Heavy sterile neutrinos - what they can be and what they can’t
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We review current astrophysical bounds on MeV sterile neutrinos, and then we discuss why a sterile keV neutrino is a natural warm dark matter candidate.

1. Introduction

The experimental evidence for neutrino masses and mixing is overwhelming [1], and if all the present day experiments are correct, then there exists at least one sterile neutrino besides the normal 3 active ones. The mass differences are found to be in the sub-eV range, but one could naturally imagine additional sterile neutrinos, and since both masses and mixing angles essentially are free parameters, one naturally asks the question, which ones are already excluded - or are there indications that heavy sterile neutrinos exist?

Let us first review the current bounds on MeV sterile neutrinos, and then later discuss how a keV sterile neutrino naturally could be produced in the early universe in just the right amount to be a warm dark matter candidate.

2. MeV neutrinos

Measuring MeV sterile neutrinos is very difficult, and NOMAD [2] exclude mixing angles for $\langle \nu_s - \nu_{\tau} \rangle$ mixing, which go from $\sin^2\theta = 1$ to $10^{-3}$, when the mass goes from 10 to 200 MeV. The question is naturally if astrophysical bounds will overlap this excluded region, and hence completely exclude the possibility of MeV sterile neutrinos? Let us first consider the bounds from SN1987A, and then later discuss how the early universe can provide us with bounds.

The duration of SN1987A gives us an upper limit on the amount of energy which could have escaped through an “invisible” channel, such as a sterile neutrino carrying away energy and hence shortening the burst. The sterile neutrinos are produced in the SN by free neutral current scattering, and their production rate is

$$\Gamma_s = \frac{1}{2} \sin^22\theta \Gamma_{NC}. \quad (1)$$

Comparing the energy carried away with the observational bound thus leads to the limit: $\sin^22\theta < 3 \cdot 10^{-8}$ for $\langle \nu_s - \nu_{\tau} \rangle$ mixing [3]. For $\langle \nu_s - \nu_{\tau} \rangle$ mixing the bound is somewhat stronger [4,5]: $\sin^22\theta < 10^{-10}$. If the mixing angle is too big the sterile neutrino will never leave the SN, and hence the found bound will not apply, this translates into: $\sin^22\theta > 0.1$. By comparison with the terrestrial bound one thus finds a small allowed region for masses $M < 40$ MeV and mixing angles $\sin^22\theta > 0.1$.

Big bang nucleosynthesis (BBN) also gives rather strong limits on MeV neutrinos. Such a neutrino will increase the total energy density, leading to faster expansion, and one will eventually produce too much helium. For a fully thermal species (e.g. the tau neutrino) one finds that masses $M > 0.35$ MeV (corresponding to 0.3 extra neutrino species) are excluded [6]. Such strong bound can be avoided by letting the neutrino decay [7], where one even can reach minus 2 extra neutrinos for a 4 MeV tau neutrino decaying with a lifetime about 0.1 sec into a majoron and an electron neutrino. The bound will naturally be different for mixed neutrinos. With non-zero mass and mixing angle the neutrino will decay: $\nu_s \rightarrow \nu_{\tau} + l + \bar{l}$, where $l$ is any of the light leptons. This translates into a lifetime of the sterile...
neutrino 

\[ \tau_s \approx \frac{1 \text{sec}}{\left( \frac{M}{10 \text{MeV}} \right)^5 \sin^2 2\theta} \]  

(2)

and therefore one also has to include the effects of a non-thermal spectrum of the electron-neutrino, which enters directly in the neutron-proton reactions. In particular a bump in the high energy part of the electron-neutrino spectrum will again lead to overproduction of helium. First one must write down the momentum dependent evolution equations for the distribution function of the sterile neutrino, \( f_s \), [3,8]

\[ \partial_x f_s = \frac{f_s^{eq} - f_s}{\tau_s} \text{[Decay + Collision]}, \]  

(3)

where \( x = a \cdot 1 \text{ MeV} \) is the expansion parameter of the universe, and Decay and Collision describe all the possible processes. Similarly one writes down the equations for the active neutrinos, and for the temperature evolution. Eventually one find the light element abundance, and upon comparison with observations one can exclude regions of the mass-mixing parameter space (see fig. 1).

It is now straight forward to translate such bounds into mixing angle through eq. (2), and one finds, that for large mixing angles this only improves the SN bound slightly, and this only for small masses, \( m = \text{few MeV} \).

