Non-abelian energy loss in quark gluon plasmas is shown to lead to novel hadron ratio suppression patterns in ultrarelativistic nuclear collisions. We apply recent (GLV) estimates for the gluon radiative energy loss, \( \Delta E(E,R) \sim E(R/6 \text{fm})^2 \), that increases linearly with the jet energy up to \( E < 20 \text{ GeV} \) and depends quadratically on the nuclear radius, \( R \). The \( K^-/K^+ \) and \( K^+/{\pi}^+ \) ratios are found to be most sensitive to the initial density of the plasma.

\( \text{PACS numbers: 12.38.Mh; 24.85.+p; 25.75.-q} \)

Energy loss of high energy quark and gluon jets penetrating dense matter produced in ultrarelativistic heavy ion collisions leads to jet quenching and thus probes the quark-gluon plasma formed in those reactions [1,2]. The non-abelian radiative energy loss, \( \Delta E(E,L) \), suppresses the moderate 2–3 GeV \( p_T < 10–15 \text{ GeV} \) distributions of hadrons in a way that can also influence the jet fragmentation pattern into different flavor hadrons. This is because quark and gluon jets suffer different energy losses proportional to their color Casimir factors (4/3, 3). First estimates [1,2] suggested that \( \Delta E \approx 1 – 2 \text{GeV}(L/\text{fm}) \) would depend linearly on the plasma thickness, \( L \), as in abelian electrodynamics. In BDMS[3], however, non-abelian (radiated gluon final state interaction) effects were shown to lead to a quadratic dependence on \( L \) with a much larger magnitude of \( \Delta E \). The analysis of jet-quenching in \( \text{Pb} + \text{Pb} \) at \( \sqrt{s} = 20 \text{ AGeV} \), however, indicated a negligible energy loss at that energy [4,5]. In GLV[6,7] finite kinematic constraints were found to reduce greatly the energy loss at moderate jet energies. In this letter we apply the GLV energy loss to estimate more quantitatively the effect of jet quenching on the hadronic ratios at moderate \( p_L < 10 \text{ GeV} \) at the Relativistic Heavy Ion Collider (RHIC) energy, \( \sqrt{s} \approx 130 \text{ AGeV} \).

An advantage of looking into particle ratios is that uncertainties in the absolute normalization due to acceptance tend to cancel. The disadvantage is of course that high \( p_L \) particle identification is increasingly difficult. As we show below, the kinematically suppressed GLV energy loss turns out to depend approximately linearly on the jet energy, \( E \). This linear dependence, \( \Delta E \propto E \), leads unfortunately to only a very weakly \( p_L \) dependent suppression of the transverse momentum distributions for the kinematic range accessible experimentally at RHIC. Our focus here is to investigate whether the \( p_L \) dependence of the particle ratios can help further pin down the jet quenching mechanisms. Our primary candidates are the measurable \( K^+/{\pi}^+ \) and \( K^-/K^+ \) ratios.

Non-abelian energy loss in pQCD has been calculated analytically in two limits. In the “thick plasma” limit, the mean number of jet scatterings, \( \bar{n} = L/\lambda_g \), is assumed to be much greater than one. For asymptotic jet energies the eikonal approximation applies and the resummed energy loss (ignoring kinematic constraints) reduces to the following simple form [3,8,9]

\[
\Delta E_{\text{BDMS}} = \frac{C_{R\alpha_s}}{4} \frac{L^2 \mu^2}{\lambda_g} \tilde{v},
\]

where \( C_R \) is the color Casimir of the jet (= \( N_c \) for gluons), and \( \mu^2/\lambda_g \propto \alpha_s^2 \rho \) is a transport coefficient of the medium proportional to the parton density, \( \rho \). The factor, \( \tilde{v} \sim 1–3 \) depends logarithmically on \( L \) and the color Debye screening scale, \( \mu \). It is the radiated gluon mean free path, \( \lambda_g \), that enters above.

In the “thin plasma” approximation, GLV [6] employed the opacity expansion derived in [7] to express the first order energy loss in the form

\[
\Delta E_{\text{GLV}}^{(1)} = \frac{2C_{R\alpha_s}}{\pi} \frac{EL}{\lambda_g} \int_0^1 dx \int_{k_{\perp}^\text{max}}^{k_{\perp}^\text{eff}} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_0^{q_{\perp}^\text{max}} \frac{d^2q_{\perp} \mu_{\text{eff}}^2}{\pi(q_{\perp}^2 + \mu^2)^2} \cdot \frac{2k_{\perp} \cdot q_{\perp}(k - q_{\perp})^2 L^2}{16x^2E^2 + (k - q_{\perp})^4 L^4},
\]

