Lepton number violating processes and Majorana neutrinos\textsuperscript{1}

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Abstract

We discuss some generic properties of lepton number violating ($\mathcal{L}$) processes and their relation to different entries of the Majorana neutrino mass matrix. Present and near future experiments searching for these processes, except the neutrinoless double beta decay, are unable to probe light (eV mass region) and heavy (hundred GeV mass region) neutrinos. On the other hand due to the effect of a resonant enhancement, some of $\mathcal{L}$ decays can be very sensitive to the intermediate mass neutrinos with typical masses in hundred MeV region. These neutrinos may appear as admixtures of the three active and an arbitrary number of sterile neutrino species. We analyze the experimental constraints on these massive neutrino states and discuss their possible cosmological and astrophysical implications.

\textsuperscript{1}Talk presented by S.Kovalenko at the International Workshop on Neutrino Physics (NANAPino), JINR, Dubna, RUSSIA, July 19-22, 2000
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1 Introduction

Recent evidences for neutrino oscillations leave nearly no room for doubts that neutrinos are massive particles. After all this point of view is becoming conventional. Solar neutrino deficit, atmospheric neutrino anomaly and results of LSND neutrino oscillation experiment all can be explained in terms of neutrino oscillations implying non-zero neutrino masses and mixings [1]. On the theoretical side [2] there are also many indications in favour of non-zero neutrino masses following from almost all phenomenologically viable models of the physics beyond the standard model (SM). These models typically predict Majorana type neutrino masses suggesting that neutrinos are truly neutral particles.

The neutrino oscillation searches fix both the neutrino mass square difference $\delta m_{ij}^2 = m_i^2 - m_j^2$ and the neutrino mixing angles, leaving the overall mass scale and the CP-phases arbitrary. Since the latter has no effect on neutrino oscillations, the important question of whether neutrinos are Majorana or Dirac particles cannot be answered by these searches.

Majorana masses violate total lepton number conservation by two units $\Delta L = 2$. Thus lepton number violating ($L=$) processes represent a most appropriate tool to address the question of the Majorana nature of neutrinos. A celebrated example of $\mathcal{L}=$-process, most advanced experimentally and theoretically, is the neutrinoless nuclear double beta ($0\nu\beta\beta$) decay (for a review see [3, 4]). The $0\nu\beta\beta$-experiments achieved unprecedented sensitivity to the so called effective Majorana neutrino mass $\langle m_\nu \rangle_{ee} = \sum U_{ei}^2 m_{ei} \leq 0.2\text{eV}$ [5], where $m_{ei}$ and $U_{ei}$ are the neutrino masses and mixing matrix elements. In the presence of only light neutrinos the effective Majorana mass coincides with the entry of the neutrino mass matrix $\langle m_\nu \rangle_{ee} = M_{ee}^{(\nu)}$. Information on the other entries can be inferred from the other $\mathcal{L}=$ processes. Many of them have been studied in the literature in this respect from both theoretical and experimental sides. Among them there are the decay $K^+ \rightarrow \mu^+ \mu^- \pi^-$ [6, 7, 8, 9, 10, 11], the nuclear muon to positron [12] or to antimuon [13] conversion, tri-muonium production in neutrino muon scattering [14], and the process $e^+ p \rightarrow \bar{\nu} l_1^+ l_2^+ X$ relevant for HERA [15], as well as direct production of heavy Majorana neutrinos at various colliders [16]. Unfortunately sensitivities of the current experiments searching for these processes are much less than in case of $0\nu\beta\beta$-decay. The analysis made in the literature [8, 17] leads to the conclusion that if these processes are mediated by Majorana neutrino exchange then, except $0\nu\beta\beta$-decay, they can hardly be observed experimentally. This analysis relies on the current neutrino oscillation data, and on certain assumptions related to the neutrino mass matrix. We will show that, despite the above conclusion being true
for contributions of the neutrino states much lighter or much heavier than the
typical energy of a certain $L$ process, there are still special windows in the neutrino
sector which can be efficiently probed by searching for some of these processes.
In the case of $K^+ \rightarrow \mu^+\mu^+\pi^-$ decay this window lies in the neutrino mass range
$245 \text{ MeV} \leq m_{\nu_{ij}} \leq 389 \text{ MeV}$, where the s-channel neutrino contribution to the
$K^+ \rightarrow \mu^+\mu^+\pi^-$ decay is resonantly enhanced, therefore making this decay very
sensitive to neutrinos in this mass domain. If neutrinos with these masses exist,
then from the present experimental data we can extract stringent limits on their
mixing with $\nu_\mu$.

Recently some phenomenological, cosmological and astrophysical issues of the
intermediate mass neutrinos in the MeV mass region have been addressed [18].
This was stimulated by the attempts of explanation of the KARMEN anomaly
in terms of these massive neutrino states [19]. Although recent data of the KAR-
MEN collaboration [20] have not confirmed this anomaly, the possible existence of
these massive neutrinos remains open, motivated by the idea of sterile neutrinos
$\nu_s$ required for the explanation of all the neutrino oscillation data including the
LSND results. The sterile species $\nu_s$ may mix with the active ones $\nu_{e,\mu,\tau}$ to form
massive states with a priori arbitrary masses. Their existence is the subject of
experimental searches as well as cosmological and astrophysical constraints.

