THE HELIUM ABUNDANCE PROBLEM AND NON-MINIMALLY COUPLED QUINTESSENCE

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There is a tension between observed Helium abundance and the prediction of the standard Big Bang Nucleosynthesis. We show that non-minimally quintessence model may help to reduce this tension between theory and observation.

Recently, it has been discovered that the expansion of the Universe is accelerating\(^1\). This requires the existence of a dark energy component in the Universe with an equation of state \( p = w \rho \), \( w < 0 \). One example of such a component is a cosmological constant. However, it is difficult to understand in the present framework of particle physics why it is so small. A ﬁne tuning of \( 10^{-120} \) is required if the cosmological constant arises from Planck scale physics.

Quintessence models\(^2\) were suggested as an alternative to cosmological constant. In quintessence models, a scalar ﬁeld provides the dark energy which drives the accelerated expansion of the Universe. The evolution of the scalar ﬁeld is such that its equation of state mimics the dominant component of the Universe, thus explains why it is so small at present time.

In Non-minimally Coupled (NMC) quintessence models\(^3\) the scalar ﬁeld couples to the gravitational constant. The action of NMC can be written as

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} FR - \frac{1}{2} \phi^{\mu}\phi_{\mu} - V(\phi) + L_{\text{fluid}} \right],
\]

where

\[
F(\phi) = 1 - \xi(\phi^2 - \phi_0^2), \quad V(\phi) = V_0 \phi^{-\alpha}.
\]

Such coupling could arise if for example, the scalar ﬁeld is a dilaton in superstring theory. One of the motivations for introducing such non-minimal coupling is to address the “coincidence problem”: why does the dark energy component happens to become dominant at the present epoch? Had this occurred at an earlier epoch, growth of structure due to gravitational instability would be inhibited, and any life form would be impossible to exist. By introducing a coupling between curvature and the quintessence ﬁeld, it was hoped that the scalar ﬁeld dominance could be triggered automatically shortly after
the Universe becomes matter dominant. Unfortunately, for the NMC models discussed here, it was found that the trigger mechanism does not work\textsuperscript{3,4}, nonetheless, the coupling to gravity could have other interesting consequences. Here, I show that NMC models provides a possible solution of the "helium problem" in the big bang nucleosynthesis.

The standard model of Big Bang Nucleosynthesis (BBN) is an enormously successful theory. The predicted abundances of the light elements, which ranges ten orders of magnitude, were found to be consistent with observations\textsuperscript{6}. In particular, the BBN prediction of $^4$He abundance ($Y_p \approx 0.25$) provides the first evidence of a hot big bang beginning of the Universe.

The $^4$He abundance were mostly influenced by two factors, the expansion rate of the Universe during BBN, and the baryon to photon ratio $\eta$ (see Fig. 1). The helium abundance increases with the expansion rate for two reasons. First, BBN starts when the weak interaction which converts proton to neutron ceases to be effective. This occurs when $\Gamma \sim H$, where $\Gamma, H$ are the reaction rate and expansion rate, respectively. For a higher $H$ at a given scale factor $a = 1/(1 + z)$, BBN starts earlier, when the neutron fraction is higher. Second, this also meant a shorter interval for the neutrons to decay before it is combined in subsequent nuclear fusion. Both of these two effects enhance the neutron fraction. Since most of these neutrons ended up in $^4$He, a faster expanding universe would yield more helium. Thus, once $\eta$ is determined from either deuterium abundance or other methods such as cosmic microwave background (CMB) anisotropy, the $^4$He abundance could be used to constrain the expansion rate during BBN.

The helium abundance in extragalactic HII (ionized hydrogen) regions could be obtained by observation of the $\text{HeII} \rightarrow \text{HeI}$ recombination lines. Since $^4$He is also produced in stars along with heavy elements such as Oxygen, it is expected that the primordial $^4$He abundance could be obtained by extrapolation to zero Oxygen abundance. Using this technique, Oliver and Steigman obtained\textsuperscript{7}

$$Y_p = 0.234 \pm 0.003\,\text{(stat.)},$$

while Izotov and Thuan obtained a higher value\textsuperscript{8}

$$Y_p = 0.244 \pm 0.002\,\text{(stat.)}.$$

Clearly these two data sets are statistically inconsistent with each, due to large systematic errors. Below, we adopt a midway value of

$$Y_p = 0.239 \pm 0.005,$$

or, $0.229 < Y_p < 0.249$ at 95% C.L.
Figure 1. Helium abundance vs. $\eta$. The solid curve is the prediction of standard BBN with 3 neutrinos, the two short-dashed curves are standard BBN with 2 and 4 neutrinos, and the long-dashed curve is the result for a NMC model. The horizontal lines mark the center and $2\sigma$ limits of helium in our “midway” approach. The vertical lines mark the value of $\eta$ determined from combined COBE+Boomerang+Maxima data.