where the opacity factor \( L/\lambda_g \) is the average number of final state interactions that the radiated gluons suffer in the plasma. The upper transverse kinematic limit is

\[
k_{\perp}^\text{max} = \text{min} \left[ 4E^2x^2, 4E^2(1 - x) \right],
\]

and the upper kinematic bound on the momentum transfer is \( q_{\perp}^2_{\text{max}} = s/4 \approx 3E\mu \). Furthermore, \( \mu_{\text{eff}}^2/\mu^2 = 1 + \mu^2/q_{\perp}^2_{\text{max}} \). For SPS and RHIC energies, these finite limits cannot be ignored as shown in [6,7]. The integral averages over a screened Yukawa interaction with scale \( \mu \). The integrand is for an exponential density profile, \( \rho \propto \exp(-2z/L) \), with the same mean thickness, \( L/2 \), as a uniform slab of plasma of width \( L \).

It was shown in [6,7] that in the asymptotic \( E \rightarrow \infty \) limit, the first-order expression (2) reduces to the BDMS result [8] up to a logarithmic factor \( \log(E/\mu) \). Numerical solutions revealed that second and third order in opacity corrections to eq. (2) remain small (<%20) in the kinematic range of interest.
For applications to RHIC energies ($\sqrt{s} \sim 130$ AGeV), a lower bound on the rapidity density of gluons is estimated from HIJING [10] to be $\sim 200$. This results in an initial gluon plasma density, $\rho \sim 10/fm^3$. As shown in [11] this gluon density is consistent with the first PHOBOS data [12] on the charged hadron rapidity density ($\sim 550$). While considerably higher density initial conditions are also consistent with the data, we illustrate here the jet quenching effects for a generic plasma with an average screening scale $\mu = 0.5$ GeV, $\alpha_s = 0.3$, and an average gluon mean free path $\lambda_g = 1$ fm. The numerical results for the first order energy loss $\Delta E$, taking into account the finite kinematic bounds are displayed in Fig. 1 for different opacities $\bar{n} = L/\lambda_g = 1-4$. Since $\Delta E \propto C_R$, quark jet energy loss is simply $4/9$ of the gluon energy loss shown in Fig. 1. As noted in Refs. [6,7], the first order opacity result reproduces the characteristic BDMS description reproduces data on pion and kaon production in $p + p$ collision. Our results are based on a leading order (LO) pQCD analysis. Detailed discussion of the formalism is published elsewhere [13,14]. Next to leading order calculations are in progress [15].

Our pQCD calculations incorporate the parton transverse momentum (“intrinsic $k_T$”) via a Gaussian transverse momentum distribution $g(\vec{k}_T)$ (characterized by the width $\langle k_T^2 \rangle$) [13,16] as:

$$E_h \frac{d\sigma_{pp}}{dp} = \sum_{abcd} \int dx_1 dx_2 d^2k_{T,a} d^2k_{T,b} g(\vec{k}_{T,a})g(\vec{k}_{T,b}) \times$$

$$f_{a/p}(x_1, Q^2)f_{b/p}(x_2, Q^2) \frac{d\sigma}{dt} \frac{D_{h/c}(z_c, Q^2)}{\pi z_c} . \quad (5)$$

Here we use LO parton distribution functions (PDF) from the MRS98 parameterization [17] and a LO set of fragmentation functions (FF) [18]. The applied scales are $Q = p_c/2$ and $\hat{Q} = p_T/2z_c$.

![Figure 1](image1.png)

**FIG. 1.** Absolute and relative energy loss of a gluon jet at different opacities, $\bar{n} = L/\lambda_g = 1, 2, 3, 4$.

The most surprising feature of Fig. 1 is that unlike in the asymptotic BDMS case, where $\Delta E/E \propto 1/E$ decreases rapidly with energy, the finite kinematic bounds in the GLV case give rise to an approximate linear energy dependence of $\Delta E$ in the energy range, $E = 2 - 10$ GeV. For this plasma, characterized by the transport coefficient $\mu^2/\lambda_g = 0.25$ GeV$^2$/fm, the approximately constant fractional energy loss of gluon jets is

$$\Delta E_{GLV}/E \approx \left( \frac{\rho}{10 \text{ fm}^3} \right) \left( \frac{L}{6 \text{ fm}} \right)^2 . \quad (4)$$

For the energy range displayed, $\Delta E_{BDMS}/E \approx (40 \text{GeV}/E)(L/6 \text{ fm})^2$ for $\bar{n} = 2$, exceeds unity throughout this energy range. The GLV expression approximates the asymptotic BDMS result from below only beyond the range of RHIC experiments.

