The dependence of \( K^+ \) production on the nuclear equation of state is investigated in heavy ion collisions. An increase of the excitation function of \( K^+ \) multiplicities obtained in heavy \((Au + Au)\) over light \((C + C)\) systems when going far below threshold which has been observed by the KaoS Collaboration strongly favours a soft equation of state. This observation holds despite of the influence of an in-medium kaon potential predicted by effective chiral models which is necessary to reproduce the experimental \( K^+ \) yields.

From the very beginning kaons have been considered as one of the best probes to study dense and hot nuclear matter formed in relativistic heavy ion collisions [1]. In particular at incident energies below the corresponding production thresholds in free space \( K^+ \) mesons are created in the early and high density phase of such reactions and – due to strangeness conservation – are not reabsorbed by the nuclear environment. Furthermore, there exist strong evidences that kaons change their properties inside the nuclear medium as predicted by effective chiral models [2,3]. The investigation of a partial restoration of chiral symmetry in dense matter probed by effective momentum dependent part of the nuclear interaction \([19]\) is well tested, contains a controlled momentum dependence and provides a reliable description of the reaction dynamics in the SIS energy range, expressed e.g. by collective nucleon flow observables as well as particle production. Also the EOS predicted by microscopic approaches \((G\text{-matrix})\) \([23]\) is similar to the soft version of the Skyrme interaction for densities up to \( 2\rho_{\text{sat}} \).

We further consider the influence of an in-medium
kaon potential based on effective chiral models [2–4,9,25].

The $K^+$ mean field consists of a repulsive vector part $V_\mu = 3/8 f_\pi^2 j_\mu$ and an attractive scalar part $\Sigma_S = m_K - m_K^2 = m_K - \sqrt{m_K^2 - \frac{1}{4} \rho_s + V_\mu V^\mu}$. Here $j_\mu$ is the baryon vector current and $\rho_s$ the scalar baryon density and $\Sigma_{KN} = 450$ MeV. Following [25] in the vector field the pion decay constant in the medium $f_\pi^2 = 0.6 f_\pi^2$ is used. However, the enhancement of the scalar part using $f_\pi^2$ is compensated by higher order contributions in the chiral expansion [25], and therefore here the bare value is used, i.e. $\Sigma_{KN}/\rho_s/f_\pi^2$. Compared to other chiral approaches [3,4] the resulting kaon dispersion relation shows a relatively strong density dependence. The increase of the in-medium $K^+$ mass $\tilde{m}_K$, Eq. (2), with this parameterisation is still consistent with the empirical knowledge of kaon-nucleus scattering and allows to explore in-medium effects on the production mechanism arising from zero temperature kaon potentials. For the kaon production via pion absorption $\pi B \rightarrow Y K^+$ the elementary cross section of [24] are used. For the $NN \rightarrow BY K^+$ channels we apply the cross sections of Ref. [17] which give a good fit to the COSY-data close to threshold. For the case of $N\Delta \rightarrow BY K^+$ and $\Delta\Delta \rightarrow BY K^+$ reactions experimental are rare. Thus we rely on the model calculation of ref. [18]. In the case that a $N^*$ resonance is involved in the reaction we used the same cross section as for nucleons. In the presence of scalar and vector fields the kaon optical potential in nuclear matter has the same structure as the corresponding Schroedinger equivalent optical potential for nucleons [23]

$$U_{opt}(\rho, k) = -\Sigma_S + \frac{1}{m_K} k_\mu V^\mu + \frac{\Sigma_S^2 - V^2}{2m_K}. \quad (1)$$

and leads to a shift of the thresholds conditions inside the medium. To fulfill energy-momentum conservation the optical potential is absorbed into an newly defined effective mass

$$\tilde{m}_K(\rho, k) = \sqrt{m_K^2 + 2m_K U_{opt}(\rho, k)} \quad (2)$$

which is a Lorentz scalar and sets the canonical momenta on the mass-shell $0 = k_\mu^2 - \tilde{m}_K^2$. Thus, e.g., the threshold condition for $K^+$ production in baryon induced reactions reads $\sqrt{s} \geq \tilde{m}_B + \tilde{m}_V + \tilde{m}_K$ with $\sqrt{s}$ the centre-of-mass energy of the colliding baryons. For a consistent treatment of the thresholds and vector baryon mean fields entering into eq. (2) are determined from two versions of the non-linear Walecka model with $K$=200/380 MeV, respectively [13]. The hyperon field is thereby scaled by $2/3$ which yields also a good description of the $\Lambda$ flow [26]. Since the parameterisations chosen for the non-linear Walecka model yield the same EOS as the Skyrme ones, the overall energy is conserved. The kaon production is treated perturbatively and does generally not affect the reaction dynamics [6].