The paper is organized as follows. In section 2 we discuss a model with
sterile neutrinos and possible spectrum of massive neutrino states. Section 3 is
devoted to some general features of constraints on the neutrino mass matrix,
derivable from the $L$ processes. In section 4 we give the theoretical framework
for $K^+ \rightarrow \mu^+\mu^+\pi^-$ decay, and then in section 5 discuss expected rates of this and
other $L$ processes in the light of the present neutrino observations. In section 6
we study the possible contribution of massive neutrinos in the resonant domain
of the $K^+ \rightarrow \mu^+\mu^+\pi^-$ decay and derive the constraints on the mixing of these
neutrinos with $\nu_\mu$. Astrophysical and cosmological implications of these massive
hundred MeV neutrinos are shortly addressed.

2 Majorana neutrino mass matrix and neutrino
counting experiments.

Consider an extension of the SM with the three left-handed weak doublet neutrini-
os $\nu'_{Li} = (\nu'_{Le}, \nu'_{L\mu}, \nu'_{L\tau})$ and $n$ species of the SM singlet right-handed neutrinos
\( \nu'_{Ri} = (\nu'_{R1}, \ldots, \nu'_{Rn}) \). The general mass term for this set of fields can be written as

\[
- \frac{1}{2} \overline{\nu'} M^{(\nu)} \nu' + \text{H.c.} = - \frac{1}{2} (\overline{\nu'}_{L}, \overline{\nu'}_{R}) \left( M_{L} L M_{D}^{T} M_{R} \right) \left( \nu'_{L}, \nu'_{R} \right) + \text{H.c.} =
\]

\[
- \frac{1}{2} \sum_{i=1}^{3+n} m_{\nu i} \overline{\nu'}_{i} \nu_{i} + \text{H.c.} \quad (1)
\]

Here \( M_{L}, M_{R} \) are \( 3 \times 3 \) and \( n \times n \) symmetric Majorana mass matrices, \( M_{D} \) is \( 3 \times n \) Dirac type matrix. Rotating the neutrino mass matrix by the unitary transformation

\[
U^{T} M^{(\nu)} U = \text{Diag}\{m_{\nu i}\} \quad (2)
\]

to the diagonal form we end up with \( n + 3 \) Majorana neutrinos \( \nu_{i} = U_{ki}^{*} \nu'_{k} \) with the masses \( m_{\nu i} \). In special cases there may appear among them pairs with masses degenerate in absolute values. Each of these pairs can be collected into a Dirac neutrino field. This situation corresponds to conservation of certain lepton numbers assigned to these Dirac fields.

The considered generic model must contain at least three observable light neutrinos while the other states may be of arbitrary mass. In particular, they may include hundred MeV neutrinos, which we will consider in section 6. Presence or absence of these neutrino states is a question for experimental searches.

Let us point out that the presence of more than three light neutrinos are not excluded by the neutrino counting LEP experiments measuring the invisible \( Z^{\nu\nu} \) width \( \Gamma_{\text{inv}} \). Actually it counts not the number of light neutrinos but the number of active flavours. To see this let us write down the \( Z^{\nu\nu} \) interaction term as

\[
Z^{\mu} \sum_{\alpha=e,\mu,\tau} \overline{\nu'}_{\alpha} \gamma_{\mu} \nu'_{\alpha} = Z^{\mu} \sum_{\alpha=e,\mu,\tau} \sum_{m,n=1}^{n+3} U_{am} U_{an}^{*} \overline{\nu}_{m} \gamma_{\mu} \nu_{m} \equiv \sum_{m,n=1}^{n+3} \mathcal{P}_{mn} \overline{\nu}_{m} \gamma_{\mu} \nu_{m}, \quad (3)
\]

where the last two expressions are written in the mass eigenstate basis. For the case of only three massive neutrinos one has \( \mathcal{P}_{mn} = \delta_{mn} \) as a consequence of unitarity of \( U_{\alpha m} \). In general \( \mathcal{P}_{mn} \) is not a diagonal matrix and flavor changing neutral currents in the neutrino sector become possible at tree level. However if all the neutrinos are significantly lighter than \( Z \)-boson with the mass \( M_{Z} \) their contribution to the invisible \( Z \)-boson width is

\[
\Gamma_{\text{inv}} = \sum_{m,n=1}^{n+3} |\mathcal{P}_{mn}|^{2} \Gamma_{\nu}^{SM} = \Gamma_{\nu}^{SM} \sum_{\alpha,\beta=e,\mu,\tau} \delta_{\alpha\beta} \delta_{\alpha\beta} = 3 \Gamma_{\nu}^{SM}, \quad (4)
\]
where $\Gamma_{\nu}^{SM}$ is the SM prediction for the partial Z-decay width to one pair of light neutrinos. This chain of equalities follows again from the unitarity of $U_{an}$. Thus, independently of the number of light neutrinos with masses $m_\nu << M_Z/2$ the factor 3 in the last step counts the number of weak doublet neutrinos. This conclusion is changed in the presence of heavy neutrinos $N$ with masses $M_N > M_Z/2$ which do not contribute to $\Gamma_{inv}$. In this case the unitarity condition is no longer valid and the factor 3 is changed to a smaller value.

