Three–Body Decays of Top and Bottom Squarks

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Abstract

We investigate the decays of third generation scalar quarks in the Minimal Supersymmetric extension of the Standard Model, focusing on the three–body modes. We calculate the partial widths of the decays of heavier top and bottom squarks into the lighter ones and a fermion pair [through virtual vector boson, Higgs boson or gaugino exchanges] and the partial widths of the three–body decays of both top squarks into bottom quarks and a pair of fermion and scalar fermion [we consider the case of lighter $\tilde{\tau}$ or $\tilde{b}$ states] and into a bottom quark, the lightest neutralino and a $W$ or a charged Higgs boson $H^{\pm}$. Some of these decay modes are shown to have substantial branching ratios in some areas of the parameter space.
1. Introduction

In the Minimal Supersymmetric extension of the Standard Model (MSSM) [1], the spin–zero partners of third generation standard chiral fermions can be significantly lighter than the corresponding scalar partners of first and second generation fermions. This is essentially due to the relatively large values of third generation fermion Yukawa couplings which enter in the non–diagonal entries of the sfermion mass matrices, the diagonalization of which turn the left– and right–handed current eigenstates $\tilde{f}_L$ and $\tilde{f}_R$ into the mass eigenstates $\tilde{f}_1$ and $\tilde{f}_2$ [2]. The mixing can generate a sizeable splitting between the masses of the two physical states and leads to a lighter sfermion $\tilde{f}_1$ with a mass possibly much smaller than the masses of the other sfermions. The situation can be even more special in the case of the lightest top squark, $\tilde{t}_1$, whose mass can be smaller than the one of its partner the top quark, $m_{\tilde{t}_1} \lesssim m_t$, to be compared with the experimental lower bound on the masses of the first and second generation squarks, $m_{\tilde{q}} \simeq O(250 \text{ GeV})$ [3].

The fact that the top quark is heavy leads to distinct phenomenological features for the decays of its scalar partners. Indeed, while the other squarks can decay directly into (almost) massless quarks and the lightest neutralino $\chi^0_1$, which is always kinematically accessible since in the MSSM the neutralino $\chi^0_1$ is assumed to be the lightest supersymmetric particle (LSP), the decay channels $\tilde{t}_i \to t\chi^0_1$ are kinematically closed for $m_{\tilde{t}_i} \leq m_t + m_{\chi^0_1}$. If, in addition, $m_{\tilde{t}_i} \leq m_b + m_{\chi^+_1}$ with $\chi^+_1$ being the lightest chargino, the decay modes $\tilde{t}_i \to b\chi^+_1$ are not accessible and the only two–body decay channel which would be allowed is the loop induced and flavor changing decay into a charm quark and the LSP, $\tilde{t}_i \to c\chi^0_1$ [4]. The other possible mode is the four–body decay channel into a bottom quark, the LSP and two massless fermions, $\tilde{t}_i \to b\chi^+_1f\bar{f}'$, which occur through virtual exchange of top quarks, charginos and scalar fermions [5].

For relatively heavier top squarks, the three–body decay channels

$$\tilde{t}_i \to bW^+\chi^0_1 , \ bH^+\chi^0_1$$

where $H^\pm$ is the MSSM charged Higgs boson, can be accessible; Fig. 1a-b. These decays have been discussed in Ref. [6, 7] in the case of the lightest top squark and have been shown to be [at least for the one with $W$ boson final states] often dominant in the case where $m_{\tilde{t}_i} \leq m_t + m_{\chi^0_1}$ and $m_b + m_{\chi^+_1}$. In addition, if sleptons are lighter than squarks [as is often the case in models with a common scalar mass at the GUT scale such as the minimal Supergravity (mSUGRA) model] the modes\footnote{This mode has also been discussed in Ref. [8] for first and second generation slepton decays into lighter $\tau$ sleptons in the context of gauge mediated Supersymmetry breaking models [9].}

$$\tilde{t}_i \to bl^+\bar{\nu}_l \text{ and/or } bl^+\nu_l$$

become possible [4, 7, 10], Fig. 1c. In the case of the lightest top squarks, they can be largely dominating over the loop induced $c\chi^0_1$ mode.

In this paper, we point out that these three–body decay modes are important not only for the lightest top squark, but also for the heavier one. In addition, we investigate a new
possibility which is the decay of the top squarks into a fermion–antifermion pair and the lightest $b$ state, which is mediated by the virtual exchange of $W$ and $H^+$ bosons:

$$\tilde{t}_i \rightarrow \tilde{b}_1 f \bar{f}'$$

(3)

$\tilde{b}_1$ can become the lightest scalar quark in the case where the ratio of the vacuum expectation values of the two–Higgs doublet fields in the MSSM, $\tan \beta$, is large$^2$.

For the heavier top squark, $\tilde{t}_2$, another possibility would be the three–body decay into the lightest top squark and a fermion pair [with $f \neq b$] through the exchange of the $Z$ and the MSSM neutral Higgs bosons [the CP–even $h, H$ and the CP–odd $A$ bosons],

$$\tilde{t}_2 \rightarrow \tilde{t}_1 f \bar{f}$$

(4)

These modes apply also for the charged decays of heavier bottom squarks into top squarks (and vice–versa) which, as in eq. (3), occur through $W$ and $H^+$ boson exchanges$^3$

$$\tilde{b}_2 \rightarrow \tilde{b}_1 f \bar{f}'$$

(5)

For $b\bar{b}$ final states, one needs to include in the case of $\tilde{t}_2 \rightarrow \tilde{t}_1 b\bar{b}$ the contributions of the exchange of the two charginos states $\chi^\pm_{1,2}$; Fig. 1d–e. This is also the case of the decay mode, $\tilde{b}_2 \rightarrow \tilde{b}_1 b\bar{b}$, where one has in addition, the virtual exchange of neutralinos and gluinos, Fig. 1e, which have to be taken into account. The latter process is a generalization [since the mixing pattern is more complicated] of the decay modes of first and second generation squarks into light scalar bottoms discussed in Ref. [14], and would be in competition with at least the two–body mode $\tilde{b}_2 \rightarrow b\chi^0_1$. The latter channel is always open since $\chi^0_1$ is the LSP, but the $b\tilde{b}_2\chi^0_1$ coupling can be small, leaving the possibility to the three–body mode to occur at a sizeable rate.

In this paper we analyze all the three–body decay modes, eqs. (2–5), discussed above. We will give complete analytical expressions for the Dalitz plot densities in terms of the energies of the final fermions as well as the fully integrated partial decay widths. In addition, we investigate the $\tilde{t}_1$ and $\tilde{t}_2$ decay modes of eq. (1) which have been already discussed in Ref. [7] for the lightest top squark $\tilde{t}_1$. In this case however, only the Dalitz densities will be given; the more complete and lengthy formulae can be found in Ref. [15].

The rest of the paper is organized as follows. In the next section, we will discuss the main properties of top and bottom squarks and summarize their two–body decay modes for completeness. In sections 3 and 4, we analyze respectively, the decay modes of top and bottom squarks into lighter sfermions and fermion pairs, and the decays of the two top squarks into neutralinos, $b$ quarks and $W$ or $H^+$ bosons. A numerical illustration is given in section 5 and a brief conclusion in section 6.

$^2$The scenario with large values of $\tan \beta$, $\tan \beta \sim m_t/m_b$, is favored in models with Yukawa coupling unification at the GUT scale [11]; the other possible solution, with $\tan \beta \sim 1.5$, seems to be ruled out by the negative searches of MSSM Higgs bosons at LEP2 [12].

$^3$If the mass splitting between the initial and final scalar eigenstates is large enough, the gauge and Higgs bosons become real, and we have the two–body decays into gauge and Higgs bosons which have been recently analyzed in Ref. [13].
Figure 1: Feynman diagrams for the three-body decay modes of top and bottom squarks.
2. The Two–Body Decay Modes

In this section, we will summarize for completeness the two–body decays of scalar quarks. This will give us the opportunity to exhibit the various couplings of squarks to charginos, neutralinos, Higgs and gauge bosons which will be needed later on, and to discuss the third generation sfermion mass spectrum and mixing pattern.

2.1 Sfermion masses and mixing

As mentioned earlier, the left–handed and right–handed sfermions of the third generation \( \tilde{f}_L \) and \( \tilde{f}_R \) [the current eigenstates] can strongly mix to form the mass eigenstates \( \tilde{f}_1 \) and \( \tilde{f}_2 \); the mass matrices which determine the mixing are given by

\[
M^2_f = \begin{bmatrix}
     m^2_{LL} & m_f \tilde{A}_f \\
     m_f \tilde{A}_f & m^2_{RR}
\end{bmatrix}
\]  

with, in terms of the soft SUSY–breaking scalar masses \( m_{\tilde{f}L} \) and \( m_{\tilde{f}R} \), the trilinear coupling \( A_f \), the higgsino mass parameter \( \mu \) and \( \text{tan} \beta = v_U/v_D \), the ratio of the vacuum expectation values of the two–Higgs doublet fields

\[
\begin{align*}
m^2_{LL} &= m_f^2 + m_{\tilde{f}L}^2 + (I_f^3 - e_f s_W^2) \cos 2\beta M_Z^2 \\
m^2_{RR} &= m_f^2 + m_{\tilde{f}R}^2 + e_f s_W^2 \cos 2\beta M_Z^2 \\
\tilde{A}_f &= A_f - \mu (\text{tan} \beta)^{-2} I_f^3
\end{align*}
\]  

with \( e_f \) and \( I_f^3 \) the electric charge and weak isospin of the sfermion \( \tilde{f} \) and \( s_W^2 = 1 - c_W^2 \equiv \sin^2 \theta_W \). In the stop sector, the mixing is strong for large values of the trilinear coupling \( A_t \) and/or for large values of \( \mu \) with small values of \( \text{tan} \beta \). In the case of the scalar bottom and tau lepton, the mixing is large when \( \text{tan} \beta \) and the parameter \( \mu \) are large.

