Atomic Radiative Transitions in Thermo Field Dynamics

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Abstract

In this work we study the energy exchange between an atomic system and a thermal radiation field, using the Dalibard, Dupont-Roc and Cohen-Tannoudji (DDC) construct, incorporating temperature effects to the eigenstates of the radiation field operator through the electromagnetic propagator of Thermo Field Dynamics in the Coulomb gauge. We also discuss the stability of the atomic system at finite temperature.
I. Introduction

Since the 70’s, it has been argued[1][2] that the physical interpretation of radiative phenomena, in particular the shift in atomic energy levels, rely upon different choices in the ordering of atomic and field operators in the interaction Hamiltonian.

Latter, Dalibard, Dupont-Roc and Cohen-Tannoudji (DDC)[3] considered the interaction between a nonrelativistic atomic electron and the quantized electromagnetic field, showing that such arbitrariness can be removed by requiring that the corresponding variation rates must be Hermitian, if we want them to have a “physical meaning”. They generalized this procedure to the case of a small system $S$ interacting with a large reservoir $R$ (which may be in thermal equilibrium). This construct allowed them to separate the physical processes in two categories, those where $R$ fluctuates and polarizes $S$ (effects of reservoir fluctuations), and those where $S$ polarizes $R$ (effects of self-reaction or radiation reaction).

In the present work we are interested in analyzing the implementation of temperature in the context of DDC formalism, where the statistical functions, which are defined from two-point functions of physical observables, play a fundamental role. These functions enable us to obtain expressions, up to second order in perturbation theory, in terms of products of correlation functions and susceptibilities[4].

The implementation of temperature[3] can be made directly in such statistical functions using the equipartition theorem, leading to a finite temperature description of the relevant phenomena.

In an alternative way, we shall study the theory using Umezawa’s formalism, known as Thermo Field Dynamics (TFD)[5]. In TFD, the quantum statistical average of an observable in a given ensemble is identified with its expectation value in a thermal vacuum. In this approach, the temperature is incorporated from the beginning, in the eigenstate of the number operator associated to the radiation field.

Our idea is to investigate the thermal propagator of electromagnetic field in the Coulomb gauge and to identify the correlation functions and susceptibilities of DDC formalism. We compare our results in the case of energy exchange between an atomic system and a thermal reservoir, analysing their implication to atomic stability.
II. Radiation considered as a Reservoir

In the Dalibard, Dupont-Roc and Cohen-Tannoudji[3] formalism, the interaction between an atom and the free electromagnetic field can be seen as the interaction of a microscopic system $S$ with a large reservoir $R$, in the sense that $R$ has many degrees of freedom and the correlation time between the observables of $R$ is small, allowing a perturbative treatment of the effect due to the coupling of $S$ and $R$. In this context, the Hamiltonian of the global system $S + R$, can be written as

$$ H = H_s + H_R + V, $$

where $H_s$ is the Hamiltonian of $S$, $H_R$ the reservoir Hamiltonian and $V$ the interaction Hamiltonian, which we assume to be of the form $V = -\sum_i R_i S_i$, where $R_i$ and $S_i$ are Hermitian observables of $R$ and $S$.

In the interaction representation with respect to $H_s + H_R$, the density operator of the global system $\rho(t)$ evolve according to

$$ \frac{d}{dt} \hat{\rho}(t) = \frac{1}{i\hbar} [\hat{V}(t), \hat{\rho}(t)]. $$

(2)

Here, the reservoir stands for the radiation field. In the case we are interested, $R$ represents the electromagnetic field so that we may choose the observables $R_i$ as the space components of the vector potential of the electromagnetic field $A_i, i = x, y, z$. Under these circumstances, it is easy to verify that the average value of $R_i$ in a state $\sigma_R$ of the reservoir is zero, i.e.,

$$ Tr [\sigma_R R(t)] = Tr \left[ \sigma_R \hat{R}(t) \right] = 0, $$

(3)

since $R_i$ and $A_i$ are linear combinations of emission and absorption operators of radiation quanta.