3. Sterile neutrino as warm dark matter

The sterile neutrinos were absent in the very early universe, and they were subsequently produced in collisions. The production rate is often approximated as

\[ \frac{\Gamma}{H} = \frac{\sin^2 2\theta_M}{2} \left( \frac{T}{T_W} \right)^3, \]  

(4)

where \( H \) is the Hubble parameter, \( \theta_M \) is the mixing angle in matter, and \( T_W \) is the decoupling temperature of the active neutrinos, \( T_W \approx 3 \text{ MeV} \). In this way one sees, that the production rate increases as \( T^3 \) when going to higher temperatures. The mixing angle is, however, also temperature dependent [9], and drops very fast for large temperatures

\[ \sin 2\theta_M \approx \frac{\sin 2\theta}{1 + 0.8 \cdot 10^{-19} T^6 m^{-2}}, \]  

(5)

where both temperature and mass are measured in MeV. One can plot the production rate as a function of \( 1/T \), see fig. 2, and one clearly sees, that for smaller mixing angles there will be produced fewer sterile neutrinos, and for smaller masses likewise. Now one must simply integrate the Boltzmann equation for the distribution function of the sterile neutrino

\[ H_x \partial_x f_s = \frac{\sin^2 2\theta_M}{2} \Gamma_W f_s, \]  

(6)

in order to find the produced sterile neutrinos [10]. The simplified approach described above agrees within a factor of 2 with the more detailed analysis [11], where also the slight departure of the sterile neutrino distribution function from the equilibrium form is described.
Figure 2. The production rate $\Gamma/H$ as a function of inverse temperature. With a smaller mixing angle there will be produced fewer sterile neutrinos, and with a smaller mass likewise.

Now, taking the Hubble parameter $h = 0.65$, and dark matter density $\Omega_{DM} = 0.3$, one finds the relation [11]

$$\sin^2 \theta \approx 10^{-14} \cdot \left( \frac{\text{MeV}}{m} \right)^2,$$

which describes a line in the mass-mixing parameter space, where the sterile neutrino must lie in order to be a good DM candidate.

3.1. Observational constraints

We saw above how SN data helped reducing the allowed parameter space for MeV sterile neutrinos, however, the bound becomes less restrictive for smaller masses. This is because the mixing angle inside the SN also is temperature dependent [12]

$$\sin^2 2\theta_M \approx \frac{\sin^2 2\theta}{\sin^2 2\theta + \left( \cos 2\theta + 10^3 \left( \frac{\text{keV}^2}{m} \right) \right)^2},$$

and hence for masses smaller than $m \sim 40$ keV the limit weakens substantially.

One must naturally demand that the sterile neutrinos are stable on time-scales of the universe age, $\tau_s > 4 \cdot 10^{17}$ sec. From eq. (2) we can thus find the corresponding relationship between mass and mixing angle: $m(\text{MeV})^5 \sin^2 2\theta < 2.5 \cdot 10^{-13}$. A stronger bound is, however, obtained by considering the radiative decay

$$\nu_s \rightarrow \nu_\tau + \gamma.$$  

(9)

By comparing with the observations of the diffuse gamma background one finds the much stronger bound [11]

$$m(\text{MeV})^5 \sin^2 2\theta < 2.5 \cdot 10^{-18},$$

(10)

which upon comparison with eq. (7) gives us

$m < 40 \text{keV}$ and $\sin^2 2\theta > 10^{-11},$

(11)

when the sterile neutrino is mixed with $\nu_\tau$ (or $\nu_\mu$), and $m < 30 \text{keV}$ when mixed with $\nu_e$.

3.2. How to detect or reject?

The best detection would be searching for a peak in the diffuse gamma background, which would have the peak energy near $m/2$. A more careful analysis of the present day data could also strengthen the bounds found in (11).

The analysis of a future nearby SN will also strengthen the bounds from the energy loss argument indicated above, and thus potentially cut away the low mass region. Several other SN aspects will also be affected by a keV neutrino [13].

Finally, the real mass of the dark matter particle will be determined from the analysis of large scale structure formation. At present the N-body simulations [14,15] point towards a mass about 0.5 keV, which in this sterile neutrino picture corresponds to a mixing angle about $\sin^2 2\theta \approx 10^{-7}$.

REFERENCES

1. See a discussion of experimental results e.g. in the Neutrino 2000 summary by J. Ellis, hep-ph/0008334.
2. NOMAD Collaboration, paper in preparation.
13. G. Fuller, lectures at SLAC Summer Institute 2000.