The mass matrices eq. (6) are diagonalized by \( 2 \times 2 \) rotation matrices of angle \( \theta_f \)

\[
R \tilde{f} = \begin{pmatrix}
c_{\theta_f} & s_{\theta_f} \\
-s_{\theta_f} & c_{\theta_f}
\end{pmatrix}
\]

\( c_{\theta_f} \equiv \cos \theta_f \) and \( s_{\theta_f} \equiv \sin \theta_f \)

(8)

The mixing angle \( \theta_f \) and the sfermion eigenstate masses are then given by

\[
\begin{align*}
c_{\theta_f} &= \frac{-m_f \tilde{A}_f}{\sqrt{(m^2_{LL} - m^2_{\tilde{f}L})^2 + m_f^2 \tilde{A}_f^2}} \quad \text{and} \quad s_{\theta_f} = \frac{m^2_{LL} - m^2_{\tilde{f}L}}{\sqrt{(m^2_{LL} - m^2_{\tilde{f}L})^2 + m^2_f \tilde{A}_f^2}} \\
m^2_{\tilde{f}1,2} &= \frac{1}{2} \left[ m^2_{LL} + m^2_{RR} \pm \sqrt{(m^2_{LL} - m^2_{RR})^2 + 4m^2_f \tilde{A}_f^2} \right]
\end{align*}
\]  

(9)  

(10)
2.2 Two–body decays into neutralinos and charginos

If the scalar quarks $\tilde{q}_i$ are heavy enough, their main decay modes will be into their partner quarks and neutralinos, $\tilde{q}_i \rightarrow q\chi^0_j \ [j=1–4]$, and quarks and charginos, $\tilde{q}_i \rightarrow q'\chi^\pm_j \ [j=1–2]$. The partial decay widths are given at the tree–level by

$$\Gamma(\tilde{q}_i \rightarrow q\chi^0_j) = \frac{\alpha \lambda^2 (m_{\tilde{q}_i}^2, m_q^2, m_{\chi^0_j}^2)}{4m_{\tilde{q}_i}^3} \left[ (a_{ij}^2 + b_{ij}^2)(m_{\tilde{q}_i}^2 - m_q^2 - m_{\chi^0_j}^2) - 4a_{ij}b_{ij}m_q m_{\chi^0_j} \epsilon_{\chi_j} \right]$$

$$\Gamma(\tilde{q}_i \rightarrow q'\chi^\pm_j) = \frac{\alpha \lambda^2 (m_{\tilde{q}_i}^2, m_q^2, m_{\chi^\pm_j}^2)}{4m_{\tilde{q}_i}^3} \left[ (c_{ij}^2 + d_{ij}^2)(m_{\tilde{q}_i}^2 - m_q^2 - m_{\chi^\pm_j}^2) - 4c_{ij}d_{ij}m_q m_{\chi^\pm_j} \epsilon_{\chi_j} \right]$$

(11)

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$ is the usual two–body phase space function and $\epsilon_{\chi_j}$ is the sign of the eigenvalue of the neutralino $\chi^0_j$. The couplings $a_{ij}$ and $b_{ij}$ for the neutral decay are given by

$$\begin{align*}
\begin{cases} a_{ij}^f = -m_f r_f \sqrt{2} M_W s_W \left\{ s_{\theta_f} \right\} & - e_{L_j}^f \left\{ c_{\theta_f} \right\} \\
b_{ij}^f = -m_f r_f \sqrt{2} M_W s_W \left\{ c_{\theta_f} \right\} & - e_{R_j}^f \left\{ s_{\theta_f} \right\}
\end{cases}
\end{align*}$$

(12)

with $r_t = N_{j4}/\sin \beta$ and $r_r = r_b = N_{j3}/\cos \beta$ and

$$\begin{align*}
e_{L_j}^f &= \sqrt{2} \left[ e_f N_j^2 + \left( I_{j}^3 - e_f s_W^2 \right) \frac{1}{c_W s_W} N_j^2 \right] \\
e_{R_j}^f &= -\sqrt{2} e_f \left[ N_{j1}^2 - \frac{s_W}{c_W} N_{j2}^2 \right]
\end{align*}$$

(13)

while the couplings $c_{ij}$ and $d_{ij}$ for the charged decay mode are given, for $\tilde{t}_i$ decays, by:

$$\begin{align*}
\begin{cases} c_{ij}^t = \frac{m_b U_{j2}}{\sqrt{2} M_W s_W \cos \beta} \left\{ c_{\theta_t} \right\} & \frac{m_t V_{j2}}{\sqrt{2} M_W s_W \sin \beta} \left\{ s_{\theta_t} \right\} \\
d_{ij}^t = \frac{V_{j1}}{s_W} \left\{ -c_{\theta_t} \right\} + \frac{m_t V_{j2}}{\sqrt{2} M_W s_W \sin \beta} \left\{ s_{\theta_t} \right\}
\end{cases}
\end{align*}$$

(14)

and

$$\begin{align*}
\begin{cases} c_{ij}^b = \frac{m_t V_{j2}}{\sqrt{2} M_W s_W \cos \beta} \left\{ c_{\theta_b} \right\} & \frac{m_b U_{j2}}{\sqrt{2} M_W s_W \sin \beta} \left\{ s_{\theta_b} \right\} \\
d_{ij}^b = \frac{U_{j1}}{s_W} \left\{ -c_{\theta_b} \right\} + \frac{m_b U_{j2}}{\sqrt{2} M_W s_W \cos \beta} \left\{ s_{\theta_b} \right\}
\end{cases}
\end{align*}$$

(15)

4The QCD corrections to these decay modes have been calculated in Ref. [16].
for $\tilde{b}_i$ state decays [in the case of $\tau$ sleptons, one has to replace in the previous equations, $b$ by $\tau$ and to set $m_\tau = 0$ and $\theta_\tau = 0$. In these equations, $N$ and $U/V$ are the diagonalizing matrices for the neutralino and chargino states [17] with

$$N'_{j1} = c_W N_{j1} + s_W N_{j2} \quad , \quad N'_{j2} = -s_W N_{j1} + c_W N_{j2} . \quad (16)$$

In the case of top squarks, these decays might be not accessible kinematically, and the only allowed two–body decay will be the loop induced and flavor changing decay mode into a charm quark and the lightest neutralino, $\tilde{t}_i \rightarrow c\chi^0$. To a good approximation, the partial decay widths [in mSUGRA] are given by [4]:

$$\Gamma(\tilde{t}_i \rightarrow c\chi^0) = \frac{\alpha^3}{64\pi^2 m_{\tilde{t}_i}} \left( 1 - \frac{m^2_{\chi^0}}{m^2_{\tilde{t}_i}} \right)^2 \left| e'_{Lj} \right|^2 \left[ \frac{V_{tb}^* V_{cb} m_b^2}{2 M_W^2 s_W^2 \cos^2 \beta} \log \left( \frac{\Lambda_{\text{GUT}}^2}{M_W^2} \right) \right] \times \left[ \frac{\Delta_i}{m^2_{\tilde{t}_i} - m^2_{\tilde{t}_j}} \right]^2 \quad (17)$$

$$\Delta_1 = -c_{\theta_i} (m^2_{\tilde{c}_L} + m^2_{\tilde{b}_R} + m^2_{H_1} + A^2_b) + s_{\theta_i} m_t A_b$$

$$\Delta_2 = s_{\theta_i} (m^2_{\tilde{c}_L} + m^2_{\tilde{b}_R} + m^2_{H_1} + A^2_b) + c_{\theta_i} m_t A_b \quad (18)$$

The widths are suppressed by the CKM matrix element $V_{cb} \sim 0.05$ and the (running) $b$ quark mass squared $m_b^2 \sim (3 \text{ GeV})^2$, but very strongly enhanced by the term log ($\Lambda_{\text{GUT}}^2/M^2_W$) with $\Lambda_{\text{GUT}} \sim 2 \cdot 10^{16}$ GeV. Assuming proper electroweak symmetry breaking, the Higgs scalar mass $m_{H_1}$ can be written in terms of $\mu, \tan \beta$ and the pseudoscalar Higgs boson mass $M_A$ as $m^2_{H_1} = M_A^2 \sin^2 \beta - \cos 2\beta M^2_W - \mu^2$.

### 2.3 Two–body decays of $\tilde{q}_2$ into gauge and Higgs bosons

If the mass splitting between two squarks of the same generation is large enough, the heavier squark can decay into a lighter one plus a gauge boson $V = W, Z$ or a Higgs boson $\Phi = h, H, A, H^\pm$. The partial decay widths are given at the tree–level by:

$$\Gamma(\tilde{q}_i \rightarrow \tilde{q}_j' V) = \frac{\alpha}{4m_{\tilde{q}_i}^3 M_V^2} g^2_{\tilde{q}_i \tilde{q}_j' V} \lambda^{3/2}(m_{\tilde{q}_i}^2, M_V^2, m_{\tilde{q}_j}^2) \quad (19)$$

$$\Gamma(\tilde{q}_i \rightarrow \tilde{q}_j' \Phi) = \frac{\alpha}{4m_{\tilde{q}_i}^3} g^2_{\tilde{q}_i \tilde{q}_j' \Phi} \lambda^{1/2}(m_{\tilde{q}_i}^2, M_\Phi^2, m_{\tilde{q}_j}^2) \quad (20)$$

In these equations, the couplings of the Higgs bosons to squarks, $g_{\tilde{q}_i \tilde{q}_j' \Phi}$, read in the case of neutral Higgs bosons:

$$g_{\tilde{q}_i \tilde{q}_j h} = \frac{1}{4s_W M_W} \left[ M_Z^2 s_2 q_0 (2I_3^j - 4 e_q s^2_W) \sin(\alpha + \beta) + 2 m_q c_2 q_0 (A_q r_2^q + 2 I_3^3 \mu r_3^q) \right] \quad (21)$$

$$g_{\tilde{q}_i \tilde{q}_2 H} = \frac{1}{4s_W M_W} \left[ - M_Z^2 s_2 q_0 (2I_3^j - 4 e_q s^2_W) \cos(\alpha + \beta) + 2 m_q c_2 q_0 (A_q r_1^q - 2 I_3^3 \mu r_2^q) \right] \quad (22)$$