Expression (3) is a one-time average. Now consider a two-time average

$$ g(t', t'') = Tr \left[ \sigma_R \hat{R}(t') \hat{R}(t'') \right]. $$

(4)

This two-point function represents an average in a state $\sigma_R$ of a product of two observables taken at two different times $t'$ and $t''$. In fact, such two-point function depends only on
Because, due to the cyclic permutation property of the trace,
\[ g(t', t'') = Tr_R \left[ \sigma_R \hat{R}(t) \hat{R}(0) \right] = g(t). \] (5)

Assuming that \( \sigma_R \) is a stationary state, it follows that \([\sigma_R, H_R] = 0\). As a consequence, we can expand \( \sigma_R \) as
\[ \sigma_R = \sum_{\mu} p_{\mu} |\mu\rangle \langle \mu|, \] (6)
where \( |\mu\rangle \) is an eigenstate of \( H_R \) with eigenvalue \( E_{\mu} \) and \( p_{\mu} \) is a given statistical weight. Note that, when the whole system is in thermal equilibrium at temperature \( T \), we may assume that the equipartition theorem applies and take
\[ p_{\mu} = \frac{\exp[E_{\mu}/k_B T]}{\sum_{\mu} \exp[E_{\mu}/k_B T]}, \] (7)
where \( k_B \) is the Boltzmann constant. Substituting (6) into (5), we obtain
\[
g(\tau) = Tr \sum_{\mu} \left\{ p_{\mu} |\mu\rangle \langle \mu| \hat{R}(\tau) \hat{R}(0) \right\} \\
= \sum_{\mu} p_{\mu} \langle \mu| \hat{R}(\tau) \hat{R}(0) |\mu\rangle \\
= \sum_{\mu, \nu} p_{\mu} R_{\mu\nu}^2 e^{i\omega_{\mu\nu}\tau}, \] (8)
where \( R_{\mu\nu} = \langle \mu| R |\nu\rangle \), \( \omega_{\mu\nu} = \omega_{\mu} - \omega_{\nu} \) and \( \omega_{\mu} = E_{\mu}/\hbar \).

Equation (8) is a superposition of exponentials oscillating at different Bohr frequencies \( \omega_{\mu\nu} \) of \( \mathcal{R} \). Because \( \mathcal{R} \) is a reservoir, it has a very dense ensemble of energy levels and, consequently, a quasi-continuous spectrum of Bohr frequencies, so that the exponentials in (8) interfere destructively once \( \tau \) becomes large enough.

The hypotheses made about \( \mathcal{R} \) are equivalent to assuming that \( \mathcal{R} \) is in a stationary state and exerts on \( \mathcal{S} \) a “force” fluctuating about a zero average value with a short correlation time \( \tau_c \).

A. The Statistical Functions

The function \( g(\tau) \) defined in (4) is not real, even to Hermitian operators \( R \), because, in general, \( \hat{R}(\tau) \) and \( \hat{R}(0) \) do not commute. In order to separate the real and imaginary parts of \( g(\tau) \) we
write

\[ g(\tau) = \frac{1}{2} \langle \{ \tilde{R}(\tau), \tilde{R}(0) \} \rangle_R + \frac{i}{2} \langle \{ \tilde{R}(\tau), \tilde{R}(0)/i \} \rangle_R, \]  

(9)

where \( \langle ., . \rangle_R \) indicates an average on the reservoir state defined by \( \sigma_R \). The first term in (9) corresponds to the symmetric correlation function and the second is related to the linear susceptibilities of the reservoir. The symmetric correlation function of the observable \( R \),

\[ C_R(\tau) = \frac{1}{2} \langle \{ \tilde{R}(t'), \tilde{R}(t'') \} \rangle_R, \]  

(10)

is real and tends to the ordinary correlation function in the classical limit. It gives a physical description of the dynamics of the fluctuations of the observable \( R \) in the state \( \sigma_R \).

The explicit expression for the quantum correlation function defined by (10) is given by the real part of expression (8) for \( g(\tau) \),

\[ C_R(\tau) = \sum_{\mu,\nu} p_\mu |R_{\mu\nu}|^2 \cos(\omega_{\mu\nu}\tau). \]  

(11)

In the frequency space (11) becomes

\[ \hat{C}_R(\omega) = \sum_{\mu,\nu} p_\mu \pi |R_{\mu\nu}|^2 [\delta(\omega + \omega_{\mu\nu}) + \delta(\omega - \omega_{\mu\nu})]. \]  

(12)