Having these arguments in mind we introduce in section 6 neutrino states with masses in the hundred MeV region. These states can be composed of sterile and active neutrino flavors as described in the present section.

3 Constraints from $\mathcal{L}$ -processes. General pattern.

Let us examine some generic features of those constraints on the Majorana neutrino mass matrix which can be derived from $\mathcal{L}$ -processes.

Majorana neutrino mass terms in Eq. (1) violate lepton number conservation by two units $\Delta L = 2$ and, thus, can induce $\mathcal{L}$ -processes with $\Delta L = 2n$. W-boson loops can be used to convert $\Delta L = 0$ from the neutrino to the charged lepton sector. Therefore $\mathcal{L}$ -processes offer one of the most straightforward ways to test the Majorana nature of neutrinos and extract information on the neutrino mass matrix $\mathcal{M}^{(\nu)}$.

Note that at energies below the new physics thresholds only $\Delta L = 2n$ can be realized provided that baryon number is conserved $\Delta B = 0$. This is a simple consequence of the Lorentz invariance and the fact that the spinor SM fields are represented only by leptons and quarks. To prevent $\Delta B = 0$ one has to contract the Lorentz indices of the external lepton fields only with each other without involving quark fields. Thus only $\mathcal{L}$ -processes with even number of external leptons, i.e. $\Delta L = 2n$ processes, can proceed at these energies. This means that any $\mathcal{L}$ -process in this energy domain is related to the Majorana neutrino mass receiving contributions from virtual Majorana neutrino exchange. Certainly only $\Delta L = 2$ processes are of practical interest. The Majorana neutrino exchange contribution to the rate of a $\Delta L = 2$ process with two external leptons $l_i l_j$ can
be written schematically as

$$\Gamma_{ij} = c \int_{s_i^-}^{s_i^+} ds \sum_k \left| U_{ik} U_{jk} m_{\nu k} \right|^2 \frac{G(s/m_0^2) + \ldots}{s \pm m_{\nu k}^2}$$

The function in the absolute value brackets originates from the $\mathcal{L}$ Majorana neutrino propagator $\langle 0|T(\nu(x)\nu^T(y))|0 \rangle$. Since the only source of $\mathcal{L}$ in the neutrino sector is given by the Majorana neutrino masses $m_{\nu k}$ the decay rate (5) vanishes when $m_{\nu k} = 0$. The ellipsis in this equation denote terms whose explicit form is irrelevant for the present general discussion. In Eq. (5) $G(z)$ is a smooth positively definite smearing function which depends on the particular process, $s_i^\pm$ are the limits of integration determined by the masses of the external particles involved in the process. The sign $+(-)$ in the denominator corresponds to the t-(s-)channel neutrino exchange. For the s-channel contribution the total neutrino width $\Gamma_{\nu}$ has to be taken into account if masses in the resonant region of s-channel exchange $s_1^- \leq m_{\nu} \leq s_1^+$ are considered. The non-zero neutrino decay widths $\Gamma_{\nu k}$ can be introduced via the substitution $m_{\nu k} \rightarrow m_{\nu k} - (i/2)\Gamma_{\nu k}$.

From the form of $\Gamma_{ij}$ one can infer that as a function on neutrino masses $m_{\nu k}$ it has a maximal value $\Gamma_{ij}^{\text{max}}$ for certain configuration of neutrino masses. This observation leads to the conclusion that the sensitivity $\Delta^{\text{Exp}}$ of a concrete experiment searching for $\mathcal{L}$ must satisfy the condition $\Delta^{\text{Exp}} \leq \Gamma_{ij}^{\text{max}}$ otherwise no information on neutrino contribution is derivable. The experiment having passed this condition provides certain constraints. Assume that neutrino fields can be divided into light $\nu_i$ and heavy $N_i$ states with masses $m_{\nu \ell} << \sqrt{s_i^-}$ and $\sqrt{s_1^+} << M_N$ respectively. Then in this “light-heavy” neutrino scenario the Eq. (5) can be approximately rewritten as

$$\Gamma_{ij} = |\langle m_{\nu} \rangle_{ij}|^2 m_0^{-1} A_{\nu} + \left| \frac{1}{M_N} \right|_{ij}^2 m_0^2 A_N \pm \text{Re} \left[ \langle m_{\nu} \rangle_{ij} \left( \frac{1}{M_N} \right)_{ij} \right] m_0 A_{\nu N}$$

where the dimensionless coefficients $A_i$ can be obtained for a concrete process from the formula such as Eq. (5). In the above equations $m_0 \sim \sqrt{s_1^+}$ is a typical scale of the $\mathcal{L}$ process under consideration.

The average masses in Eq. (6) are determined in the standard way

$$\langle m_{\nu} \rangle_{ij} = \sum_{k=\text{light}} U_{ik} U_{jk} m_{\nu k}, \quad \langle \frac{1}{M_N} \rangle_{ij} = \sum_{k=\text{heavy}} \frac{U_{ik} U_{jk}}{M_{Nk}}$$

6
Summation over light and heavy neutrinos implies masses $m_{\nu k} << \sqrt{s_1}$ and $M_{N k} >> \sqrt{s_1^+}$ respectively.