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5The QCD corrections to these decay modes have also been calculated and can be found in Ref. [18].
\[ g_{\tilde{q}_1 \tilde{q}_2 A} = -g_{\tilde{q}_2 \tilde{q}_1 A} = \frac{-m_q}{2 s_W M_W} \left[ \mu + A_q (\tan \beta) - 2 l^3_q \right] \]  

(23)

with the coefficients \( r_{1,2}^q \) as \( \alpha \) is a mixing angle in the CP–even Higgs sector of the MSSM, and at the tree-level, can be expressed only in terms of \( M_A \) and \( \tan \beta \)

\[ r_1^q = \frac{\sin \alpha}{\sin \beta}, \quad r_2^q = \frac{\cos \alpha}{\sin \beta}, \quad r_1^b = \frac{\cos \alpha}{\cos \beta}, \quad r_2^b = -\frac{\sin \alpha}{\cos \beta}. \]  

(24)

In the case of the charged Higgs boson, the couplings to squarks are given by

\[ g_{\tilde{q}_1 \tilde{q}_2 H^\pm} = \frac{1}{2 s_W M_W} \sum_{k,l=1}^2 \left( R^\dagger \right)_{ik} C_{\tilde{q} q_l H^\pm}^{k l} \left( R \right)_{lj} \]  

(25)

with the matrix \( C_{\tilde{q} q H^\pm} \) summarizing the couplings of the \( H^\pm \) bosons to the squark current eigenstates; it is given by

\[ C_{\tilde{t} b H^\pm} = \sqrt{2} \begin{pmatrix} m_t^2 \tan \beta + m_t^2 / \tan \beta - M_W^2 \sin 2 \beta & m_b (A_b \tan \beta + \mu) \\ m_t (A_t / \tan \beta + \mu) & 2 m_t m_b / \sin 2 \beta \end{pmatrix} \]  

(26)

Turning to the couplings of squarks to the \( W \) and \( Z \) gauge bosons, one has

\[ g_{\tilde{q}_1 \tilde{q}_2 Z} = -g_{\tilde{q}_2 \tilde{q}_1 Z} = \frac{2 I^3_q s_{2 \theta_Q}}{4 s_W c_W} \]  

(27)

\[ g_{\tilde{q}_1 \tilde{q}_2 W} = \frac{1}{\sqrt{2} s_W} \begin{pmatrix} c_{\theta_Q} c_{\theta_Q} & -s_{\theta_Q} c_{\theta_Q} \\ -s_{\theta_Q} c_{\theta_Q} & s_{\theta_Q} \end{pmatrix} \]  

(28)

Finally, for the next section, we will need the couplings of the \( W, Z \) gauge bosons and the four Higgs bosons \( h, H, A \) and \( H^\pm \) to fermions \( [r_{1,2}^f] \) are defined above:

\[ v_{ffZ} = \frac{2 I^3_f - 4 e_f s_W}{4 c_W s_W}, \quad a_{ffZ} = \frac{2 I^3_f}{4 c_W s_W}, \quad v_{ffW} = a_{ffW} = \frac{1}{2 \sqrt{2} s_W} \]  

(29)

\[ g_{ffh} = \frac{m_f r_2^f}{2 s_W M_W}, \quad g_{ffH} = \frac{m_f r_1^f}{2 s_W M_W}, \quad g_{ffA} = \frac{m_f (\tan \beta)^{-2 l^3_f}}{2 s_W M_W} \]  

(30)

\[ g_{udH^\pm} = \frac{m_d \tan \beta + m_d \cot \beta}{2 \sqrt{2} s_W M_W}, \quad g_{udH^\pm} = \frac{m_d \tan \beta - m_d \cot \beta}{2 \sqrt{2} s_W M_W} \]  

(31)

and the couplings of \( W \) and \( H^\pm \) bosons to chargino/neutrino pairs:

\[ g_{\chi^0_i \chi^+_j W^-} = \frac{1}{\sqrt{2} s_W} [-N_{i4} V^{*}_{j2} + \sqrt{2} N_{i2} V_{j1}^*], \quad g_{\chi^0_i \chi^+_j W^-} = \frac{1}{\sqrt{2} s_W} [N_{i3} U_{j2} + \sqrt{2} N_{i2} U_{j1}] \]  

(32)

\[ g_{\chi^0_i \chi^+_j H^-} = \frac{\cos \beta}{s_W} \left[ N_{i4} V^{*}_{j1} + \frac{1}{\sqrt{2}} (N_{i2}^* + \tan \theta_W N_{i1}^* ) V^{*}_{j2} \right] \]  

(33)
3. Decays into scalar fermion final states

In this section, we will analyze the decay modes of top and bottom squarks into lighter scalar fermions, their quark partners and bottom quarks. We will neglect for simplicity the masses of the final state fermions [except in the Yukawa couplings] since, even in the case of the bottom quark and tau lepton, it is a very good approximation for the $\tilde{t}$ and $\tilde{b}$ masses of the order $\mathcal{O}(100 \text{ GeV})$ that we are considering. In the case of top quark final states, the $t$–mass effects have of course to be taken into account and they can be found in Ref. [15]. However, in this case and for top squarks for instance, the two–body decays $\tilde{t}_i \rightarrow t\chi_1^0$ are kinematically allowed and will dominate over all other decays. Thus, throughout this paper, we will not consider $t$–quark final states and take all the fermions to be massless.

3.1 Scalar top decays into lighter sleptons

We start by considering the decay of top squarks\(^6\) into lighter sleptons, $\tilde{t}_i \rightarrow bl\tilde{l}_j$ with $\tilde{l}_j$ being either a sneutrino $\tilde{l}_j \equiv \tilde{\nu}_l$ or a charged slepton eigenstate $\tilde{l}_{1,2} [l = e, \mu, \tau]$, which occurs through the exchange of the two chargino states, $\chi^\pm_{1,2}$. In terms of the reduced energies of the final state particles and the reduced slepton mass defined as:

$$x_1 = \frac{2p_i \cdot p_h}{m_{\tilde{t}_i}^2}, \quad x_2 = \frac{2p_i \cdot p_t}{m_{\tilde{t}_i}^2}, \quad x_3 = \frac{2p_i \cdot p_{\tilde{l}_j}}{m_{\tilde{t}_i}^2} = 2 - x_1 - x_2, \quad \mu_i = \frac{m_{\tilde{t}_i}^2}{m_{\tilde{t}_i}}$$

(34)

the Dalitz density of the decay mode reads:

$$\frac{d\Gamma(\tilde{t}_i \rightarrow bl\tilde{l}_j)}{dx_1dx_2} = \frac{\alpha^2}{16\pi} m_{\tilde{t}_i} \sum_{k,l=1}^2 \left[ (\epsilon_{kl}^G)^2 \frac{dG_{1kl}^i}{d\mu_i} + \sqrt{\mu_{\chi_k}\mu_{\chi_l}}(\epsilon_{kl}^G)^2 \frac{dG_{2kl}^i}{d\mu_i} \right]$$

(35)

with the two functions $dG_{1kl}^i$ and $dG_{2kl}^i$ are given by\(^7\)

$$dG_{1ij}^i = \frac{(1 - x_1)(1 - x_2) - \mu_j}{(1 - x_1 - \mu_{\chi_l})(1 - x_1 - \mu_{\chi_j})}$$

$$dG_{2ij}^i = \frac{x_1 + x_2 - 1 + \mu_j}{(1 - x_1 - \mu_{\chi_l})(1 - x_1 - \mu_{\chi_j})}$$

(36)

\(^6\)The expressions that we will write in this section are also valid, with the proper change of the couplings and masses, in the case of bottom squark decays which occur through neutralino exchange. This decay is of importance in models where SUSY is broken by gauge interactions (the so–called GMSB models [9]), and where the lightest SUSY particle, the gravitino, couples very weakly to matter. The $\tilde{b}$ states will then mainly decay through this channel (but with neutralino exchange) into the tau slepton which is in general the next–to–lightest SUSY particle; see Ref. [9].

\(^7\)These functions are the same as the ones appearing in the simpler case of first and second generation squark decays given in eq. (5) of Ref. [14]. Note that there is a typographical error in the first part of the latter equation: $\mu_i$ has to be replaced by $-\mu_i$; the integrated form, eq. (8) of Ref. [14], based on the correct Dalitz density is the same as eq. (38) of the present paper.
Integrating the functions over the phase space, with boundary conditions:

\[ 1 - x_1 - \mu_f \leq x_2 \leq 1 - \frac{\mu_f}{1 - x_1}, \quad 0 \leq x_1 \leq 1 - \mu_f \]  

(37)

one obtains the functions \( G_{i1ij} \) and \( G_{2ij} \) which read:

\[
G_{i1ij}^{\tilde{f}} = \frac{1}{4} \left\{ \frac{(\mu_f - 1)(3\mu_f + 3 - 2\mu_1^2 - 2\mu_2^2) - 2\mu_f^2}{\mu_1^2 \mu_2^2} \log \mu_f \right. \\
- \frac{2(\mu_f - \mu_1)^2(\mu_2 - 1)^2}{\mu_1^2(\mu_1 - \mu_2)} \log \mu_2 - \frac{2(\mu_f - \mu_2)^2(\mu_1 - 1)^2}{\mu_2^2(\mu_1 - \mu_2)} \log \mu_1 \right\} 
\]

(38)

\[
G_{2ij}^{\tilde{f}} = \frac{1}{\mu_1^2 \mu_2^2} \left\{ \mu_f \left[ 1 + \mu_f - \frac{\mu_f(\mu_1 + \mu_2)}{2\mu_1^2 \mu_2^2} \right] \log \mu_f + \frac{1}{2}(1 - \mu_f)(\mu_f + \mu_1, \mu_2) \right. \\
- \frac{\mu_1^2(\mu_f - \mu_1^2)(\mu_2 - 1)^2}{2\mu_1 \mu_2 (\mu_1 - \mu_2)^2} \log \mu_2 - \frac{\mu_2^2(\mu_f - \mu_2^2)(\mu_1 - 1)^2}{2\mu_1 \mu_2 (\mu_1 - \mu_2)^2} \log \mu_1 \right\} 
\]

(39)