The other statistical function is the linear susceptibility \( \chi_R(\tau) \), which characterizes the reservoir response to an external perturbation,

\[ \chi_R(\tau) = \frac{i}{\hbar} \theta(\tau) \langle [\tilde{R}(t'), \tilde{R}(t'')] \rangle_R \]

\[ = \frac{2}{\hbar} \theta(\tau) \text{Im} g(-\tau), \]  

(13)

where \( \theta(\tau) \) is the step function. Using (8),

\[ \chi_R(\tau) = -\frac{2}{\hbar} \sum_{\mu,\nu} p_\mu |R_{\mu\nu}|^2 \theta(\tau) \sin \omega_{\mu\nu}\tau. \]  

(14)

In the frequency space we have

\[ \hat{\chi}_R(\omega) = \hat{\chi}'_R(\omega) + i \hat{\chi}''_R(\omega), \]  

(15)

where

\[ \hat{\chi}'_R(\omega) = \frac{1}{\hbar} \sum_{\mu,\nu} p_\mu |R_{\mu\nu}|^2 \left[ \frac{1}{\omega_{\mu\nu} + \omega} + \frac{1}{\omega_{\mu\nu} - \omega} \right], \]  

\[ \hat{\chi}''_R(\omega) = \frac{\pi}{\hbar} \sum_{\mu,\nu} p_\mu |R_{\mu\nu}|^2 [\delta(\omega_{\mu\nu} + \omega) - \delta(\omega_{\mu\nu} - \omega)]. \]  

(16)
In (16) $P$ denotes the principal value. The above expression characterize, respectively, the response in phase and in quadrature at the frequency $\omega$.

**B. Atomic Transition**

Let $S$ be an atom fixed at the origin $0$ of the coordinate system and $R$ an homogeneous and isotropic broadband radiation field. The radiation density operator is, according to (6), a statistical mixture of the eigenstates $|n_1 \ldots n_k \ldots \rangle$ of $H_R$, representing $n_1$ quanta in the mode $1, \ldots, n_k$ quanta in the mode $k, \ldots$, with a weight $p(n_1 \ldots n_k \ldots)$,

$$\sigma_R = \sum_{\{n_k\}} p(\ldots n_k \ldots) |\ldots n_k \ldots \rangle \langle \ldots n_k \ldots|.$$  \hspace{1cm} (18)

The average number of quanta in the mode $k$ is, therefore, given by

$$\langle n_k \rangle = \sum_{\{n_k\}} n_k p(n_1 \ldots n_k \ldots).$$  \hspace{1cm} (19)

Since it depends only on $\omega_k$, we hereafter use the notation $\langle n(\omega_k) \rangle$.

In order to simplify the problem, let us consider a model where an atom with a single electron, moving in a spherically symmetric potential around the center ($r = 0$). Further, assuming that the electron is inside a volume having small dimensions compared with the wavelength of the incident radiation, we can make use of the long wavelength approximation to all modes whose frequency is below a cutoff $\omega_M$. In this case, the Hamiltonian of the global system is given by (1) and the interaction Hamiltonian between the atom and the field reduces to the expression

$$V = -\sum_i \left( \frac{e}{m} p_i \right) A_i(0),$$  \hspace{1cm} (20)

where $i = x, y, z$.

We can now verify directly that expression (12) for $\hat{C}_R(\omega)$ and (15) for $\hat{\chi}_R(\omega)$ have the same form, i.e.,

$$S_{\pm}(\omega) = \sum_{\mu, \nu} p_{\mu} |R_{\mu\nu}|^2 f^\pm(\omega_{\mu\nu}, \omega).$$  \hspace{1cm} (21)

\footnote{In the long wavelength approximation, the $A^2$ term is associated to a correction for the electron kinetic energy.}
where $f^\pm(\omega_{\mu\nu}, \omega)$ is a function of a given parity with regard to $\omega_{\mu\nu}$: + for $\hat{C}$ and $-$ for $\hat{\chi}$. In another notation,

$$S^\pm_i(\omega) = \sum_{\{n\}} p(n_1, \ldots, n', \ldots) \times$$

$$\times \sum_j |\langle \ldots, n_j, \ldots | A_{ij}(0)| \ldots, n_j + 1, \ldots \rangle|^2 f^\pm(-\omega_j, \omega) +$$

$$+ |\langle \ldots, n_j, \ldots | A_{ij}(0)| \ldots, n_j - 1, \ldots \rangle|^2 f^\pm(\omega_j, \omega)$$