An experimental constraint on the rate of certain $\mathcal{L}$ process derived from its non-observation at the experimental sensitivity $\Gamma_{\text{Exp}}$ can be translated with the aid of Eq. (6) into the bounds:

$$\Gamma_{ij} \leq \Gamma_{\text{Exp}} \rightarrow \left\{ \begin{array}{l} \langle m_{\nu} \rangle_{ij} \leq \text{Exp}(\nu) \equiv \sqrt{m_0 \Gamma_{\text{Exp}}/A_{\nu}}, \\
\langle M_{S} \rangle_{ij} \leq \text{Exp}(N) \equiv \sqrt{\Gamma_{\text{Exp}}/(m_0^3 A_{\nu})}. \end{array} \right. \quad (8)$$

However this is only possible if the experimental sensitivity satisfies the consistency conditions

$$\text{Exp}(\nu) << \sqrt{s_1} \sim m_0, \text{Exp}(N)^{-1} >> \sqrt{s_1^+} \sim m_0. \quad (9)$$

Otherwise experimental data cannot be translated into the form (8) as it was done, for instance, in Refs. [10, 22, 15]. If the consistency conditions (9) are not satisfied one has to use the initial formula (5).

The following remark is in order. If all the neutrino states are light satisfying $m_{\nu k} << \sqrt{s_1}$ then the following relation takes place

$$\langle m_{\nu} \rangle_{ij} = M_{ij}^{(\nu)} \quad (10)$$

This relation is not true if there are heavy neutrino states with masses not satisfying the condition $m_{\nu} << \sqrt{s_1}$. According to Eq. (7) they do not contribute to $\langle m_{\nu} \rangle_{ij}$ measured in some $\mathcal{L}$ process. Therefore a concrete $\mathcal{L}$ process can give direct information on the entry $M_{ij}^{(\nu)}$ of the Majorana neutrino mass matrix only under the assumption that all the neutrino masses are small compared to a typical scale of this process $m_{\nu k} << m_0 \sim \sqrt{s_1^+}$, or assuming that the heavy states are sterile. From this point of view $\mathcal{L}$ processes with larger typical scales $m_0$ are preferable.

In the subsequent sections we will study concrete $\mathcal{L}$ processes from the viewpoint of their ability to probe neutrino properties. We will consider the conventional neutrino spectrum with three light neutrinos plus a number of kinematically unattainable heavy states as well as a model with additional intermediate mass neutrinos in the hundred MeV domain.

### 4 K-meson neutrinoless double muon decay

Here we shortly outline the theoretical framework for the $K^+ \rightarrow \mu^+\mu^+\pi^-$ decay. Recently this $\mathcal{L}$ process attracted attention [9, 11] as a possible probe of the
neutrino sector complimentary to other known processes.

In the SM extension with Majorana neutrinos there are two lowest order diagrams, shown in Fig. 1, which contribute to the $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay. We concentrate on the s-channel neutrino exchange diagram in Fig. 1(a) which plays a central role in our analysis. The t-channel diagram in Fig. 1(b) requires in general a detailed hadronic structure calculation. In Ref. [7] this diagram was evaluated in the Bethe-Salpeter approach and shown to be an order of magnitude smaller than the diagram in Fig. 1(a), for light and intermediate mass neutrinos.

The contribution from the factorizable s-channel diagram in Fig. 1(a) can be calculated in a straightforward way, without referring to any hadronic structure model. A final result for the $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay rate is given by [11]

$$\Gamma(K^+ \rightarrow \mu^+ \mu^+ \pi^-) = c \int \frac{ds_1}{s_1} \left| \sum_k \frac{U_{\mu k}^2 m_{\nu k}}{s_1 - m_{\nu k}^2} \right|^2 G\left(\frac{s_1}{m_K^2}\right) +$$

$$2 \frac{c}{m_K^2} \text{Re} \left[ \int \frac{ds_1}{s_1} \frac{U_{\mu k}^2 m_{\nu k}}{s_1 - m_{\nu k}^2} \int \frac{ds_2}{s_2} \left( \frac{U_{\mu n}^2 m_{\nu n}}{s_2 - m_{\nu n}^2} \right)^* H\left(\frac{s_1}{m_K^2}, \frac{s_2}{m_K^2}\right) \right].$$

The unitary mixing matrix $U_{ij}$ relates $\nu'_i = U_{ij}\nu_j$ weak $\nu'$ and mass $\nu$ neutrino eigenstates. The numerical constant in Eq. (11) is

$$c = (G_F^4/32)(\pi)^{-3} f_{\pi}^2 f_{\pi}^2 m_K^5 |V_{ud}|^2 |V_{us}|^2,$$

where $f_K = 1.28 f_\pi$, $f_\pi = 0.668$ $m_\pi$ and $m_K = 494$ MeV is the K-meson mass. The functions $G(z)$ and $H(z_1, z_2)$ in Eq. (11) after the phase space integration can be written in an explicit algebraic form

$$G(z) = \frac{\phi(z)}{z^2} \left[ h_{+-}(z) h_{--}(z) - x_\pi^2 h_{+-}(z) \right] \left[ x_\mu^2 + z - (x_\mu^2 - z)^2 \right]$$

$$H(z_1, z_2) = h_{--}(z_1) h_{--}(z_2) + x_\pi^2 r_+(z_1, z_2) - x_\mu^2 t(z_1, z_2, 1) - r_-(z_1, z_2) t(z_1, z_2, x_\mu).$$