In the case where \( \chi_j = \chi_i = \chi \) [i.e. for the squared terms], the expressions simplify to:

\[
G_{i1ii}^{\tilde{f}} = \frac{1}{4} \left\{ (\mu_f - 1) \left[ 5 - 6\mu_f + 5\mu_f - 2\mu_f^2 \right] - \frac{2\mu_f^2}{\mu_f^2} \log \mu_f \right. \\
+ \frac{2(\mu_f - \mu_1^2)(\mu_2 - 1)(\mu_f + \mu_1 \mu_2 + \mu_1 - \mu_f) \log \mu_1 - \mu_f \right\} 
\]

(40)

\[
G_{2ii}^{\tilde{f}} = \frac{1}{\mu_1^2 \mu_2^2} \left\{ \frac{1}{2}(\mu_f - 1)(\mu_1^2 - 2\mu_1^2 + 2\mu_f + \mu_1 \mu_2) + \mu_f + \mu_f - \frac{\mu_f}{\mu_1^2} \log \mu_f \right. \\
+ \frac{(\mu_f - \mu_1^2)(\mu_f - \mu_2^2)}{\mu_1^2 \mu_2^2} \frac{3}{\mu_1^2} \log \mu_1 - \mu_f \right\} 
\]

(41)

3.2 Scalar top and bottom decays into lighter squarks and fermion pairs \( f \neq b \)

The neutral decays \( \tilde{q}_2 \rightarrow \tilde{q}_1 f \tilde{f} \), with \( \tilde{q} = \tilde{t} \) or \( \tilde{b} \), are mediated only by Z boson and \( h, H \), A boson exchanges if the final state fermion is not a partner of the decaying squark. The Dalitz density, with the reduced energies \( x_1 = 2(p_{\tilde{q}_2} \cdot p_f)/m_{\tilde{q}_2}^2 \) and \( x_2 = 2(p_{\tilde{q}_2} \cdot p_{\tilde{f}})/m_{\tilde{q}_2}^2 \) and the reduced mass squared \( \mu_q = m_{\tilde{q}_1}/m_{\tilde{q}_2} \), is given by:

\[
\frac{d\Gamma(\tilde{q}_2 \rightarrow \tilde{q}_1 f \tilde{f})}{dx_1 dx_2} = \frac{\alpha^2 N_C}{8\pi} m_{\tilde{q}_2} \left[ \sum_{\Phi, \Phi' = h, H} g_{\tilde{q}_1 \tilde{q}_2 \Phi} g_{\tilde{q}_1 \tilde{q}_2 \Phi'} g_{ff \Phi} g_{ff \Phi'} dV_{\Phi \Phi'} + g_{\tilde{q}_1 \tilde{q}_2 A}^2 dF_{AA}^\tilde{q} + 4 g_{\tilde{q}_1 \tilde{q}_2}^2 (v_{ffZ}^2 + a_{ffZ}^2) dF_{ZZ}^{\tilde{q}} \right] 
\]

(42)

For the charged decay mode, \( \tilde{q}_i \rightarrow \tilde{q}_j f \tilde{f}' \) [i.e. those of the decays \( \tilde{t}_{1,2} \rightarrow \tilde{b}_{1,2} f \tilde{f} \) and \( \tilde{b}_{1,2} \rightarrow \tilde{t}_{1,2} f \tilde{f} \) which are allowed by phase space], mediated by W and \( H^+ \) boson [if the
final fermion $f$ is not a partner of the squark $\tilde{q}'_j$, the Dalitz density reads

\[
\frac{d\Gamma(\tilde{q}_i \to \tilde{q}'_j f f')}{dx_1 dx_2} = \frac{\alpha^2 N_C}{8\pi} m_{\tilde{q}_i} \left[ 4g^2_{\tilde{q}'_j H} (v_{ff} + a^2_{ff}) \mathrm{d}F_{WW}^{\tilde{q}'_j} + g^2_{\tilde{q}_j H} (\sigma_{HH}^S + \sigma_{HH}^P) \mathrm{d}F_{HH}^{\tilde{q}_i} \right]
\]

(43)

where $x_1$ and $x_2$ are as above and the reduced mass is now $\mu_{\tilde{q}} = m_{\tilde{q}_i}^2/m_{\tilde{q}_i}^2$; $N_C$ is the color factor of the fermion $f$, $N_C = 3$ for quarks and $N_C = 1$ for leptons.

The two functions for the exchange of gauge bosons and scalar bosons $\mathrm{d}F_{VV}^{\tilde{q}}$ [$V = Z, W$] and $\mathrm{d}F_{\Phi\Phi'}^{\tilde{q}}$ [$\Phi, \Phi' = h, H, A, H^\pm$] are given by:

\[
\mathrm{d}F_{VV}^{\tilde{q}} = \frac{(1 - x_1)(1 - x_2) - \mu_{\tilde{q}}}{(x_1 + x_2 - 1 + \mu_{\tilde{q}} - \mu_v)^2}
\]

(44)

\[
\mathrm{d}F_{\Phi\Phi'}^{\tilde{q}} = \frac{x_1 + x_2 - 1 + \mu_{\tilde{q}}}{(x_1 + x_2 - 1 + \mu_{\tilde{q}} - \mu_\Phi)(x_1 + x_2 - 1 + \mu_{\tilde{q}} - \mu_{\Phi'})}
\]

(45)

Integrating over the phase with boundary conditions as in eq. (37), and using the phase space function,

\[
\lambda(\mu_X, \mu_Y) = -1 + 2\mu_X + 2\mu_Y - (\mu_X - \mu_Y)^2
\]

(46)

one obtains the integrated functions\(^8\) for the partial decay widths [which have to be multiplied by the same factors as in eqs. (42,43)]

\[
F_{VV}^{\tilde{q}} = \frac{1}{4} \left\{ \frac{1}{3} (1 - \mu_{\tilde{q}}) \left[ 5(1 + \mu_{\tilde{q}}) - 4\mu_v + \frac{2}{\mu_v} \lambda(\mu_v, \mu_{\tilde{q}}) \right] + \left( \lambda(\mu_v, \mu_{\tilde{q}}) - 2\mu_{\tilde{q}} \right) \log \mu_{\tilde{q}} \right\}
\]

(47)

\[
F_{\Phi\Phi'}^{\tilde{q}} = (\mu_{\tilde{q}} - 1) + \frac{1}{2} (1 + \mu_{\tilde{q}} - \mu_\Phi - \mu_{\Phi'}) \log \mu_{\tilde{q}}
\]

\[
- \frac{\mu_\Phi \sqrt{\lambda(\mu_{\tilde{q}}, \mu_\Phi)}}{\mu_{\Phi} - \mu_{\Phi'}} \arctan \left[ \frac{(1 - \mu_{\tilde{q}}) \sqrt{\lambda(\mu_\Phi, \mu_{\tilde{q}})}}{\mu_{\Phi}(1 - \mu_{\Phi} + \mu_{\tilde{q}}) - \lambda(\mu_\Phi, \mu_{\tilde{q}})} \right] + \frac{\mu_{\Phi'} \sqrt{\lambda(\mu_{\tilde{q}}, \mu_{\Phi'})}}{\mu_{\Phi} - \mu_{\Phi'}} \arctan \left[ \frac{(1 - \mu_{\tilde{q}}) \sqrt{\lambda(\mu_{\Phi'}, \mu_{\tilde{q}})}}{\mu_{\Phi'}(1 - \mu_{\Phi'} + \mu_{\tilde{q}}) - \lambda(\mu_{\Phi'}, \mu_{\tilde{q}})} \right]
\]

(48)

The latter function reduces in the case where $\Phi = \Phi'$, i.e. for the squared terms, to

\[
F_{\Phi\Phi}^{\tilde{q}} = 2(\mu_{\tilde{q}} - 1) + \frac{1}{2} (1 + \mu_{\tilde{q}} - 2\mu_\Phi) \log \mu_{\tilde{q}}
\]

\[
- \frac{\mu_\Phi (1 - \mu_\Phi + \mu_{\tilde{q}}) + \lambda(\mu_{\tilde{q}}, \mu_\Phi)}{\sqrt{\lambda(\mu_{\tilde{q}}, \mu_\Phi)}} \arctan \left[ \frac{(1 - \mu_{\tilde{q}}) \sqrt{\lambda(\mu_\Phi, \mu_{\tilde{q}})}}{\mu_{\Phi}(1 - \mu_\Phi + \mu_{\tilde{q}}) - \lambda(\mu_\Phi, \mu_{\tilde{q}})} \right]
\]

(49)

\(^8\)Note that the function $F_{VV}^{\tilde{q}}$ is the same as the one obtained in Ref. [19] for the three–body decays of a heavy Higgs boson into a lighter Higgs boson and a fermion-antifermion pair.
3.3 Scalar top and bottom decays into lighter squarks and $\tilde{b}\tilde{b}$ pairs

In the case of the decays $\tilde{t}_2 \rightarrow \tilde{t}_1 \tilde{b}\tilde{b}$, there are additional contributions with the exchange of charginos, while in the case of the decay $\tilde{b}_2 \rightarrow \tilde{b}_1 \tilde{b}\tilde{b}$ one has to include the contributions of virtual neutralinos and gluinos. The Dalitz density eqs. (42,43) have then to be transformed according to:

$$\frac{d\Gamma(\tilde{t}_2 \rightarrow \tilde{t}_1 \tilde{b}\tilde{b})}{dx_1 dx_2} \rightarrow \frac{d\Gamma(\tilde{b}_2 \rightarrow \tilde{b}_1 \tilde{b}\tilde{b})}{dx_1 dx_2} \bigg|_{\tilde{q}=\tilde{t}} + \frac{\alpha^2}{16\pi} m_{\tilde{b}_2} \left\{ \sum_{k,l=1}^{2} \left[ (c_{1k}^t c_{2k}^t c_{2l}^t + d_{1k}^t d_{2k}^t d_{2l}^t) dG_{1kl}^{\tilde{t}_1} + \sqrt{\mu_{\chi_0^+} + \mu_{\chi_0^-}} (c_{1k}^t c_{2l}^t a_{2k}^t a_{2l}^t + d_{1k}^t d_{2l}^t c_{2k}^t c_{2l}^t) dG_{2kl}^{\tilde{t}_1} \right] \\
-4 \sum_{k=1}^{2} \left[ g_{i_1 i_2} (\epsilon_{1k} c_{1k}^t (v_{bbZ} - a_{bbZ}) + d_{2k}^t a_{2k}^t (v_{bbZ} + a_{bbZ})) dG_{V_k}^{\tilde{t}_1} \right] -2 \sum_{\Phi} g_{\tilde{b}_1 \tilde{b}_2 \tilde{t}_1} (\epsilon_{1k} c_{2k}^t d_{1k}^t + d_{2k}^t c_{1k}^t) \sqrt{\mu_{\chi_0^+} dG_{\Phi_k}^{\tilde{t}_1}} \right\} \right\} (50)$$

$$\frac{d\Gamma(\tilde{b}_2 \rightarrow \tilde{b}_1 \tilde{b}\tilde{b})}{dx_1 dx_2} \rightarrow \frac{d\Gamma(\tilde{q}_2 \rightarrow \tilde{q}_1 \tilde{f}\tilde{f})}{dx_1 dx_2} \bigg|_{\tilde{q}=\tilde{b}} + \frac{\alpha^2}{3\pi} m_{\tilde{b}_2} \left[ 2s_{\theta_{1b}}^2 c_{\theta_{1b}}^2 dG_{1gg}^{\tilde{b}_1} + (c_{\theta_{1b}}^4 + s_{\theta_{1b}}^4) \mu_g dG_{2gg}^{\tilde{b}_1} \right] \right\} \right\} (51)$$