(22)

where $j$ represents a given mode $(k,r)$ and

$$A_{ij}(0) = \left( \frac{\hbar}{2\varepsilon_0 L^3 \omega_j} \right) \epsilon_{ij} [a_j + a_j^\dagger].$$

(23)

Evaluating the matrix element in (22), we obtain, after replacing the sum in the modes by a sum in the polarizations and an integral in $k$,

$$S^\pm_i(\omega) = \int_0^\omega \! d\omega' \left( \frac{\hbar \omega'}{6\pi^2 \varepsilon_0 c^3} \right) \left[ \pm \langle n(\omega') + 1 \rangle + \langle n(\omega') \rangle \right] f^\pm(\omega', \omega),$$

(24)

where the angular part has been already performed.

Choosing $i = x$ in the above expression, we obtain the corresponding correlation function and susceptibilities for the $x$ component of the field:

$$\hat{C}_{xx}^R(\omega) = \frac{1}{3\pi\varepsilon_0 c^3} \int_0^\omega \! d\omega' \hbar \omega' \langle \langle n(\omega') \rangle + 1/2 \rangle \delta(\omega' - \omega) + \delta(\omega' + \omega) \rangle$$

$$= \frac{1}{3\pi\varepsilon_0 c^3} \hbar |\omega| \langle |\omega| \rangle + 1/2 \rangle,$$

(25)

$$\hat{\chi}_{xx}^R(\omega) = \frac{1}{6\pi^2 \varepsilon_0 c^3} \int_0^\omega \! d\omega' \omega' \left[ P \frac{1}{\omega' + \omega} + P \frac{1}{\omega' - \omega} \right],$$

(26)

$$\hat{\chi}''_{xx}^R(\omega) = \frac{-1}{6\pi^2 \varepsilon_0 c^3} \int_0^\omega \! d\omega' \omega' \left[ \delta(\omega' + \omega) - \delta(\omega' - \omega) \right]$$

$$= \frac{1}{6\pi^2 \varepsilon_0 c^3} \omega.$$

(27)

The correlation function for the atomic variable $(e p_x/m)$ and the corresponding susceptibilities, when the atom is in a given state $|a\rangle$, are obtained in an analogous way and are given by

$$\hat{C}_{xx}^{Aa}(\omega) = \sum_b \frac{e^2}{m^2} |\langle a | p_x | b \rangle|^2 \pi \left[ \delta(\omega_{ab} + \omega) + \delta(\omega_{ab} - \omega) \right].$$

(28)
\[
\chi_{AA}^{xx}(\omega) = \sum_{b} \frac{-e^2}{\hbar m^2} |\langle a | p_x | b \rangle|^2 \left[ \mathcal{P} \frac{1}{\omega_{ab} + \omega} + \mathcal{P} \frac{1}{\omega_{ab} - \omega} \right]
\]

(29)

\[
\chi_{AA}^{xx}(\omega) = \sum_{b} \frac{-e^2}{\hbar m^2} |\langle a | p_x | b \rangle|^2 \pi [\delta(\omega_{ab} + \omega) - \delta(\omega_{ab} - \omega)].
\]

(30)

In order to study phenomena at finite temperature, we may substitute the average number of radiation quanta \( \langle n \rangle \), which appear in (24), by the Bose-Einstein distribution function[3]. This procedure is justified by the use of (7) and accounts for the equipartition theorem for the modes of the radiation field.

### III. Thermal Correlation Functions and Susceptibilities

In this section we study the thermal propagator of electromagnetic field in the context of Thermal Field Dynamics (TFD). Our idea is to obtain the statistical functions \( C_R \) and \( \chi_R \), implementing temperature in a criterious way. We start by writing the space components of the electromagnetic potential \( A_i(t) \) as

\[
A_i(t) = A_i^{(+)}(t) + A_i^{(-)}(t),
\]

(31)

where \( A_i^{(+)}(t) \) and \( A_i^{(-)}(t) \) are the components with positive and negative frequency, defined, respectively, as

\[
A_i^{(+)}(t) = \sum_{k,r} \alpha_k \mathbf{e}_r(k) a_k^r e^{-i\omega_k t},
\]