Here we defined $x_i = m_i/m_K$ and introduced the functions

$$h_{\pm \pm}(z) = z \pm x_\pi^2 \pm x_\mu^2,$$

$r_\pm(z_1, z_2) = z_1 z_2 \mp x_\pi^2 \pm x_\mu^4$,

$$t(z_1, z_2, z_3) = z_1 + z_2 - 2z_3,$$

$$\phi(z) = \lambda^{1/2}(1, x_\mu^2, z) \lambda^{1/2}(z, x_\mu^2, x_\pi^2)$$

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$. 8
The integration limits in Eq. (11) are
\[ s_1^- = m_K^2 (x^2 + x^2_\mu), \quad s_1^+ = m_K^2 (1 - x^2_\mu), \]
\[ s_2^+ = \frac{m_K^2}{2y} \left[ 2y(1 + x^2_\mu) - (1 + y - x^2_\mu)h_+(y) \right] \]
with \( y = s_1/m_K^2 \).

Assuming that neutrinos can be separated into light \( \nu_k \) and heavy \( N_k \) states, with masses \( m_{\nu_k} \ll \sqrt{s_1} \) and \( \sqrt{s_1} \ll M_{N_k} \) we can rewrite Eq. (11) in the approximate form (6) with the dimensionless coefficients
\[ A_\nu = 4.0 \times 10^{-31}, \quad A_N = 7.0 \times 10^{-32}, \quad A_{\nu N} = 1.7 \times 10^{-31}. \] (16)

With these numbers we can estimate the current upper bound on the \( K^+ \rightarrow \mu^+ \mu^+ \pi^- \) decay rate from the experimental data on other processes.

5 “Light-Heavy” neutrino scenario

Here we assume that all the neutrino mass eigenstates can be divided into very light \( \nu_i \) and very heavy \( N_i \) states with masses, respectively, much smaller and much larger than the characteristic energy scale \( m_0 \) of the studied \( \mathcal{L} \) process. Let us consider in this “Light-Heavy” neutrino scenario several typical examples of \( \mathcal{L} \) processes and estimate their ability to constraint the average masses \( \langle m_\nu \rangle_{ij}, \langle 1/M_N \rangle_{ij} \) as well as their possible rates.

At present the highest experimental sensitivity to the Majorana neutrino contribution has been achieved in neutrinoless double beta decay \( (0\nu\beta\beta) (A, Z) \rightarrow (A, Z + 2) + 2e^- \). A typical scale of this process is set by the nucleon Fermi momentum \( m_0 \sim p_F \approx 100\text{MeV} \). The current constraints from \( 0\nu\beta\beta \) decay are \[ [5, 4] \]
\[ \langle m_\nu \rangle_{ee} \leq 0.2\text{eV}, \quad \langle 1/M_N \rangle_{ee} \leq (9.0 \cdot 10^7\text{GeV})^{-1}. \] (17)

The first constraint, assuming that all the neutrinos are much lighter than 100 MeV, provides a direct constraint on the \( M_{ee}^{(0)} \) entry of the Majorana neutrino mass matrix. Evidently these constraints satisfy the consistency conditions (9).

Experiments searching for the other \( \mathcal{L} \) processes have not yet reached enough sensitivity to establish meaningful constraints directly on the neutrino mass matrix elements.
For instance, experiments on the muon to positron nuclear conversion \( \mu^- + (A, Z) \rightarrow e^+ + (A, Z - 2) \) in \(^{48}\)Ti give at current sensitivity the following upper bound on the branching ratio [21]

\[
R(\mu^- \rightarrow e^+) = \frac{\Gamma(\text{Ti} + \mu^- \rightarrow \text{Ca} + e^+)}{\Gamma(\text{Ti} + \mu^- \rightarrow \text{Sc} + \nu_\mu)} \leq 1.7 \cdot 10^{-12} \quad (90\%\text{CL})
\]  

(18)

Assuming that all the neutrinos are much lighter that the typical energy scale \( m_0 \sim m_\mu = 105\text{MeV} \) of this reaction one finds the bound

\[
\langle m_\nu \rangle_{\mu e} \leq 17(80)\text{MeV}
\]  

(19)

for the proton pairs of the final nucleus in the singlet(triplet) state[12]. This constraints are marginal from the viewpoint of the consistency condition (9).

Direct searches for \( K^+ \rightarrow \mu^+ \mu^+ \pi^- \) decay by E865 experiment at BNL [23] give

\[
R_{\mu \mu} = \frac{\Gamma(K^+ \rightarrow \mu^+ \mu^+ \pi^-)}{\Gamma(K^+ \rightarrow \text{all})} \leq 3.0 \times 10^{-9} \quad (90\%\text{CL}) .
\]  

(20)

Applying to this case the approximate formula (6) one gets the limit

\[
\langle m_\nu \rangle_{ij} \leq 500\text{GeV}
\]  

(21)

which makes no sense because it does not satisfy the condition (9) with the typical energy scale of this process \( m_0 \sim m_\nu = 494\text{MeV} \). Thus the approximate formula (6) is not applicable to the present experimental situation for the \( K^+ \rightarrow \mu^+ \mu^+ \pi^- \) searches. A similar picture holds for all known \( \mathcal{L} \) processes searched for in various experiments (for other examples see Refs. [17]).