Note that the sums run only on the virtual states; for instance, in the case of the decay $\tilde{b}_2 \rightarrow \tilde{b}_1 \tilde{b}\tilde{b}$ one has to discard the exchange of the lightest neutralino $\chi_0^0$ [since it is the LSP and the decay $\tilde{b}_2 \rightarrow \tilde{b}\chi_0^0$ always occurs at the two–body level if $m_b = 0$] and add the two–body partial width $\Gamma(\tilde{b}_1 \rightarrow \tilde{b}\chi_0^0)$ to the total decay width. Note also that the gluino exchange diagram does not interfere with the other diagrams due to color conservation.

The functions $dG_{1kl}^{\tilde{t}_1}$ and $dG_{2kl}^{\tilde{t}_1}$ have been given previously, while the new functions $dG_{V_k}^{\tilde{t}_1}$ and $dG_{\Phi_k}^{\tilde{t}_1}$ are given by:

$$dG_{V_k}^{\tilde{t}_1} = \frac{1 - x_1(1 - x_2) - \mu_{\tilde{q}}}{(x_1 + x_2 - 1 + \mu_{\tilde{q}} - \mu_V)(1 - x_1 - \mu_{\chi_1})}$$

$$dG_{\Phi_k}^{\tilde{t}_1} = \frac{x_1 + x_2 - 1 + \mu_{\tilde{q}}}{(x_1 + x_2 - 1 + \mu_{\tilde{q}} - \mu_{\Phi})(1 - x_1 - \mu_{\chi_1})}$$

If the finite widths of the exchanged particles are consistently included in the expressions, one can use them also for on–shell exchanged particles. However, in the case of the decay $\tilde{b}_1 \rightarrow \tilde{b}\chi_0^0$ this procedure has always to be done since $\chi_0^0$ is stable.
When integrating over the phase space, one obtains:

\[
G_{Y_{1}}^{t} = (\mu_{\bar{q}} - 1) \left[ \frac{1}{4} (1 + \mu_{\bar{q}} + 2 \mu_{V}) \right] + \log \frac{\mu_{X} - \mu_{\bar{q}}}{\mu_{X_i} - 1} \]

\[
(\mu_{\bar{q}} - \mu_{X_i})(\mu_{X_i} - 1) - \frac{1}{4} [\lambda(\mu_{V}, \mu_{\bar{q}}) + 2 \mu_{X_i} (1 - \mu_{V} + \mu_{\bar{q}}) - 6 \mu_{\bar{q}}] \log \mu_{\bar{q}}
\]

\[
+ \frac{1}{2} \sqrt{\lambda(\mu_{V}, \mu_{\bar{q}})} (1 + \mu_{\bar{q}} - 2 \mu_{X_i} - \mu_{V}) \arctan \left( \frac{(1 - \mu_{\bar{q}}) \sqrt{\lambda(\mu_{V}, \mu_{\bar{q}})}}{\lambda(\mu_{V}, \mu_{\bar{q}}) + \mu_{V}(\mu_{V} - 1 - \mu_{\bar{q}})} \right)
\]

\[
+ [\mu_{X_i} (1 + \mu_{\bar{q}} - \mu_{X_i} - \mu_{V}) - \mu_{\bar{q}}] f(\mu_{V}, r_{+}^{Y_{1}}, r_{-}^{Y_{1}})
\]

(53)

\[
G_{\Phi i}^{t} = (\mu_{\bar{q}} - 1) - \frac{\mu_{\bar{q}}}{\mu_{X_i}} \log \mu_{\bar{q}} + \left( \frac{\mu_{\bar{q}} - \mu_{X_i}}{\mu_{X_i} - 1} \right) \log \frac{\mu_{X_i} - 1}{\mu_{X_i} - \mu_{\bar{q}}} + \mu_{\Phi} \tilde{f}(\mu_{\Phi}, r_{+}^{\Phi}, r_{-}^{\Phi})
\]

(54)

with the function \( \tilde{f} \), with arguments \( r_{\pm} = \frac{1}{2} [1 + \mu_{X} - \mu_{\bar{q}} \pm \sqrt{-\lambda(\mu_{X}, \mu_{\bar{q}})}] \), defined as:

\[
\tilde{f}(z, u, v) = - f(1) + f(u) + f(v) + \log z \log \frac{\mu_{X_i} - \mu_{\bar{q}}}{\mu_{X_i} - 1}
\]

(55)

where in terms of the Spence functions defined as, \( \text{Li}_2(x) = \int_{x}^{1} \text{d} t \frac{1}{t} \log(1 - t) \), one has:

\[
f(x) = \log (\mu_{X_i} - 1 + x) \log \frac{1 - \mu_{X_i}}{\mu_{\bar{q}} - \mu_{X_i}} + \text{Li}_2 \left( \frac{\mu_{X_i} - \mu_{\bar{q}}}{\mu_{X_i} - 1 + x} \right) - \text{Li}_2 \left( \frac{\mu_{X_i} - 1}{\mu_{X_i} - 1 + x} \right)
\]

(56)

4. Scalar top decays into \( W, H^+ \) bosons and the LSP

In this section we will analyze the three–body decay modes of top squarks, \( \tilde{t}_{i} \to b\chi_{j}^{0}B \) with \( B = W, H^{+} \). This is a (straightforward) generalization of the modes \( \tilde{t}_{1} \to b\chi_{1}^{0}W, H^{+} \) discussed in Refs. [6, 7] since here, we will consider both top squarks in the initial state and any neutralino in the final state. Here again, we will neglect the \( b \)-quark mass in the amplitude squared and in the phase space, as well as the finite widths of the exchanged particles [the latter can be easily included in the propagators]; the complete expressions with a finite \( m_{b} \) value in the propagators and in the phase space [which gives a better approximation for stop masses of order 100 GeV] can be found in Ref. [15].

In terms of the reduced energies of the final particles \( x_{1} = 2(p_{t_{i}} \cdot p_{b})/m_{t_{i}}^{2}, x_{2} = (p_{t_{i}} \cdot p_{X})/m_{t_{i}}^{2} \) and \( x_{3} = (p_{t_{i}} \cdot p_{B})/m_{t_{i}}^{2} \), and for the reduced masses \( \mu_{X} = p_{X}^{2}/m_{t_{i}}^{2} \) [we will drop the index for \( \chi_{j}^{0}, \mu_{X} \equiv \mu_{X_{1}}^{j} \)], and introducing the new scaled variables:

\[
y_{1} = \frac{p_{b} \cdot p_{X}}{m_{2}^{2}, y_{2} = \frac{p_{b} \cdot p_{B}}{m_{2}^{2}}, y_{3} = \frac{p_{X} \cdot p_{B}}{m_{2}^{2}}
\]

(57)

the Dalitz densities for the decay modes \( \tilde{t}_{i} \to b\chi_{j}^{0}B \) are given by:

\[
\frac{d\Gamma}{dx_{1}dx_{2}} = \frac{\alpha^2}{16\pi} \left[ \Gamma_{BB}^{B} + \Gamma_{X}^{B} \Gamma_{X}^{B} + 2\Gamma_{BB}^{B} + 2\Gamma_{BB}^{B} + 2\Gamma_{BB}^{B} \right]
\]

(58)

where the terms correspond to the square of the contributions of the sbottom, chargino and top quark exchange diagrams and the interference terms.
4.1 The decay $\tilde{t}_i \to b\chi_j^0W$