(32)

\[
A_i^{(-)}(t) = \sum_{k,r} \alpha_k \mathbf{e}_r(k) a_k^{r\dagger} e^{i\omega_k t}
\]

(33)

with

\[
\alpha_k = \left( \frac{\hbar}{2\varepsilon_0 L^3 \omega_k} \right)^{1/2}.
\]

(34)

In TFD we double the field degrees of freedom introducing the tilde conjugated of \( A_i(t) \) [5][6]. Using the thermal doublet notation[6][7], we obtain

\[
\mathbf{A}_i(t) = \left( \begin{array}{c} A_i(t) \\ \tilde{A}_i(t) \end{array} \right) \quad \mathbf{\tilde{A}}_i(t) = ( A_i(t), - \tilde{A}_i(t) )
\]

(35)

\footnote{As in the last section, we assume that the atom is at rest at the origin of the coordinate system (\( r = 0 \)) and that we are using the dipole approximation.}

8
where (−) denotes the transposed and

\[
A_i(t) = \sum_{k,r} \alpha_k e^r(k) (a_k^r e^{-i\omega_k t} + a_k^r e^{i\omega_k t})
\]

\[
= A_i^{(+)}(t) + A_i^{(-)}(t),
\]

(36)

\[
\tilde{A}_i(t) = \sum_{k,r} \alpha_k e^r(k) (\tilde{a}_k^r e^{i\omega_k t} + \tilde{a}_k^r e^{-i\omega_k t})
\]

\[
= \tilde{A}_i^{(+)}(t) + \tilde{A}_i^{(-)}(t).
\]

(37)

By construction, both fields \(A_i\) and \(\tilde{A}_i\) are independent; the corresponding absorption and emission operators satisfy the algebra[6]

\[
[a_k^r, a_{k'}^{s\dagger}] = [\tilde{a}_k^r, \tilde{a}_{k'}^{s\dagger}] = \delta_{k,k'} \delta_{r,s}.
\]

(38)

At zero temperature, the vacuum state is given by the direct product \(|0\rangle_A \otimes |0\rangle_{\tilde{A}} = |0\rangle\). Using (40), it follows that

\[
A_i^{(+)} |0\rangle = 0, \quad \tilde{A}_i^{(+)} |0\rangle = 0.
\]

(39)

In order to find the thermal propagator associated with the statistical functions, we must calculate the commutator

\[
[A_i(t'), \tilde{A}_j(t'')]_{\mu\nu} = \Delta_{ij}^{\mu\nu}(t' - t'')
\]

(40)

where \(\mu, \nu = 1,2\) and \(i,j = x,y,z\). The anti-diagonal components of the above quantity are identically zero when we calculate their expectation value in the \(|0\rangle\) state. The component \(\mu = \nu = 1\) can be written as

\[
\Delta_{ij}^{11}(t' - t'') = \Delta_{ij}^{11}(+) (t' - t'') + \Delta_{ij}^{11}(+) (t' - t''),
\]

(41)

where

\[
\Delta_{ij}^{11}(+) (t' - t'') = [A_i^{(+)}(t'), A_j^{(-)}(t'')],
\]

(42)

\[
\Delta_{ij}^{11}(+) (t' - t'') = [A_i^{(-)}(t'), A_j^{(+)}(t'')].
\]

(43)

Now, using (38), (39) and (40), we calculate explicitly these comutators,

\[
\Delta_{ij}^{11}(+) (t) = \sum_{k,r} \alpha_k^2 e^r(k) e^r(k) e^{-i\omega_k t'},
\]

(44)
\[ \Delta_{ij}^{11(-)}(\tau) = -\sum_{k,r} \alpha_k^2 \epsilon_i^r(k) \epsilon_j^r(k) e^{i\omega_k \tau}, \] (45)

where \( \tau = t' - t'' \). From (44) and (45), we can define two functionals:

\[ \Delta_{ij}^{11}(|\text{ret}|)(\tau) \doteq \theta(\tau)\Delta_{ij}^{11(+)}(\tau) + \theta(\tau)\Delta_{ij}^{11(-)}(\tau) = \Delta_{ij}^{11}(|\text{ret}|)(*)(\tau) + \Delta_{ij}^{11}(|\text{ret}|)(-)(\tau), \] (46)

and

\[ \Delta_{ij}^{11}(1)(\tau) \doteq \Delta_{ij}^{11(+)}(\tau) - \Delta_{ij}^{11(-)}(\tau). \] (47)