In order to compare theory with observation, we also need to determine $\eta$. $\eta$ could be determined from BBN. Burles and Tytler\(^9\) obtained $D/H = (3.3 \pm 0.25) \times 10^{-5}$, corresponding to a lower bound on $\eta$ at $2\sigma$ level

$$\eta_{10} \equiv 10^{10} \eta < 6.3.$$  \hfill (6)

CMB anisotropy provides another way of measuring baryon density. Recently, an analysis of the combined data from Boomerang and Maxima yields a higher baryon density. The best fit model with a flat universe yields\(^10\)

$$\Omega_0 h^2 = 0.030 \pm 0.004, \quad \eta_{10} = 8.2 \pm 1.0,$$  \hfill (7)
If we assume that there are three neutrino species, and adopt $\eta \approx 4.5$ as inferred from the deuterium abundance, then the standard BBN $^4\text{He}$ abundance is in disagreement with the results of Oliver and Steigman. It is in marginal agreement with the “midway” result of Eq. 5, but still at high the end. If we adopt the $\eta$ inferred from CMB, then even the “midway” limit is exceed (see Fig. 1).

Furthermore, in addition to the three standard model neutrinos, a sterile neutrino may be needed to explain the results from neutrino oscillation experiments$^{11}$. If either or both of these were confirmed, or if there is any other light particle in the Universe, the breach between theory and observation on $^4\text{He}$ would become even wider.

How could we make a model which produce less helium? If the expansion of the Universe is slower at the time of BBN, then the helium abundance is reduced. In the standard BBN model, the expansion rate is given by

$$H^2 = \frac{8\pi G}{3} \rho,$$  \hspace{1cm} (8)

Thus, $\rho$ becomes greater with the introduction of each new particle species.

In quintessence models, $\rho_{tot} = \rho_f + \frac{1}{2} \dot{\phi}^2 + V(\phi)$, is it possible to introduce a negative $V$ to reduce $\rho$? Unfortunately, this would not work. To see this, note that if a negative potential is introduced, the minima of the potential must have $V < 0$, the Universe would fall to this potential well. Since $\rho_f$ is a decreasing function of $\alpha$, sooner or later we would reach to the point that $\rho_{tot} = 0$, further expansion is not possible, and the Universe would begin to contract. Such a contraction of the Universe in the future is not ruled out, however, if we hope to use a negative potential to reduce helium production, the negative potential must become sub-dominant at BBN era, and then become dominant well before the current era, which is incompatible with observation.

With the NMC model introduced in Eq. 1, however, it is possible to obtain a lower helium abundance, because now we have

$$H^2 = \frac{8\pi G}{3 F} \left( \rho + \frac{1}{2} \dot{\phi}^2 + V(\phi) - 3 \dot{\phi} H \right).$$  \hspace{1cm} (9)

For $F > 1$ the value of $H$ could be lowered.

The Helium abundance in such a model could be estimated. To a first approximation,

$$Y = (0.2378 + 0.0073 \ln \eta_{10})(1 - 0.058/\eta_{10})$$
$$+0.013(N_\nu - 3) + 2 \times 10^{-4}(\tau_n - 887).$$  \hspace{1cm} (10)
The speed up factor
\[ \zeta = \frac{H(a)}{H_{\text{sm}}(a)} \]  
(12)
is related to the neutrino number by
\[ \zeta^2 = 1 + \frac{7}{43}(N_{\nu} - 3). \]  
(13)
So we have
\[ \Delta Y = 0.08(\zeta^2 - 1). \]  
(14)

The differential speed up factor \( \zeta - 1 \) for a number of models is plotted in Fig. 2. As an example, let us consider \( \alpha = 10, \xi = 0.004, \) and \( Q_0 = 5.5 \)
which satisfies the solar system limit $|\xi| < 0.022Q_0^{-1}$. The helium abundance could be reduced by as much as $0.096\%$. In Fig. 1, the helium abundance for this NMC model is plotted. It lies much more comfortably within the allowed range. Alternatively, the helium bound on neutrino number could be relaxed. If we apply this to the current cosmological limit on neutrino number $^{12}$, which is $1.7 < N_\nu < 4.3$ at 95% C.L., the upper limit of $N_\nu$ could be lifted to 5.

In summary, I have shown that in some NMC models, the BBN helium abundance could be reduced, thus alleviate the marginal disagreement between theory and observation, and make more room for new neutrinos or other new particles. Whether such a reduction is necessary depends on the result of future observations.

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References

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