In the case of the decay $\tilde{t}_i \to b\chi_j^0W$, one has for the various terms:

$$\Gamma^W_{bb} = 8 \sum_{k,l=1}^2 C^0_{kl} \frac{y_1[\mu_W^{-1}(y_2 + y_3)^2 - \mu_\chi - 2y_1]}{(1 - x_3 + \mu_W - \mu_{b_k})(1 - x_3 + \mu_W - \mu_{b_l})}$$  (59)

$$\Gamma^W_{XX} = \sum_{k,l=1}^2 \frac{2}{(1 - x_1 - \mu_\chi^+(1 - x_1 - \mu_\chi^+))} \left\{ C^1_{kl} \left[ 4y_3(y_1 + y_2 + \mu_W^{-1}y_1y_3) + y_1(\mu_\chi - \mu_W) + 2\mu_\chi y_2(1 - \mu_W^{-1}y_3) \right] 
+ C^1_{kl} \sqrt{\mu_\chi^+(1 - x_1 - \mu_\chi^+)(y_1 + 2\mu_W^{-1}y_2y_3)} - 3\sqrt{\mu_\chi}(y_1 + y_2) \left[ \sqrt{\mu_\chi^+} C^2_{kl} + \sqrt{\mu_\chi^+} C^2_{lk} \right] \right\}$$  (60)

$$\Gamma^W_{tt} = \frac{2}{(1 - x_2 - \mu_\chi - \mu_t)^2} \left\{ C^3_{ij} \left[ 4y_2y_1(\mu_W^{-1}y_2 + 1) - \mu_W y_1 + 4y_2y_3 \right] 
+ C^3_{ij} \mu_t[y_1 + 2y_2y_3\mu_W^{-1}] - 4\sqrt{\mu_\chi^+} C^4_{ij}y_2[3 + 2\mu_W^{-1}y_2] \right\}$$  (61)

$$\Gamma^W_{bb} = \sum_{k,l=1}^2 \frac{-4}{(1 - x_3 + \mu_W - \mu_{b_k})(1 - x_1 - \mu_\chi^+)} \left\{ C^5_k \left[ (y_2 + y_3)(\mu_\chi y_2 - 2y_1y_3)\mu_W^{-1} \right] 
+ y_1(2y_1 + y_2 - y_3 + \mu_\chi) + \mu_\chi y_2 \right\} + C^6_k \sqrt{\mu_\chi^+} \left[ y_1 - \mu_W^{-1}y_2(y_2 + y_3) \right]$$  (62)

$$\Gamma^W_{tt} = \sum_{k=1}^2 \frac{-2}{(1 - x_3 + \mu_\chi - \mu_{b_k})(1 - x_2 + \mu_\chi - \mu_t)} \left\{ \sqrt{\mu_\chi^+} C^7_k \left[ y_1 - y_2\mu_W^{-1}(y_2 + y_3) \right] 
+ C^7_k \left[ y_1y_2 \left( -1 + 2(y_2 + y_3)\mu_W^{-1} \right) + \mu_\chi y_2 - y_1y_3 - 2y_1^2 \right] \right\}$$  (63)

$$\Gamma^W_{tt} = \sum_{k=1}^2 \frac{-2}{(1 - x_1 - \mu_\chi^+)(1 - x_2 + \mu_\chi - \mu_t)} \left\{ \sqrt{\mu_\chi^+} C^8_k \left[ y_1 + 2\mu_W^{-1}y_2y_3 \right] 
+ C^8_k \left[ y_1(2y_2 + 4y_1 - \mu_W) + y_2(4y_3 + \mu_\chi) - 2\mu_W^{-1}y_2(y_2y_3 - \mu_\chi y_2) \right] 
- 3\sqrt{\mu_\chi^+} C^9_k \left[ y_1 + y_2 \right] - \sqrt{\mu_\chi^+} C^9_k \left[ y_2(3 + 2\mu_W^{-1}y_2) \right] \right\}$$  (64)

The various combinations of couplings $C_{0,9}^6$ read as follows:

$$C^6_{lk} = g_i^b b_W^a g_i^b b_W^a (a^b_{kj} a^b_{ij} + b^b_{kj} b^b_{ij})$$

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In the case of the decay amplitude square of the chargino exchange contribution, the term $2\sum_{k,l=1}^{2}D_{kl}^{0} \frac{y_{1}}{1-x_{3}+\mu_{H}-\mu_{b_{k}}}(1-x_{3}+\mu_{H}-\mu_{b_{l}})}$ should be absent. Furthermore, in the first square bracket of eq. (19) for the amplitude squared of the top quark exchange diagram, $4/M_{W}^{2} \times (p_{\chi_{j}^{0}} \cdot p_{b})(p_{b} \cdot p_{W})^{2}$ should be replaced by $4/M_{W}^{2} \times (p_{\chi_{j}^{0}} \cdot p_{b})(p_{b} \cdot p_{W})^{2}$.

Note that there are two typographical errors\textsuperscript{10} in the corresponding expressions for these amplitudes in terms of the four–momenta, given in Ref. [6]. In eq. (18) for the amplitude square of the chargino exchange contribution, the term $2m_{\tilde{t}_{i}}^{2}D_{ij}$ should be replaced by $4\sum_{k,l=1}^{2}D_{kl}^{0} \frac{y_{1}}{1-x_{3}+\mu_{H}-\mu_{b_{k}}}(1-x_{3}+\mu_{H}-\mu_{b_{l}})}$ should be absent.

\subsection*{4.2 The decay $\tilde{t}_{i} \rightarrow b\chi_{j}^{0}H^{+}$}

In the case of the decay $\tilde{t}_{i} \rightarrow b\chi_{j}^{0}H^{+}$, one has for the various terms:

$$\Gamma_{bb}^{H^{+}} = 2 \sum_{k,l=1}^{2} D_{kl}^{0} \frac{y_{1}}{1-x_{3}+\mu_{H}-\mu_{b_{k}}}(1-x_{3}+\mu_{H}-\mu_{b_{l}})}$$

$$\Gamma_{xx}^{H^{+}} = 2 \sum_{k,l=1}^{2} \frac{y_{1}}{1-x_{1}+\mu_{x_{k}^{+}}}(1-x_{1}+\mu_{x_{l}^{+}}) \left\{ D_{kl}^{1-} \frac{\mu_{x_{k}^{+}}^{2}}{\sqrt{\mu_{x_{k}^{+}}^{2}+\mu_{x_{l}^{+}}^{2}}-\nu_{1}} \right\}$$

\textsuperscript{10}We thank Werner Porod for his cooperation in resolving this issue.
\[
\Gamma_{tt}^{H^+} = \frac{2}{(1-x_2 + \mu_\chi - \mu_t)^2} \left\{ D_{ij}^{3+}(-\mu_H y_1 + 2y_2 y_3) + D_{ij}^{3-} \mu_t y_1 - 2\sqrt{\mu_\chi \mu_H} D_{ij}^4 y_2 \right\} \tag{68}
\]

\[
\Gamma_{b\chi}^{H^+} = -2 \sum_{k,l=1}^2 \frac{D_{ij}^5 \sqrt{\mu_\chi}(y_1 + y_2) + D_{ij}^6 \sqrt{\mu_\chi^+} y_1}{(1-x_3 + \mu_H - \mu_b)(1-x_1 - \mu_\chi^+)} \tag{69}
\]

\[
\Gamma_{bt}^{H^+} = 2 \sum_{k=1}^2 \frac{\sqrt{\mu_\chi} D_{ij}^{7+} y_1 - D_{ij}^{7-} \sqrt{\mu_\chi} y_2}{(1-x_3 + \mu_\chi - \mu_b)(1-x_2 + \mu_\chi - \mu_t)} \tag{70}
\]

\[
\Gamma_{\chi t}^{H} = \sum_{k=1}^2 \frac{2}{(1-x_1 - \mu_\chi^+)(1-x_2 - \mu_\chi - \mu_t)} \left\{ -\sqrt{\mu_\chi \mu_\chi^+} D_{ij}^6 y_1 + \left[ D_{ij}^{8+}(-\mu_H y_1 + 2y_2 y_3 + \mu_\chi y_2) - \sqrt{\mu_\chi \mu_\chi^+} D_{ij}^{9+} y_1 + \sqrt{\mu_\chi \mu_\chi^+} D_{ij}^{9-} y_2 \right] \right\} \tag{71}
\]

with the various combinations of couplings \(D_{0,9}\) given by:

\[
\begin{align*}
D_{lk}^0 &= g_{t\bar{b}_k H} g_{t\bar{b}_l H} \left( a_{kj}^b \bar{a}_{ij}^\dagger + b_{kj}^b \bar{b}_{ij}^\dagger \right) \\
D_{lk}^{1+} &= \bar{a}_{ik}^\dagger d_{ij}^L G_{jlH}^L G_{jlH}^L + \bar{c}_{ik}^\dagger c_{ij}^R G_{jlH}^R G_{jlH}^R \\
D_{lk}^{1-} &= \bar{a}_{ik}^\dagger d_{ij}^R G_{jlH}^L G_{jlH}^R + \bar{c}_{ik}^\dagger c_{ij}^L G_{jlH}^R G_{jlH}^L \\
D_{lk}^{2+} &= \bar{a}_{ik}^\dagger d_{ij}^R G_{jlH}^L G_{jlH}^R + \bar{c}_{ik}^\dagger c_{ij}^L G_{jlH}^R G_{jlH}^L \\
D_{lk}^{3+} &= (a_{ij}^\dagger)^2 (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P)^2 + (b_{ij}^\dagger)^2 (g_{b\bar{b}H^+} - g_{b\bar{b}H^+}^P)^2 \\
D_{lk}^{3-} &= (a_{ij}^\dagger)^2 (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P)^2 + (b_{ij}^\dagger)^2 (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P)^2 \\
D_{lk}^4 &= a_{ij}^\dagger b_{ij}^\dagger \left[ (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P)^2 + (g_{b\bar{b}H^+} - g_{b\bar{b}H^+}^P)^2 \right] \\
D_{lk}^5 &= (a_{kj}^b \bar{a}_{ij}^\dagger + b_{kj}^b \bar{b}_{ij}^\dagger) \left[ g_{i\bar{b}_l H} + g_{i\bar{b}_l H}^P \right] \\
D_{lk}^6 &= (b_{kj}^b \bar{a}_{ij}^\dagger + a_{kj}^b \bar{b}_{ij}^\dagger) \left[ g_{i\bar{b}_l H} + g_{i\bar{b}_l H}^P \right] \\
D_{lk}^{7+} &= \left[ a_{kj}^b \bar{a}_{ij}^\dagger (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) + b_{kj}^b \bar{b}_{ij}^\dagger (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) \right] g_{i\bar{b}_l H} \\
D_{lk}^{7-} &= \left[ b_{kj}^b \bar{a}_{ij}^\dagger (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) + a_{kj}^b \bar{b}_{ij}^\dagger (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) \right] g_{i\bar{b}_l H} \\
D_{lk}^{8+} &= a_{ij}^\dagger c_{ik}^R G_{jkH}^R (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) + b_{ij}^\dagger c_{ik}^L G_{jkH}^L (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) \\
D_{lk}^{8-} &= a_{ij}^\dagger c_{ik}^R G_{jkH}^R (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) + b_{ij}^\dagger c_{ik}^L G_{jkH}^L (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) \\
D_{lk}^{9+} &= a_{ij}^\dagger c_{ik}^R G_{jkH}^R (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) + b_{ij}^\dagger c_{ik}^R G_{jkH}^R (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) \\
D_{lk}^{9-} &= a_{ij}^\dagger c_{ik}^R G_{jkH}^R (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) + b_{ij}^\dagger c_{ik}^R G_{jkH}^R (g_{b\bar{b}H^+} + g_{b\bar{b}H^+}^P) \tag{72}
\end{align*}
\]
5. Numerical illustrations

A Fortran code called SDECAY [20] has been developed for the numerical analysis; all the partial decay widths and the branching ratios for the two–body and three–body decay modes of scalar quarks [as well as the decays of charginos, neutralinos, gluinos, sleptons and the four–body decays of the lightest to squark] have been implemented. It has been interfaced with the programs SUSPECT [21] for the calculation of the supersymmetric particle spectrum [including the renormalization group equations for the evolution of the SUSY parameters and the implementation of radiative electroweak symmetry breaking] and the program HDECAY [22] for the Higgs boson spectrum and couplings [17, 23] where the renormalization improved two–loop radiative corrections in the MSSM Higgs sector [24] and the QCD corrections to the Higgs couplings [25] have been incorporated.