In order to investigate the influence of the GLV energy-dependent radiative energy loss on hadron production, we apply a perturbative QCD (pQCD) based description of $Au + Au$ collisions, including energy loss prior to hadronization. First, we check that the applied pQCD description reproduces data on pion and kaon production in $p + p$ collision. Our results are based on a leading order (LO) pQCD analysis. Detailed discussion of the formalism is published elsewhere [13,14]. Next to leading order calculations are in progress [15].

Our pQCD calculations incorporate the parton transverse momentum (“intrinsic $k_T$”) via a Gaussian transverse momentum distribution $g(\vec{k}_T)$ (characterized by the width $\langle k_T^2 \rangle$) [13,16] as:

$$E_h \frac{d\sigma_{pp}}{dp} = \sum_{abcd} \int dx_1 dx_2 d^2k_{T,a} d^2k_{T,b} g(\vec{k}_{T,a})g(\vec{k}_{T,b}) \times$$

$$f_{a/p}(x_1, Q^2)f_{b/p}(x_2, Q^2) \frac{d\sigma}{dt} \frac{D_{h/c}(z_c, Q^2)}{\pi z_c} . \quad (5)$$

Here we use LO parton distribution functions (PDF) from the MRS98 parameterization [17] and a LO set of fragmentation functions (FF) [18]. The applied scales are $Q = p_c/2$ and $\hat{Q} = p_T/2z_c$.

![Figure 2](image2.png)

**FIG. 2.** $\pi^-/\pi^+$ and $K^-/K^+$ ratios in $p + p \rightarrow h + X$ collisions in the energy region $19.4 \leq \sqrt{s} \leq 130$ GeV. Data are from the references listed in the bottom panel.

Utilizing available high transverse-momentum ($2 < p_T < 10$ GeV) $p + p$ data on pion and kaon production at $19 < \sqrt{s} < 63$ GeV we can determine the best fitting energy-dependent $\langle k_T^2 \rangle$ parameter (see Ref. [13] for further details on $\langle k_T^2 \rangle$). Fig. 2 shows a general agreement between the data and our calculations for the $\pi^-/\pi^+$ and the $K^-/K^+$ ratios within the present errors for $p_T \geq 3$ GeV. We also show predictions for $\sqrt{s} = 130$ (56) GeV, with $\langle k_T^2 \rangle = 3.5$ (3.0) GeV$^2$ for $\pi^+$, $\pi^-$ and $K^-$. For $K^+$ we used a smaller phenomenological value, $\langle k_T^2 \rangle = 2.5$ (2.0) GeV$^2$, as at lower energies [14,15].

Fig. 3 displays the results for $K^-/\pi^-$ and $K^+/\pi^+$. One can conclude that, while agreement is reasonable for
$p_T \geq 4$ GeV, there is a systematic discrepancy at smaller transverse momenta. High statistics $p+p$ data at RHIC will be essential to establish an accurate baseline to which $A+A$ must be compared.

\[ f_{a/A}(x_1, Q^2) f_{b/A}(x_2, Q^2) \frac{d\sigma_{z_c}^+ D_{h/c}(z_c^+, Q^2)}{dt} \frac{dz_c}{\pi z_c}, \]  

With these approximations the invariant cross section of hadron production in central $A+A$ collision is given by

\[ E_h \frac{d\sigma_{AA}^A}{d^3p} = B_{AA}^{(0)} \int dx_1 dx_2 d^2k_{T,a} d^2k_{T,b} g(k_{T,a}) g(k_{T,b}) \]

where $B_{AA}^{(0)} = 2\pi \int_0^{b_{max}} b \ dB_{AA}(b)$ with $b_{max} = 3$ fm for central $Au+Au$ collision and $T_{AA}(b)$ is the thickness function. The factor $z_c^+ / z_c$ appears because of the in-medium modification of the fragmentation function \[19\]. Thus, the invariant cross section (7) will depend on the average opacity or collision number, $\bar{n} = L/\lambda_c$. The calculated spectra for pions and kaons are displayed for $\bar{n} = 0, 2, 3, 4$ in Fig. 4 (top panels), with their ratios to the non-quenched spectra (bottom panels). (For $\pi^-$ and $K^-$ one obtains similar spectra and ratios.)

FIG. 3. $K^-/\pi^-$ and $K^+/\pi^+$ ratios in $p+p \rightarrow h+X$ collisions in the energy region $19.4 \leq \sqrt{s} \leq 130$ GeV. Data are from the references listed in the bottom panel of Figure 2.

