absorbed in counterterms of the theory [6, 7, 8].

Weak radiative decays of hyperons have been studied before in the context of ChPT by Jenkins, Luke, Manohar and Savage [26] and Neufeld [25]. Jenkins et al. and Neufeld calculated the amplitude up to the one-loop level. These loop diagrams give contributions to the amplitudes which are at least of order \( O(E^2) \) in the chiral expansion. However, tree-level direct emission diagrams from the next-to-leading order weak Lagrangian, which give contribution of order \( O(E) \) to the amplitudes, were not considered [6, 7]. In a series of paper [6, 7, 8] we consistently calculated the leading-order amplitude of weak radiative decays of hyperons in ChPT. At this order, no loop contribution need to be considered. However, one does need to take into account the higher-order terms in the weak chiral Lagrangian. We showed that it gives rise to violation of Hara’s theorem. As a consequence
the decay rates for the charged decays $\Sigma^+ \to p + \gamma$ and $\Sigma^0 \to \Sigma^- \gamma$ can be accounted for consistently. We showed that, in leading order, ChPT predicts the ratios of the decay amplitudes of all the neutral channels as functions of the baryon masses only. We compared these predictions with the data. Furthermore, the asymmetry parameters still vanish in this leading order calculation. However, this is not necessarily inconsistent with the data in the expansion scheme of ChPT.

In particular, the diagrams contributing to the leading-order amplitude are tree diagrams given in Fig. 2. There are two kinds of diagrams: the direct emission diagrams, Fig. 2a, and the baryon pole-diagrams, Fig. 2b and Fig. 2c. Loop diagrams give rise to contributions of higher order. The pole-diagrams only contribute to the parity-conserving form factor $C$, in accordance with the Lee-Swift theorem [29]. Unfortunately as concluded in Ref.[8], within the context of simplest heavy baryon chiral perturbation theory without inclusion of any additional resonances, the understanding of many of the data in radiative hyperon decays, including the asymmetry, is still beyond reach. However, if one is willing to introduce additional resonances, together with the additional parameters that come with it, it is possible to fit the data without some models[9].

![Fig. 2](image)

Fig.2. Here circle and square represent strong and weak vertex respectively.

## 5 Magnetic Moments

The magnetic moments of the octet baryons were found to obey approximate $SU(3)$ symmetry a long time ago by Coleman and Glashow (CG)[30]. In the $SU(3)$ symmetric limit, the nine observable moments (including the transitional moment between $\Sigma^0$ and $\Lambda$) can be parameterized in terms of two parameters, and as a result obey approximate relationships. The two parameter result of CG can in fact fit the observed magnetic moments up to about the 20% level. However, since at present the moments have been
measured with an accuracy of better than 1% [10], an improved theoretical understanding is clearly desirable.

Many attempts were made trying to improve the numerical predictions of CG by including the $SU(3)$ breaking effects using ChPT [31, 32, 33, 34]. However, many of these efforts resulted in numerical fits worse than the leading order $SU(3)$ invariant one by CG. For example, Caldi and Pagels [31] found that the leading $SU(3)$ breaking corrections, in their scheme for ChPT, appear in the non-analytic forms of $\sqrt{m_s}$ and $m_s \ln m_s$. They showed that the $\sqrt{m_s}$ corrections are in fact at least as large as the $SU(3)$ invariant zeroth order terms, which casts doubt on the applicability of ChPT. Caldi and Pagels suggested that this “failure” of ChPT might be attributed to the large mass of kaon in the loops and the fact that the leading correction is of non-analytic form. Such non-analytic contributions were indeed pointed out earlier by Li and Pagels [37] and others [38], however, the non-analyticity appears only in the $SU(3)$ invariant chiral symmetry breaking mass, not in $SU(3)$ breaking parameters. More recently, similar large corrections to the baryon magnetic moments non-analytic in $m_s$ have been found, by calculating them up to the one-loop level in ChPT [32, 26, 34]. By only using the $\sqrt{m_s}$ terms Jenkins et al. [26] could improve the accuracy of the Coleman-Glashow results from 20% to about 10%. However, this could only be achieved by using in kaon loops a different value of the meson decay constant than in pion loops, with the effect that the magnitude of the kaon loops is artificially reduced. In addition, Krause [32] showed that $m_s \ln m_s$ corrections are just as important, which disagrees with Refs. [26, 34]. Also, Krause further argued that the non-analytic contributions are not a good approximation of the loop integrals at all.

### 5.1 Okubo relation

Shortly after CG, Okubo [39] derived a relation

$$6\mu_\Lambda + \mu_\Sigma - 4\sqrt{3}\mu_{\Lambda\Sigma^0} - 4\mu_n + \mu_{\Sigma^+} - 4\mu_{\Xi^0} = 0,$$

where $\mu_{\Lambda\Sigma^0}$ is the $\Sigma^0 \to \Lambda$ transition moment. This relation can be obtained if one assumes that $SU(3)$ breaking corrections to the moments are linear in the quark mass matrix $\sigma$, defined by $\sigma \equiv \text{diag}(0, 0, m_s)$. The $SU(3)$ breaking terms introduce additional 5 parameters. The resulting 7 parameter prediction can fit the current 9 high precision
Table 1: Present status of decay rates and asymmetry parameters. The numbers are the combined weighted mean from Ref. [4]. Both the decay rate and the asymmetry parameter for $\Sigma^0 \to \Lambda + \gamma$ have not been measured.

<table>
<thead>
<tr>
<th>$B_i \to B_f + \gamma$</th>
<th>$\Gamma [10^{-18} \text{ GeV}]$</th>
<th>$\alpha$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda \to n + \gamma$</td>
<td>$4.07 \pm 0.35$</td>
<td>$-$</td>
<td>[15]</td>
</tr>
<tr>
<td>$\Xi^0 \to \Lambda + \gamma$</td>
<td>$2.4 \pm 0.36$</td>
<td>$0.43 \pm 0.44$</td>
<td>[16]</td>
</tr>
<tr>
<td>$\Xi^0 \to \Sigma^0 + \gamma$</td>
<td>$8.1 \pm 1.0$</td>
<td>$0.20 \pm 0.32$</td>
<td>[17]</td>
</tr>
<tr>
<td>$\Sigma^+ \to p + \gamma$</td>
<td>$10.1 \pm 0.5$</td>
<td>$-0.76 \pm 0.08$</td>
<td>[18, 19]</td>
</tr>
<tr>
<td>$\Xi^- \to \Sigma^- + \gamma$</td>
<td>$0.51 \pm 0.092$</td>
<td>$1.0 \pm 1.3$</td>
<td>[20]</td>
</tr>
</tbody>
</table>

observables to within 1.5% [35].

Within the context of ChPT, there is no unique way of treating the $SU(3)$ breaking yet. However, the nice fit of Okubo relation can be taken as a hint. In Ref.[36], it was shown that, even with in the scheme of ChPT used in Ref.[33], while the one-loop non-analytic contributions of the form $\sqrt{m_s}$ satisfies the Okubo relation, the nonanalytic contributions of the form $m_s \ln m_s$ does not! In Ref.[35], a formulation of ChPT was suggested such that the leading $SU(3)$ breaking correction is indeed linear in $m_s$.

6 Summary

We had shown that there are still many issues that we do not understand associated with hyperon decays based only on fundamental principles. It is possible that some of the difficulties are such that, due to our inability of dealing with strong interaction, we cannot understand them without invoking some phenomenological models. It is also possible that some of the current data that makes it so hard for us to understand may be changed by improved measurement in the future.

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References