Viewing these processes from the stand point of neutrino observations one finds that except for the \( 0\nu\beta\beta \)-decay they have very small rates in the “Light-Heavy” neutrino scenario.

Atmospheric and solar neutrino oscillation data demonstrate \( \delta m^2 << (1\text{eV})^2 \) suggesting that all the neutrino mass eigenstates are approximately degenerate at the 1 eV scale [24]. This observation in combination with the tritium beta decay endpoint allows one to set upper bounds on masses of all the three neutrinos [24] \( m_{e,\mu,\tau} \leq 3\text{eV} \). Thus in the three neutrino scenario one derives

\[
\langle m_\nu \rangle_{ij} \leq 9\text{eV} \quad \text{for} \quad i,j = e, \mu, \tau.
\]  

(22)

This is much lower than the existing constraints on this quantity from \( \mathcal{L} \) processes, except for \( 0\nu\beta\beta \)-decay which gives a significantly more stringent upper bound (17). With the constrain (22) we can predict the rate of various \( \mathcal{L} \) processes.
Let us substitute the upper bound (22) into the formula (6) written for the $K^+ \to \mu^+ \mu^+ \pi^-$ decay with coefficients given in Eq. (16). This gives rise to the following extremely small branching ratio

$$\mathcal{R}_{\mu\mu} = \frac{\Gamma(K^+ \to \mu^+ \mu^+ \pi^-)}{\Gamma(K^+ \to \text{all})} \leq 3.0 \times 10^{-30}$$  \hspace{1cm} (3 light neutrino scenario). \hspace{1cm} (23)$$

Assume there exist in addition heavy neutrinos $N$ with the masses in the GeV region. Using the current LEP limit on heavy stable neutral leptons $M_N \geq 39.5$ GeV [25], we get

$$\langle M_N^{-1}\rangle_{\mu\mu} \leq n \, (39.5 \text{ GeV})^{-1} ,$$

where $n$ is the number of heavy neutrinos.

This limit being substituted in Eq. (6) together with the limit (22) results in the upper bound

$$\mathcal{R}_{\mu\mu} \leq 2.0 \times 10^{-19} \hspace{1cm} (3 \text{ light } + 1 \text{ heavy neutrino scenario}). \hspace{1cm} (25)$$

Comparison of the theoretical predictions in Eqs. (23), (25) with the experimental bound in Eq. (20) clearly shows that both cases are far from being ever detected. A similar conclusion is true for the other $\mathcal{L}$ processes except $0\nu\beta\beta$ decay.

On the other hand experimental observation of these processes at larger rates would indicate some new physics beyond the SM, or, as we will see for the case of the $K^+ \to \mu^+ \mu^+ \pi^-$ decay, the presence of an extra neutrino state $\nu_j$ with mass in the hundred MeV domain. As we discussed in section 2, the extra massive neutrino states $\nu_j$ can appear as a result of mixing of the three active neutrinos with certain number of sterile neutrinos. These massive neutrinos are at present searched for in many experiments [26]. The $\nu_j$ states would manifest themselves as peaks in differential rates of various processes, and can give rise to significant enhancement of the total rate if their masses lie in an appropriate region.

### 6 Hundred MeV neutrinos in $K^+ \to \mu^+ \mu^+ \pi^-$-decay

Assume there exists a massive Majorana neutrino $\nu_j$ with the mass $m_j$

$$\sqrt{s_1} \approx 245 \text{ MeV} \leq m_j \leq \sqrt{s_1^+} \approx 389 \text{ MeV} .$$ \hspace{1cm} (26)
In this mass range the s-channel neutrino exchange diagram in Fig. 1(a) absolutely dominates over the t-channel diagram in Fig. 1(b), independently of hadronic structure. Here the diagram in Fig. 1(a) blows up because the integrand of the first term in Eq. (11) has a non-integrable singularity at $s = m^2_j$. Therefore, in this resonant domain the total $\nu_j$-neutrino decay width $\Gamma_{\nu_j}$ has to be taken into account. This can be done by the substitution $m_j \rightarrow m_j - (i/2)\Gamma_{\nu_j}$.

The total decay width $\Gamma_{\nu_j}$ of the Majorana neutrino $\nu_j$ with mass in the resonant domain (26) receives contributions from the following decay modes:

\[ \nu_j \rightarrow \begin{cases} \begin{array}{l} e^+\pi^-, \ e^-\pi^+, \ \mu^+\pi^-, \ \mu^-\pi^+, \\ e^+e^-\nu_e, \ e^+\mu^-\nu_\mu, \ e^+e^-e^e, \ e^+\mu^-\mu^- \\ e^-\nu_e, \ e^-\mu^-\nu_\mu, \ e^-e^-e, \ e^-\mu^-\mu^- \end{array} \end{cases} \]  

(27)

Since $\nu_j \equiv \nu_j^c$ it can decay in both $\nu_j \rightarrow l^-X(\Delta L = 0)$ and $\nu_j \rightarrow l^+X^c(\Delta L = 2)$ channels. Calculating partial decay rates we obtain [11]

\[ \Gamma(\nu_j \rightarrow l\pi) = |U_{lj}|^2 \frac{G_F^2}{4\pi} f^2 \pi m^3_j F(y_1, y_\pi) \equiv |U_{lj}|^2 \Gamma_2^{(l)}, \]  