By taking the Fourier transform of (46) and (47) we obtain, respectively,

\[ \Delta_{ij}^{11}(|\text{ret}|)(\omega) = \sum_{k,r} \alpha_k^2 \epsilon_i^r(k) \epsilon_j^r(k) \left[ \left( \frac{i}{\omega - \omega_k - i\epsilon} \right) - \left( \frac{i}{\omega + \omega_k + i\epsilon} \right) \right]. \] (48)

\[ \Delta_{ij}^{11}(1)(\omega) = \sum_{k,r} \alpha_k^2 \epsilon_i^r(k) \epsilon_j^r(k) \pi [\delta(\omega + \omega_k) + \delta(\omega - \omega_k)]. \] (49)

Adopting the same procedure, we can extend the above calculation to the component \( \mu = \nu = 2 \). As a result, we obtain

\[ \Delta_{ij}^{22}(|\text{ret}|)(\omega) = \sum_{k,r} \alpha_k^2 \epsilon_i^r(k) \epsilon_j^r(k) \left[ \left( \frac{i}{\omega - \omega_k - i\epsilon} \right) - \left( \frac{i}{\omega + \omega_k + i\epsilon} \right) \right], \] (50)

\[ \Delta_{ij}^{22}(1)(\omega) = -\sum_{k,r} \alpha_k^2 \epsilon_i^r(k) \epsilon_j^r(k) \pi [\delta(\omega + \omega_k) + \delta(\omega - \omega_k)]. \] (51)

We may write expression (48) and (50) in a more compact notation, i.e.,

\[ \Delta_{ij}^{(\text{ret})}(\omega) = \sum_{k,r} \alpha_k^2 \epsilon_i^r(k) \epsilon_j^r(k) \left\{ \frac{i}{k_0 - \omega_k + i\tau_3\epsilon} - \frac{i}{k_0 + \omega_k + i\tau_3\epsilon} \right\} \] (52)

and, in the same way, we write (49) and (51) as

\[ \Delta_{ij}^{(1)}(\omega) = -\sum_{k,r} \alpha_k^2 \epsilon_i^r(k) \epsilon_j^r(k) \pi \tau_3 [\delta(\omega + \omega_k) + \delta(\omega - \omega_k)], \] (53)

where, in the last two expressions,

\[ \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \] (54)

In TFD, it is known that the propagator at zero temperature is related to the one calculated in the thermal vacuum through a Bogoliubov transformation[8]. Applying this result to (52)
and (53), we obtain, respectively,

\[
\Delta_{ij (\text{ret})}^{\mu \nu \beta}(\omega) = \{ B_k^{-1}(\beta) \Delta_{ij (\text{ret})}(\omega) B_k(\beta) \}^{\mu \nu},
\]

\[
\Delta_{ij (1)}^{\mu \nu \beta}(\omega) = \{ B_k^{-1}(\beta) \Delta_{ij (1)}(\omega) B_k(\beta) \}^{\mu \nu},
\]

where \( B_k(\beta) \) is given by

\[
B_k(\beta) = (1 - n_k)^{1/2} \left( -f_k^{1-\alpha} - f_k^{\alpha} \right),
\]

with \( \alpha = 1/2, f_k = \exp[-\hbar \omega_k \beta] \) and

\[
n_k = \frac{1}{f_k^{-1} - 1} = \frac{1}{e^{\hbar \omega_k \beta} - 1},
\]

\((\beta = 1/kT, \text{where } k \text{ is the Boltzmann constant and } T \text{ the equilibrium temperature})\). The \( \mu = \nu = 1 \) component of (55) is found to be

\[
\Delta_{11}^{11 \beta (\text{ret})}(\omega) = -i \sum_{k,r} \alpha_k^2 e_i^r(k) e_j^r(k) \left\{ \mathcal{P} \frac{1}{\omega_k - \omega} + \mathcal{P} \frac{1}{\omega_k + \omega} + i \pi \left[ \delta(\omega_k - \omega) - \delta(\omega_k + \omega) \right] (1 + 2n(\omega_k)) \right\},
\]

and, from (56),

\[
\Delta_{11}^{11 \beta (1)}(\omega) = \sum_{k,r} \alpha_k^2 e_i^r(k) e_j^r(k) \pi \left[ \delta(\omega - \omega_k) + \delta(\omega + \omega_k) \right] (1 + 2n(\omega_k)).
\]