We begin our numerical illustration with the decays of the lightest top squark \( \tilde{t}_1 \). We will concentrate on the “unconstrained” MSSM, where for simplicity, we use a common soft–SUSY breaking scalar mass \( m_\tilde{q} \) for the three generations of squarks and \( m_f \) for the three generations of sleptons, i.e. \( m_{\tilde{t}_L} = m_{\tilde{t}_R} = m_{\tilde{b}_L} = m_{\tilde{b}_R} = m_{\tilde{q}} \) and \( m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_{\tilde{\nu}_L} = m_{\tilde{\nu}_R} \). [We will also assume that the mixing between different generations is absent at the tree–level, otherwise the decay mode \( \tilde{t}_i \to c_\chi_0 \) would already occur at this stage.]

The mass splitting between the two mass eigenstates will be then only due to the different D–terms of \( m_{\tilde{j}_L} \) and \( m_{\tilde{j}_R} \), and to the off–diagonal entries of the sfermion mass matrices. The mixing is made strong in the stop sector by taking large values of \( \theta_t = \pi/2 \) (no mixing) or to \( \pm \pi/4 \) (maximal mixing) for respectively small and large values of the entry \( m_t \bar{A}_t \) compared to the diagonal entries of the mass matrix. The mixing is strong in the \( \tilde{b} \) and \( \tilde{\tau} \) sectors for large values of \( \tan \beta \) and the parameter \( \mu \), almost independently of \( A_b \) and \( A_\tau \) which will be fixed to 100 GeV. In the gaugino sector, we will make the usual assumption of the unification of the gaugino masses at the GUT scale, leading to the relation \( M_1 = \frac{5}{3} \tan^2 \theta_W M_2 \sim \frac{1}{2} M_2 \).

In Fig. 2, we show the branching ratios of the decays of the lightest top squark \( \tilde{t}_1 \) as a function of \( \tan \beta \) for large values of \( \mu = -750 \) GeV. This implies that the lightest chargino and neutralinos are gaugino like for \( M_2 \gtrsim 300 \) GeV, with masses \( m_{\chi_1^{\pm}} \simeq m_{\chi_2^0} \simeq 2 m_{\tilde{\chi}_1^0} \simeq M_2 \) [with a very small variation with \( \tan \beta \)]. We choose a common squark mass \( m_\tilde{q} \) of \( \mathcal{O}(500 \) GeV) which, for the chosen \( \mu \) and \( A_t \) values, leads to a \( \tilde{t}_1 \) with a mass between 170 and 250 GeV [depending on the value of \( \tan \beta \), \( m_{\tilde{t}_1} \) being smaller for low \( \tan \beta \) values]. In this case, \( m_{\tilde{t}_1} \) is smaller than \( m_t + m_{\chi_1^{\pm}} \) and \( m_b + m_{\chi_1^{\pm}} \) but possibly larger than \( m_{\chi_1^{\pm}} + M_W, m_{\chi_1^{\pm}} + M_{H^+}, m_{\tilde{b}_1} \), or \( m_{\tilde{\nu}_1} \), allowing to some three–body decay channels to be open kinematically. Since these three–body decay modes are of \( \mathcal{O}(\alpha^2) \), they can compete with the \( \tilde{t}_1 \to c_\chi_0 \) mode which is of \( \mathcal{O}(\alpha^3) \) modulo the large logarithm \( \log(\Lambda_{\text{GUT}}^2/M_W^2) \).

In Fig. 2a, the common slepton mass is chosen to be relatively small, \( m_\tilde{e} = 280 \) GeV, to allow for \( \tilde{t}_1 \) decays into staus and the pseudoscalar \( A \) boson mass is taken to be relatively large, \( M_A = 250 \) GeV, implying a too heavy charged Higgs boson, \( M_{H^+} = \sqrt{M_A^2 + M_W^2} \gtrsim 260 \) GeV, for the decay \( \tilde{t}_1 \to b_\chi_0 A \) to occur. \( m_\tilde{q} \) and \( A_t \) are fixed to 450 GeV and 1 TeV, respectively, while \( M_2 = 250 \) GeV. This leads to a scalar fermion spectrum, for \( \tan \beta = 5 \) (45), of \( m_{\tilde{t}_1} \sim 170 \) (230) GeV, \( m_{\tilde{b}_1} \sim 430 \) (190) GeV and \( m_{\tilde{\nu}_1} \sim 270 \) (140) GeV.
Figure 2: The branching ratios for the two–body and three–body decay modes of the lightest top squark $\tilde{t}_1$ as a function of $\tan \beta$ for $\mu = -750$ GeV.
For small values of \( \tan \beta \), \( \tan \beta \lesssim 7 \), the mixing is too strong in the stop sector and \( \tilde{t}_1 \), being too light to have three–body decays, will mainly decay into \( c\chi_1^0 \) final states. For larger values of \( \tan \beta \), the \( \tilde{t}_1 \) mass becomes larger than \( m_b + M_W + m_{\chi_1^0} \) and the channel \( \tilde{t}_1 \to b\chi_1^0 W \) becomes kinematically accessible; it will be largely dominant, with a branching ratio above \( \sim 80\% \), up to \( \tan \beta \sim 30 \). For \( \tan \beta \) close to the latter value, \( \tilde{\tau}_1 \) becomes relatively light and the decay \( \tilde{t}_1 \to b\tilde{\tau}_1^+ \nu \) opens up and becomes competitive, the branching ratio reaching a maximum at \( \tan \beta \sim 40 \). For even larger values of \( \tan \beta \), \( \tilde{b}_1 \) becomes also light and the three–body decay \( \tilde{t}_1 \to \tilde{b}_1 ff \) will be the leading decay channel. For \( \tan \beta \gtrsim 50 \), \( m_{\tilde{t}_1} \) is larger than \( M_W + m_{\tilde{b}_1} \) and the two body–decay \( \tilde{t}_1 \to \tilde{b}_1 W \) opens up and will have a branching ratio close to unity [however, at this stage \( \tilde{b}_1 \) will eventually become lighter than the LSP neutralino].

In the scenario of Fig. 2b, the common slepton mass is taken to be larger than previously, \( m_{\tilde{e}} = 500 \) GeV, leading to heavier sleptons [in particular \( \tilde{\tau} \)'s] while the pseudoscalar Higgs boson mass is chosen to be smaller, \( M_A = 100 \) GeV, leading to a lighter charged Higgs boson, \( M_{H^+} \simeq 126 \) GeV, which can thus appear in the decay modes of the \( \tilde{t}_1 \) state. The other parameters are taken to be \( m_{\tilde{q}} = 450 \) GeV and \( A_t = 800 \) GeV, while \( M_2 \) is fixed to 300 GeV. This gives a scalar fermion spectrum, again for \( \tan \beta = 5 (45) \), of \( m_{\tilde{t}_1} \sim 200 (300) \) GeV, \( m_{\tilde{b}_1} \sim 440 (190) \) GeV and \( m_{\tilde{\tau}_1} \sim 500 (440) \) GeV.

In this scenario, since the \( \tilde{t}_1 \) mass is slightly larger than previously, the channel \( \tilde{t}_1 \to b\chi_1^0 W \) is already kinematically open for small values of \( \tan \beta \), and will be the dominating decay mode until the channel \( \tilde{t}_1 \to b\chi_1^0 H^+ \) becomes kinematically accessible. The latter will be largely dominating for \( 20 \lesssim \tan \beta \lesssim 40 \), reaching a branching ratio of \( \sim 90\% \) for \( \tan \beta \sim 35 \), until the opening of the decay channel \( \tilde{t}_1 \to \tilde{b}_1 f f \) which becomes accessible for \( \tan \beta \sim 35 \). This channel becomes then quickly the dominant decay mode of \( \tilde{t}_1 \). Note that for \( \tan \beta \gtrsim 45 \), \( m_{\tilde{t}_1} \) becomes larger than \( m_{\tilde{b}_1} + M_W \) and we have the two–body decay mode \( \tilde{t}_1 \to \tilde{b}_1 W \) which has a branching ratio very close to unity.

Let us now turn our attention to the “Constrained MSSM” or minimal Supergravity model (mSUGRA) [26] where the soft SUSY breaking scalar masses, gaugino masses and trilinear couplings are universal at the GUT scale; the left– and right–handed sfermion masses are then given in terms of the gaugino mass parameter \( M_{1/2} \), the universal scalar mass \( m_0 \), the universal trilinear coupling \( A_0 \) and \( \tan \beta \). The soft SUSY breaking scalar masses and the trilinear couplings at the low energy scale are given by their Renormalization Group Equations, the one–loop approximations of which are given for instance in Ref. [23, 26]. The parameter \( \mu \) [up to its sign] is fixed by the requirement of proper electroweak symmetry breaking. In mSUGRA and in the relatively small \( \tan \beta \) regime, due to the running of the (large) top Yukawa coupling, the two top squarks can be much lighter than the other squarks, and in contrast with the first two generations one has generically a sizeable splitting between \( m_{\tilde{t}_L} \) and \( m_{\tilde{t}_R} \) at the electroweak scale. Thus, even without large mixing, \( \tilde{t}_1 \) can be much lighter than the other squarks in this scenario.