(28)

\[ \Gamma(\nu_j \rightarrow l_1l_2\nu) = |U_{l_1j}|^2 \frac{G_F^2}{192\pi^3} m^5_j H(y_1, y_2) \equiv |U_{l_1j}|^2 \Gamma_3^{l_1l_2}, \]  

(29)

where $y_i = m_i/m_j$ and

\[ F(x, y) = \lambda^{1/2}(1, x^2, y^2)[(1 + x^2)(1 + x^2 - y^2) - 4x^2], \]  

(30)

\[ H(x, y) = 12 \int_{z_1}^{z_2} \frac{dz}{z} (z - y^2)(1 + x^2 - z)\lambda^{1/2}z, \]  

(31)

The integration limits are $z_1 = y^2_1$, $z_2 = (1 - y_2)^2$ and $F(0, 0) = H(0, 0) = 1$. Summing up all the decay modes in (27) one gets for the total $\nu_j$ width

\[ \Gamma_{\nu_j} = 2|U_{lj}|^2(\Gamma_2^{(l)} + \Gamma_3^{(\mu)} + \Gamma_3^{(e)}) + 2|U_{ej}|^2(\Gamma_2^{(e)} + \Gamma_3^{(ee)} + \Gamma_3^{(e)}) \equiv |U_{lj}|^2\Gamma_\nu^{(l)} + |U_{ej}|^2\Gamma_\nu^{(e)}. \]  

(32)

In the resonant domain (26) $\Gamma_{\nu_j}$ reaches its maximum value at $m_j = \sqrt{s_1}$. Assuming for the moment $|U_{lj}| = |U_{ej}| = 1$, we estimate this maximum value to be $\Gamma_{\nu_j} \approx 4.7 \times 10^{-10}$ MeV. Since $\Gamma_{\nu_j}$ is so small in the resonant domain (26) the neutrino propagator in the first term of Eq. (11) has a very sharp maximum at $s = m^2_j$. The second term, being finite in the limit $\Gamma_{\nu_j} = 0$, can be neglected in the considered case. Thus, with a good precision we obtain from Eq. (11) [11]

\[ \Gamma^{res}(K^+ \rightarrow \mu^+\mu^-\pi^-) \approx c\pi G(z_0) \frac{m_j|U_{\mu j}|^4}{|U_{\mu j}|^2\Gamma_\nu^{(\mu)} + |U_{ej}|^2\Gamma_\nu^{(e)}} \]  

(33)
with \( z_0 = (m_j/m_K)^2 \). This equation allows one to derive, from the experimental bound of Eq. (20), constraints on the \( \nu_j \) neutrino mass \( m_j \) and the mixing matrix elements \( U_{\mu j}, U_{e j} \) in a form of a 3-dimensional exclusion plot. However one may reasonably assume that \( |U_{\mu j}| \sim |U_{e j}| \). Then from the experimental bound (20) we derive a 2-dimensional \( m_j - |U_{\mu j}|^2 \) exclusion plot given in Fig. 2. For comparison we also present in Fig. 2 the existing bounds taken from [25]. As shown in the figure, the experimental data on the \( K^+ \rightarrow \mu^+ \mu^+ \pi^- \) decay exclude a region unrestricted by the other experiments. The constraints can be summarized as

\[
|U_{\mu j}|^2 \leq (5.6 \pm 1) \times 10^{-9} \quad \text{for} \quad 245 \text{ Mev} \leq m_j \leq 385 \text{ MeV}, \quad (34)
\]

The best limit \( |U_{\mu j}|^2 \leq 4.6 \times 10^{-9} \) is achieved at \( m_j \approx 300 \text{ MeV} \). Note that these limits are compatible with our assumption that \( |U_{\mu j}| \sim |U_{e j}| \) since in this mass domain, typically \( |U_{e j}|^2 \leq 10^{-9} \) [26].

The constraints from \( K^+ \rightarrow \mu^+ \mu^+ \pi^- \) in Fig. 2 and Eq. (34) can be significantly improved in the near future by the experiments E949 at BNL and E950 at FNAL [27]. It is important to notice that in the resonant domain we have \( \Gamma_{\text{res}}(K^+ \rightarrow \mu^+ \mu^+ \pi^-) \sim |U|^2 \), while outside \( \Gamma(K^+ \rightarrow \mu^+ \mu^+ \pi^-) \sim |U|^4 \). Thus in the resonant mass domain the \( K^+ \rightarrow \mu^+ \mu^+ \pi^- \) decay has a significantly better sensitivity to the neutrino mixing matrix element. In forthcoming experiments the upper bound on the ratio in Eq. (20) can be improved by two orders of magnitude or even more. Then this experimental bound could be translated to the limit \( |U_{\mu j}|^2 < 10^{-11} \) and stronger.

### 7 Hundred MeV neutrinos in astrophysics and cosmology

It is well known that massive neutrinos may have important cosmological and astrophysical implications. They are expected to contribute to the mass density of the universe, participate in cosmic structure formation, big-bang nucleosynthesis, supernova explosions, imprint themselves in the cosmic microwave background etc. (for a review see, for instance, Ref. [28]). This implies certain constrains on the neutrino masses and mixings. Currently, for massive neutrinos in the mass region (26), the only available cosmological constraints arise from the mass density of the universe and cosmic structure formation.