Now, we are in position to defining the thermal correlation function and susceptibilities,

\[
C_{ij}^{\beta}(\omega) \equiv \Delta_{11}^{11 \beta (\text{ret})}(\omega),
\]

and

\[
\chi_{ij}^{\beta}(\omega) \equiv \frac{i}{\hbar} \Delta_{11}^{11 \beta (1)}(\omega),
\]

where

\[
\chi_{ij}^{\beta}(\omega) = \chi_{ij}^{\beta}(\omega) + i \chi_{ij}^{\prime \beta}(\omega),
\]

\[
\chi_{ij}^{\prime \beta}(\omega) = \frac{1}{\hbar} \sum_{k,r} \alpha_k^2 e_i^r(k) e_j^r(k) \left[ \mathcal{P} \frac{1}{\omega_k - \omega} + \mathcal{P} \frac{1}{\omega_k + \omega} \right],
\]

\[
\chi_{ij}^{\prime \prime \beta}(\omega) = -\frac{1}{\hbar} \sum_{k,r} \alpha_k^2 e_i^r(k) e_j^r(k) \pi (1 + 2n(\omega_k)) \left[ \delta(\omega_k + \omega) - \delta(\omega_k - \omega) \right].
\]
Choosing \( i = j = x \) and substituting the summation over modes by a polarization sum and an integral in \( k \), we obtain

\[
C^{\beta}_{xx}(\omega) = \frac{1}{3\pi\varepsilon_0 c^3} \int_0^\infty d\omega' \ h(\omega') (n(\omega') + 1/2) \left[ \delta(\omega - \omega') + \delta(\omega' + \omega) \right] \\
= \frac{1}{3\pi\varepsilon_0 c^3} h|\omega|(n(|\omega|) + 1/2)
\]

(66)

\[
\chi^{\prime\prime \beta}_{xx}(\omega) = \frac{1}{6\pi\varepsilon_0 c^3} \int_0^\infty d\omega' \omega' \left[ P \frac{1}{\omega' - \omega} + P \frac{1}{\omega' + \omega} \right]
\]

(67)

\[
\chi^{\prime \beta}_{xx}(\omega) = \frac{-1}{6\pi\varepsilon_0 c^3} \int_0^\infty d\omega' \omega' (2n(\omega') + 1) \left[ \delta(\omega' + \omega) - \delta(\omega' - \omega) \right] \\
= \frac{1}{3\pi\varepsilon_0 c^3} \omega (n(|\omega|) + 1/2).
\]

(68)

IV. Energy Exchange

In order to draw a conclusion we must compare the expressions for the statistical functions derived in sections II and III. It is clear that only the dissipative part of the susceptibilities, expression (28) and (67), are different. We must remember that \( g(\tau) \), which is the starting point in the construction of statistical functions, is defined in terms of free field eigenstates, leading to expression (29), which does not depend on the average number of photons. It can be show\[9\] that such a difference does not affect calculations related to phenomena like the Lamb shift and the AC Stark effect. Hence, the discrepancy acquires an important physical meaning if, for example, we are interested in the energy exchange between \( S \) and \( R \). To see this, we analyze the variation rate of the mean atomic energy when, initially, the system is in a given state \( a \),

\[
\frac{d}{dt} \langle H_s \rangle_a = \sum_b (E_b - E_a) \Gamma_{a\rightarrow b}.
\]

(69)

In (69), \( \Gamma_{a\rightarrow b} \) represents the transition rate between the levels \( a \) and \( b \) due to the interaction with the reservoir. It is shown in reference [3] that (69) can be written in terms of the statistical functions, giving

\[
\frac{d}{dt} \langle H_s \rangle_a = \dot{Q}^{fr} + \dot{Q}^{ir},
\]

(70)

where

\[
\dot{Q}^{fr} = \int \frac{d\omega}{2\pi} \omega \hat{C}_R(\omega) \hat{\chi}^{\prime \prime}_{AA}(\omega),
\]

(71)
\[ \dot{Q}^{rr} = -\int \frac{d\omega}{2\pi} \omega \hat{\chi}''(\omega) \dot{C}_A(\omega). \] (72)

The last two expressions have a clear meaning: (71) is associated with the absorption of energy by the system when it is affected by reservoir fluctuations and (72) is related to the damping of the atomic motion caused by the reservoir.