In Table 1, we show some of the branching ratios of the lightest top squark for the fixed values of the gaugino mass \( M_{1/2} = 250 \) GeV and sign(\( \mu \)) = − and for several values of the scalar mass \( m_0 = 100, 150, 200 \) and 300 and several values of \( \tan \beta = 4, 10, 20 \) and
30 [this leads to $m_{\tilde{\chi}_1^+} \sim 2m_{\tilde{\chi}_1^0} \sim 200$ GeV, with a slight dependence on $\tan \beta$]. The $\tilde{t}_1$ mass is fixed to approximately $m_{\tilde{t}_1} \sim 200$ GeV by varying the trilinear coupling $A_0$. One sees that the branching ratios for some of the three-body decays [the channels $\tilde{t}_1 \to b\chi_i^0W$ and $b\tau_1\nu_\tau$] are sizeable.

For small values of $\tan \beta$ and the chosen values of $m_0$, $\tilde{\tau}_1$ is rather heavy, and the only three-body decay channel which is available is $\tilde{t}_1 \to bW\chi_1^0$ and for $m_0 = 150$ GeV the branching ratio is very close to unity. For larger values of $\tan \beta$, $\tilde{\tau}_1$ becomes lighter and the phase space for the decay into $bW\chi_1^0$ is suppressed so that only the decay mode $\tilde{t}_1 \to b\tilde{\tau}_1\nu_\tau$ is largely dominating. For $\tan \beta \gtrsim 30$, $\tilde{\tau}_1$ becomes too light [with a mass below the experimental bound] and the electroweak symmetry breaking does not take place for values of $m_0, A_t$ leading to a relatively light stop. A few remarks can be made here:

- For larger values of $M_{1/2}$, the top squarks [and all other squarks] become rather heavy and the two-body decays into $b\chi_1^+$ and even to $t\chi_1^0$ are kinematically allowed and dominate. For smaller values of $M_{1/2}$, the chargino $\chi_1^+$ becomes too light and again, the two-body decay channel $\tilde{t}_1 \to b\chi_1^+$ opens up.

- In the studied examples, the parameter $\mu$ is always rather large, $|\mu| \gtrsim 500$ GeV, so that all Higgs particles [except for the $h$ boson] are relatively heavy with a mass of $O(|\mu|)$. In particular, $H^+$ is too heavy for the three-body decay $\tilde{t}_1 \to bH^+\chi_1^0$ to occur.

- $\tilde{b}_1$ is also rather heavy in the studied scenario. It is only for very large values of $\tan \beta$ that $\tilde{b}_1$ becomes light enough for the decay $\tilde{t}_1 \to \tilde{b}_1ff$ to occur. But in this case, $\tilde{\tau}_1$ is even lighter and its mass is smaller than the experimental bound of $O(100$ GeV).

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$m_0$</th>
<th>$BR(bW\chi_1^0)$</th>
<th>$BR(b\bar{l}l)$</th>
<th>$BR(c\chi_1^0)$</th>
</tr>
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<tr>
<td>4</td>
<td>150</td>
<td>0.993</td>
<td>$4 \cdot 10^{-2}$</td>
<td>$3 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>$3 \cdot 10^{-4}$</td>
<td>0.915</td>
<td>0.085</td>
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<td>20</td>
<td>250</td>
<td>0.02</td>
<td>0.81</td>
<td>0.17</td>
</tr>
<tr>
<td>30</td>
<td>300</td>
<td>0.015</td>
<td>0.63</td>
<td>0.355</td>
</tr>
</tbody>
</table>

Table 1: Some examples of branching ratios for the three-body decays of $\tilde{t}_1$ in the mSUGRA model for $M_{1/2} = 250$ GeV, sign($\mu$) = − and $m_{\tilde{t}_1} \sim 200$ GeV.

We turn now to the decays of the heavier top squark, $\tilde{t}_2$. In principle, $\tilde{t}_2$ should have the same decay modes as the lighter $\tilde{t}_1$ if the two squarks have approximately the same mass [which means that the mixing is not too strong if the left- and right-handed soft-SUSY breaking scalar masses, $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ are approximatively the same]. The branching ratios would be, however, different because of the different couplings. However, if the mass splitting between the two stop eigenstates is sizeable, the additional mode $\tilde{t}_2 \to \tilde{t}_1ff$ through $Z$ and neutral Higgs boson exchanges has to be taken into account.
This is shown in Fig. 3 where the decays of $\tilde{t}_2$ are displayed as a function of $\tan \beta$, for $\mu = -350$ GeV and $M_2 = 310$ GeV. We have taken $A_t = A_b = A_\tau = -100$ GeV and a common slepton mass $m_\tilde{l} = 200$ GeV; in the squark sector, we have used a common mass $m_\tilde{q} = 400$ GeV for the first and second generation squarks but non–universal masses $m_{\tilde{t}_L} = m_{\tilde{b}_L} = 200$ GeV and $m_{\tilde{t}_R} = 120$ GeV to allow for lighter top squarks with masses $m_{\tilde{t}_1} \sim 200$ and $m_{\tilde{t}_2} \sim 250$ GeV for $\tan \beta \sim 10$ [in this case, $m_{\tilde{t}_2}$ is lighter than the chargino $\chi_1^+$ so that the two–body decay $\tilde{t}_2 \to b\chi_1^+$ is shut].

In this scenario, $\tilde{b}_1$ and the sleptons $\tilde{\tau}_1$ and $\tilde{\nu}$ are lighter than $\tilde{t}_2$ so that the three–body decays $\tilde{t}_2 \to \tilde{b}_1 f \bar{f}$ and $\tilde{t}_2 \to b\tilde{\tau}_1 \nu_r, b\tilde{\nu} l$ [in the figure we sum the branching ratios for all sleptons] are kinematically open; the former decay channel is dominant up to values $\tan \beta \sim 20$ where the two–body decay $\tilde{t}_2 \to \tilde{b}_1 W$ opens up and reaches a branching fraction close to unity. The decays into $bW\chi_1^0$ and $bH^+\chi_1^0$, as well as the loop induced decay $\tilde{t}_2 \to c\chi_1^0$, are suppressed below the percent level [we cut the branching ratio for the decay $\tilde{t}_2 \to bW\chi_1^0$, which is mediated by sbottom exchange, when $\tilde{b}_1$ becomes on–shell since then, we have the decay chain $\tilde{t}_2 \to \tilde{b}_1 W \to \tilde{b}_1 f \bar{f}$]. The mass splitting between $\tilde{t}_2$ and $\tilde{t}_1$ is smaller than the $Z$ and Higgs boson masses and the three–body decay $\tilde{t}_2 \to \tilde{t}_1 f \bar{f}$ occurs at a sizeable rate for small and intermediate values of $\tan \beta$, reaching a branching ratio of the order of 50% for $\tan \beta \sim 15$.

Figure 3: Example of branching ratios for the two–body and three–body decay modes of the heaviest top squark $\tilde{t}_2$ as a function of $\tan \beta$ for $\mu = -350$ GeV, $M_2 = 310$ GeV, $m_\tilde{t} = 200$ GeV and trilinear couplings $A_t = A_b = A_\tau = -100$ GeV.
Finally, for completeness, we have also studied the three-body decays of bottom squarks. In this case, since $\chi_0^0$ is the LSP and $m_b$ is small, the two-body decay channels $\tilde{b}_i \rightarrow b\chi_1^0$ are always kinematically open so that three-body decays can be hardly competitive. The only situation where the latter can be sizeable is when the $\tilde{b}_i b\chi_1^0$ couplings are very tiny. This occurs when the lightest neutralinos are higgsino–like [one has then to consider both $\chi_1^0$ and $\chi_2^0$ states since in this case, $m_{\chi_1^0} \sim m_{\chi_2^0} \sim |\mu|$] so that the coupling is suppressed by $m_b/M_W$. One also needs rather small $\tan \beta$ values not to enhance the couplings which are proportional to $1/\cos \beta$. However, even in this case, the three–body decay $\tilde{b}_1 \rightarrow \tilde{t}_1 W^* \rightarrow \tilde{t}_1 f f'$ for instance has a small branching ratio: it is only in a rather limited range of the MSSM parameter space that it reaches the level of a few percent. This is shown in Fig. 4, where for the chosen set of soft–SUSY breaking parameters, $\text{BR}(\tilde{b}_1 \rightarrow \tilde{t}_1 f f')$ exceeds the percent level only for relatively small values of $\tan \beta$.

The smallness of the three–body decay rates is mainly due to the fact that in the MSSM one has to take into account the experimental constraints on the squark masses and on $\tan \beta$ from the negative Higgs boson searches at LEP. In an unconstrained MSSM, for instance without the unification of the gaugino masses at the GUT scale [see Ref. [27], for examples of models], several constraints on the SUSY parameters can be relaxed and some three–body decays might become important. In particular, as it has been recently discussed in Ref. [14], the decay $\tilde{b}_2 \rightarrow \tilde{b}_1 b\bar{b}$ through virtual gluino exchange can be competitive [since it is a strong interaction process] with the decay channel $\tilde{b}_2 \rightarrow b\chi_1^0$ if the gluino mass is not too large compared to the lightest neutralino masses, as it might be the case in the models discussed in Ref. [27].
6. Conclusions

We have performed a comprehensive analysis of the decays of third generation squarks, focusing on the three body decay modes. Because of the large value of the top quark mass and the possible large mixing in the stop and sbottom sectors which leads to sizeable splitting between the masses of the two physical states, the decay pattern of scalar top [and to a lesser extent bottom] quarks can be dramatically different from the decay pattern of first and second generation squarks, which simply decay into their almost massless partner quarks and neutralino or chargino states.

Several new decay channels, including the cascade decays of heavier squarks into lighter ones and fermion–antifermion pairs as well the decays of both top squarks into $W, H^+$ bosons and the lightest neutralinos or into leptons and lighter sleptons, are possible. In some areas of the MSSM parameter space, these additional decay modes can have sizeable branching fractions, and they can even be the dominating decay modes. These channels need therefore to be taken into account in the search of scalar top and bottom quarks at present and future colliders.

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