The contribution of stable massive neutrinos to the mass density of the universe is described by the “Lee-Weinberg” \( \Omega_\nu h^2 - m_\nu \) curve. From the requirement that the universe is not “overclosed” this leads to the two well know solutions
$m_\nu \leq 40\text{eV}$ and $m_\nu \geq 10\text{GeV}$ which seem to exclude the domain Eq. (26). However for the unstable neutrinos the situation is different. They may decay early to light particles and, therefore, their total energy can be significantly “redshifted” down to the “overclosening” limit. Constraints on the neutrino life times $\tau_{\nu_j}$ and masses $m_j$ in this scenario were found in Ref. [29]. In the mass region (26) we have an order of magnitude estimate

$$\tau_{\nu_j} < (\sim 10^{14})\text{sec} \quad \text{Mass Density limit} \quad (35)$$

Decaying massive neutrinos may also have specific impact on the cosmic structure formation introducing new stages in the evolution of the universe. After they decay into light relativistic particles the universe returns for a while from the matter to the radiation domination phase. This may change the resulting density fluctuation spectrum since the primordial fluctuations grow due to gravitation instability during the matter dominated stages. Comparison with observations leads to an upper bound on the neutrino life time [30]. In the mass region (26) one finds roughly

$$\tau_{\nu_j} < (\sim 10^{7})\text{sec} \quad \text{Structure Formation limit} \quad (36)$$

On the other hand, on the basis of formula (32), assuming $|U_{\mu j}|^2 \sim |U_{e j}|^2 \leq 4.6 \times 10^{-9}$ as in Eq. (34), we find conservatively

$$10^{-2}\text{sec} < \tau_{\nu_j}, \quad \text{Theoretical limit.} \quad (37)$$

Thus massive neutrinos with masses in the interval (26) are not yet excluded by the known cosmological constraints (35), (36) and there remains a wide open interval of allowed mixing matrix elements:

$$\sim \left(10^{-18}\right) < |U_{\mu j}|^2, |U_{e j}|^2 < (\sim 10^{-9}). \quad (38)$$

Big-bang nucleosynthesis and the SN 1987A neutrino signal may presumably lead to much more restrictive constraints [18]. Unfortunately, as yet the analysis [18] of these constraints does not involve the mass region (26). It may happen that these constraints, in combination with our constrains in Eq. (34), close the window for neutrinos with masses in the interval (26). Then the only physics left to be studied using the $K^+ \to \mu^+ \mu^+ \pi^-$ searches would be physics beyond the SM other than neutrino issues. Nevertheless, significant model dependence of all the cosmological constraints should be carefully considered before such a determining conclusion is finally drawn.
8 Conclusion

We analyzed some generic properties of $\Delta L = 2$ lepton-number violating processes and the constraints derivable from them on neutrino masses and mixing matrix elements. We discussed consistency conditions for experimental bounds when these bounds can be translated into the upper limits on the average neutrino mass $\langle m_\nu \rangle$ or the inverse average mass $\langle 1/M_N \rangle$. We found that excepting the neutrinoless double decay other $\Delta L = 2$ processes are unable to provide us with sensible constraints on these quantities. Using the neutrino oscillation data, the tritium beta decay and the LEP searches for the heavy neutral lepton we estimated constraints on their rates in the scenario with three light and several heavy neutrinos. Typical values of these rates are far from being reached experimentally in the near future.

We studied the potential of the K-meson neutrinoless double muon decay as a probe of Majorana neutrino masses and mixings. We found that this process is very sensitive to the hundred MeV neutrinos $\nu_j$ in the resonant mass range (26). We analyzed the contribution of these neutrinos to the $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay rate and derived stringent upper limits on Majorana neutrino mixing matrix element $\vert U_{\mu j} \vert^2$ from current experimental data. In Fig. 2 we presented these limits in the form of a 2-dimensional exclusion plot, and compared them with existing limits. The $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay excludes a domain previously unrestricted experimentally. We stressed that the known astrophysical and cosmological constraints do not yet exclude hundred MeV neutrinos satisfying these $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ constraints.

Finally, we notice that the decay $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ can in principle probe lepton number violating interactions beyond the standard model. However, according to recent studies [9, 31], supersymmetric interactions both with and without R-parity conservation seem to be beyond the reach of the K-decay experiments.

Acknowledgments

This work was supported in part by Fondecyt (Chile) under grants 1990806, 1000717, 1980150 and 8000017, and by a Cátedra Presidencial (Chile).

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Figure 1: The lowest order diagrams contributing to $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay.

Figure 2: Exclusion plots in the plane $|U_{\mu j}|^2 - m_j$. Here $U_{\mu j}$ and $m_j$ are the heavy neutrino $\nu_j$ mixing matrix element to $\nu_{\mu}$ and its mass respectively. Domains above the curves are excluded by various experiments according to the recent update in Ref. [25]. Region excluded by $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay [11] covers the interval $249\text{MeV} \leq m_j \leq 385\text{MeV}$ and extends down to $|U_{\mu j}|^2 \leq 4.6 \times 10^{-9}$. 