Now we turn to the calculation for $Au+Au$ collision at RHIC energy, $\sqrt{s} = 130$ AGeV. As a first approximation, let us consider slab geometry, neglecting radial dependence. We include the isospin asymmetry and the nuclear modification (shadowing) into the nuclear PDF. We consider the average nuclear dependence of the PDF, and apply a scale independent parameterization with the shadowing function $S_{a/A}(x)$ taken from Ref. [10]:

\[ f_{a/A}(x) = S_{a/A}(x) \left[ \frac{Z}{A} f_{a/p}(x) + \left( 1 - \frac{Z}{A} \right) f_{a/n}(x) \right]. \]  

The value of the $\langle k_T^2 \rangle$ of the transverse component of the PDF will be increased by multiscattering effects in $A+A$ collisions. Two limiting cases were investigated with (i) a large number of rescatterings [16] and (ii) a small number of rescatterings (“saturated Cronin effect”) [13]. To clarify the situation, further study of $p+A$ collisions is necessary. Here we concentrate on the influence of jet-quenching on the hadron spectra, leaving the inclusion of the multiscattering effect for future work.

In this simplified calculation, jet quenching reduces the energy of the jet before fragmentation. We concentrate on $y_{cm} = 0$, where the jet transverse momentum before fragmentation is shifted by the energy loss, $p_T^*(L/\lambda) = p_T - \Delta E/E, L)$. This shifts the $z_c$ parameter in the integrand to $z_c^* = z_c / (1 - \Delta E / p_T)$. The applied scale in the FF is similarly modified, $\bar{Q} = p_T / 2 z_c^*$, while for the elementary hard reaction the scale remains $Q = p_T / 2$.

FIG. 4. Top: $\pi^+$ and $K^+$ spectra in $Au+Au$ collision at $\sqrt{s} = 130$ AGeV without jet-quenching (dashed line) and with $\bar{n} = 2, 3, 4$ (full lines). Bottom: Ratios of the spectra with quenching to the “non-quenching” case for $\bar{n} = 1, 2, 3, 4$.

We note that the GLV energy-dependent energy loss leads to a rather structureless downward shift of the single inclusive yields of all hadrons because $\Delta E/E$ is approximately constant. This is in distinction to the estimates in [16], where an energy dependent fractional energy loss, $\Delta E/E = (0.5$ GeV/E)(L/fm), was used. Our new result indicates that over the entire accessible energy range, the GLV radiative energy loss reduces the $p_{T\perp}$ distribution by up to an order of magnitude for pions and somewhat less for kaons.

Fig. 4 indicates the importance of the absolutely normalized hadronic spectra to study the effect of jet-quenching. The effect on particle ratios is, however, more interesting, as shown by Fig. 5 and 6.
While the influence on $\pi^-/\pi^+$ is very small, the $K^-/K^+$ ratio is predicted to drop dramatically in the few GeV domain. The shift of solid $K^-/K^+$ curves to smaller $p_\perp$ (relative to the dotted curve) can be used to test the nonlinearity of the energy loss as a function of $L$. It also provides a constraint on the magnitude of the $\mu^2/\lambda_9$ transport coefficient. The most conspicuous quenching effect is limited in the $K^-/\pi^-$ ratio to be below 4-5 GeV. There is about a factor of 2 enhancement of the $K^+/\pi^+$ ratio for opacity $\bar{n} = 4$ throughout this range. These patterns arise from the interplay of the energy-shifted gluon and quark fragmentation functions into pions and kaons. Since the quark to gluon jet ratio at fixed $p_\perp$ depends sensitively on the beam energy for $\sqrt{s} = 20 - 200$ AGeV, these patterns are expected to shift in a systematic way with bombarding energy as well.

In summary, we applied the GLV parton energy loss [6, 7] in LO pQCD to pion and kaon production in nuclear collisions and calculated the sensitivity of the hadronic ratios to this property of the QCD plasma. The $K^+/\pi^+$ and $K^-/K^+$ ratios in the $2 < p_\perp < 10$ GeV range were found to be the most sensitive to the gluon opacity at RHIC energies.

Acknowledgments. We thank G. David and P. Stankus for useful discussions on RHIC experiments, G.G. Barnaföldi for discussion on kaon production, and I. Vitev for discussions on the GLV formalism. This work was supported by the U.S. DOE under DE-FG02-86ER40251, DO-AC03-76SF00098 and DE-FG02-93ER40764, by the U.S. NSF under INT-0000211, by the US-Hungarian Joint Fund No.652 and OTKA No. T029158. M.G. was partially supported by the NSD/LBL. P.L. was partially supported by the Bolyai Fellowship.