Using expressions (27), (32) and (71) and taking into account the summation over \(x, y\) and \(z\), we decompose \(\dot{Q}^{fr}\) in

\[ \dot{Q}^{fr} = \dot{Q}^{fr'} + \dot{Q}^{fv}, \] (73)

where

\[ \dot{Q}^{fr'} = \sum_b (E_b - E_a) \Gamma_{ab}^{sp} \langle n(|\omega_{ab}|) \rangle, \] (74)

\[ \dot{Q}^{fv} = \sum_b (E_b - E_a) \frac{\Gamma_{ab}^{sp}}{2}, \] (75)

and

\[ \Gamma_{ab}^{sp} = \frac{e^2 |\langle a | p | b \rangle|^2 |\omega_{ab}|}{3\pi\varepsilon_0 \hbar m^2 c^3}, \] (76)

the rate of spontaneous emission relative to transition between the levels \(a\) and \(b\).

Just as a random classical perturbation, the fluctuations of the radiation field transfer populations from level \(a\) to a higher or lower level \(b\). From (74), we see that the incident radiation contributes to processes with a factor proportional to \(\langle n(|\omega_{ab}|) \rangle\) per mode and, from (75), we see that the vacuum fluctuations contribute proportionally to \(1/2\).

The quantity \(\dot{Q}^{rr}\) is calculated in the same way, from (29), (30) and (72). As a result, we find

\[ \dot{Q}^{rr} = \frac{-1}{6\pi\varepsilon_0 c^3} \sum_b \frac{e^2}{m^2} |\langle a | p | b \rangle|^2 \omega_{ab}^2 \] (77)

\[ = \sum_b -|E_b - E_a| \frac{\Gamma_{ab}^{sp}}{2}. \] (78)

Considering \(E_a < E_b\), it follows that \(|E_b - E_a| = (E_a - E_b)\) and

\[ \dot{Q}^{rr} + \dot{Q}^{fv} = 0. \] (79)

Further, if the system is in thermal equilibrium at temperature \(T\), we note that

\[ \frac{d}{dt} \langle H_A \rangle_a \neq 0 \] (80)
where we have substituted (73) and (78) in (70).

On the other hand, if we use expression (66) and (68) instead (27) and (29), we obtain, after some manipulation,

\[ \dot{Q}_{fr}^\alpha = \sum_b (E_b - E_a) \Gamma_{ab}^{sp} \{ \langle n(|\omega_{ab}|) \rangle + 1/2 \} \]  (81)

and

\[ \dot{Q}_{rr}^\alpha = - \sum_b (E_b - E_a) \Gamma_{ab}^{sp} \{ \langle n(|\omega_{ab}|) \rangle + 1/2 \} , \]  (82)

which gives

\[ \frac{d}{dt} \langle H_A \rangle_a = 0. \]  (83)

Expression (83) shows that, in thermal equilibrium, the stability of the atomic system is preserved, as we should expect. Note that, for \( T = 0 \), this stability is still holds, since the effects of radiation reaction, \( \dot{Q}_{rr}^\alpha \) (resp. second term in (82)), are cancelled by the thermal vacuum fluctuation, \( \dot{Q}_{fr}^\alpha \) (resp. second term in (81)).

V. Concluding Remarks

We have discussed the issue of temperature implementation in DDC and argued that if we naively apply the equipartition theorem, essential information may be lost. This occurs because we have neglected essential physical requirements. On the other hand, in the TFD approach the temperature is introduced at an early stage, in the eigenstates of the number operator for the radiation field, through the modification of the vacuum state[6]. In fact, the eigenstates of the number operator do not satisfy the same dynamics of the original (free) field and, in this case, the Fock space of asymptotic states of the electromagnetic field must be constructed taking into account the correlations between the reservoir \( \mathcal{R} \) and an image reservoir \( \bar{\mathcal{R}} \), simulating the effects of thermal correlations[10]. In quantum electrodynamics we have an analogue situation, where a consistent construction of particle eigenstates must take into account the long range Coulomb interaction, which modifies the dynamics of these particle states in the asymptotic region[11].
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References