FIG. 1. Excitation function of the $K^+$ production cross section in $Au + Au$ (scaled by $10^{-1}$) and $C + C$ reactions. The calculations are performed with in-medium kaon potential and using a hard/soft nuclear EOS and are compared to data from the KaoS Collaboration [8,20]. For $C + C$ also calculations without kaon potential are shown.

In Fig. 1 the $K^+$ excitation function for $Au + Au$ and $C + C$ reactions starting from 0.8 A-GeV which is far below threshold ($E_{th} = 1.58$ GeV) are shown. The calculations are performed for a soft/hard EOS including the in-medium kaon potential. For both systems the agreement with the KaoS data [8,20] is very good when a soft EOS is used. In the large system there is a visible EOS effect which is absent in the light system. To estimate the influence of the in-medium kaon potential for $C + C$ also calculations without potential are shown. Already in the light system the $K^+$ yield is reduced by about 50% by the influence of the potential which is essential to reproduce the data [8].

To extract more clear information on the nuclear EOS next we consider the ratio $R$ of the kaon multiplicities obtained in $Au + Au$ over $C + C$ reactions, normalised to the corresponding mass numbers. In Fig. 2 central (b=0 fm) collisions are analysed. Going far below threshold $R$ strongly increases when a soft EOS is applied whereas the increase of $R$ is much less pronounced when using the stiff EOS and $R$ even slightly drops at the lowest energy.

Hence, this ratio reflects the higher compression achieved in the heavy system. The $C + C$ system, on the other hand, is too small to develop a significantly larger compression in the case of a soft EOS compared to the hard EOS. Moreover, in the latter case the slightly more energetic binary collisions lead even to a higher $K^+$ yield in the case of a hard EOS at 0.8 A-GeV. Remarkably, this behaviour is seen despite of the presence of an in-medium kaon potential which acts opposite to the EOS effect: a higher compression increases the kaon yield but
also the value of the in-medium kaon mass which, on the other hand, tends to lower the yield again. However, the increase of the in-medium mass goes linear with density whereas the collision rate per volume increases approximately with \( \rho^2 \). E.g. in central \( \text{Au} + \text{Au} \) reactions at 0.8 A-GeV the average density \( <\rho> \) at kaon production is enhanced from 1.47 to 1.57 \( \rho_{\text{sat}} \) switching from the hard to the soft EOS. This leads to an average shift of the in-medium mass \( \Delta \) compared to the vacuum value of 55/61 MeV applying the hard/soft EOS, i.e. a relative shift of 6 MeV between soft and hard. However, collective effects like the accumulation of energy by multiple scattering show a higher sensitivity on the compression resulting in an enhancement of the available energy \( <\sqrt{s}> \geq 90 \) MeV applying the soft EOS. For \( \text{C} + \text{C} \) this effect is reverse \( (<\sqrt{s}> = -45 \) MeV) since the system is too small to develop a significant difference in compression and more repulsive collisions enhance the \( K^+ \) yield at low energies. This effect disappears above 1.0 A-GeV. There exists thus a visible EOS dependence of the kaon multiplicities.

To obtain a quantitative picture of the explored density effects in Fig. 3 the baryon densities are shown at which the kaons are created. The energy is chosen most below threshold, i.e. at 0.8 A-GeV and only central collisions are considered where the effects are maximal. \( dM_{K^+}/d\rho \) is defined as

\[
dM_{K^+}/d\rho = \sum_i N_{K^+} \frac{dP_i}{d\rho_B(x_i,t_i)}
\]

where \( \rho_B \) is the baryon density at which the kaon \( i \) was created and \( P_i \) is the corresponding production probability. For the comparison of the two systems the curves are normalised to the corresponding mass numbers. Fig. 3 illustrates several features: Only in the case of a soft EOS the mean densities at which kaons are created differ significantly for the two different reaction systems, i.e. \( <\rho>/\rho_{\text{sat}} = 1.46/1.40 \) \( \text{(C + C)} \) and 1.47/1.57 \( \text{(Au + Au)} \) using the hard/soft EOS. Generally, in \( \text{C} + \text{C} \) densities above \( 2\rho_{\text{sat}} \) are rarely reached whereas in \( \text{Au} + \text{Au} \) the kaons are created at densities up to three times saturation density. Furthermore, for \( \text{C} + \text{C} \) the density distributions are weakly dependent on the nuclear EOS. The situation changes completely in \( \text{Au} + \text{Au} \). Here the densities profile shows a pronounced EOS dependence [13]. Moreover, the excess of kaons obtained with the soft EOS originates almost exclusively from high density matter which demonstrates that compression effects are probed.

![FIG. 3. Kaon multiplicities (normalised to the mass numbers of the colliding nuclei) as a function of the baryon density at the space-time coordinates where the \( K^+ \) mesons have been created. Central (\( b=0 \text{ fm} \)) \( \text{Au} + \text{Au} \) and \( \text{C} + \text{C} \) reactions at 0.8 A-GeV are considered. The calculations are performed with in-medium kaon potential and using a hard/soft nuclear EOS.

The comparison to the KaoS data [20] is finally made in Fig. 4. Here only calculations including the kaon potential are shown since it is already clear from Fig. 1 that without the potential one is not able to reproduce the experimental yields. The calculations are performed under minimal bias conditions with \( b_{\text{max}} = 11 \) fm for \( \text{Au} + \text{Au} \) and \( b_{\text{max}} = 5 \) fm for \( \text{C} + \text{C} \) and normalised to the experimental reaction cross sections [8,20]. Both calculations show an increase of \( R \) with decreasing incident energy down to 1.0 A-GeV. As already seen for the central collisions this increase is much less pronounced.
using the stiff EOS. In the latter case $R$ even drops for 0.8 A-GeV whereas the soft EOS leads to an unrelieved increase of $R$. At 1.5 A-GeV which is already very close to threshold the differences between the two models tend to disappear. The overall behaviour of $R$ is found to be quite independent of the various production channels with initial states $i = NN, \pi N, N\Delta, \pi\Delta, \Delta\Delta$. Ratios $R_i$ built separately for the individual channels show in both cases (soft or hard) a similar energy dependence as the total $R$ (except of $R_{\pi\Delta}$ which tends to remain large also at high energies). The transport calculations further demonstrate that the increase of $R$ is not is not caused by a trivial, i.e. EOS independent limitation of phase space at low energy in the small system. This is supported by the fact that the number of collisions which the involved particles encountered prior to the production of a kaon and which is a measure of the collectivity provided by the system does not reach a sharp limit for $C + C$ at low energies. The strong increase of $R$ can be directly related to higher compressible nuclear matter. The comparison to the experimental data from KaoS [20] where the increase of $R$ is even more pronounced strongly favours a soft equation of state.

FIG. 4. Excitation function of the ratio $R$ of $K^+$ multiplicities obtained in inclusive $Au + Au$ over $C + C$ reactions. The calculations are performed with in-medium kaon potential and using a hard/soft nuclear EOS and compared to the experimental range of $R$ (shaded area) given by the data from the KaoS Collaboration [20].

To summarise, we find that at incident energies far below the free threshold $K^+$ production is a suitable tool to study the dependence on the nuclear equation of state. Using a light system as reference frame there is a visible sensitivity on the EOS when ratios of heavy ($Au + Au$) over light ($C + C$) systems are considered. Transport calculations indicate that the $K^+$ production gets hardly affected by compressional effects in $C + C$ but is highly sensitive to the high density matter ($1 \leq \rho/\rho_{sat} \leq 3$) created in $Au + Au$ reactions. Results for the $K^+$ excitation function in $Au + Au$ over $C + C$ reactions as measured by the KaoS Collaboration, strongly support the scenario with a soft EOS. This statement is also valid when an enhancement of the in-medium kaon mass as predicted by chiral models is taken into account.

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