Effect of nuclear structure on Type Ia supernova nucleosynthesis

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The relationship among nuclear structure, the weak processes in nuclei, and astrophysics becomes quite apparent in supernova explosion and nucleosynthesis studies. In this brief article, I report on progress made in the last few years on calculating electron capture and beta-decay rates in iron-group nuclei. I also report on applications of these rates to Type-Ia nucleosynthesis studies.

1. Introduction

Nuclear physics plays an important role in stellar evolution. Fusion, proton capture, and neutron capture are all examples of the importance of the electromagnetic interaction in creating energy that powers stars. Weak interactions in nuclei, including electron capture and $\beta$-decay, play an important role in the evolution of both Type Ia and II supernovae and their nucleosynthesis. In this brief article, I will discuss recent progress in understanding the nuclear physics involved in Type Ia explosion mechanisms. I focus on progress made in accurately calculating electron-capture and $\beta$-decay rates in iron group nuclei.

In order to understand weak processes in nuclei, it becomes necessary to properly describe the nuclear structure of the relevant systems. Short of a complete solution to the many-body problem, the shell model is widely acknowledged to be the appropriate theoretical tool to describe both ground- and excited-state properties of nuclei. The shell-model requires as input a reasonable valence model space and a reliable effective two-body interaction that reproduces known properties of nuclei within the given model space. Such interactions exist for $p$-, $sd$-, and $pf$-shell nuclei, and are under development for heavier or more neutron-rich systems. In this paper, I discuss calculations made in the $pf$-shell using the two-body effective interaction KB3 [1], or slight modification thereof.

The very nature of the quantum many-body problem for fermions – its inherent computational difficulty due to the necessary antisymmetrization of the many-body wavefunction – requires significant computational capability and expertise. This is particularly true of approaches that are trying to treat the many-body problem exactly, or in extremely large shell-model spaces. Standard shell-model diagonalization techniques have recently progressed into the $pf$-shell [2] while other techniques based on Monte Carlo algorithms have also been quite successful in recent years [3].

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2. Electron capture in Type Ia supernovae

Electron capture on nuclei takes place in high density matter where the Fermi energy of a degenerate electron gas is sufficiently large to overcome the energy thresholds given by the negative Q-values of such reactions. Type Ia supernovae appear to be thermonuclear explosions of white dwarfs in binary systems. The high accretion rates of the white dwarf permits relatively stable H- and He-shell burning and leads to a growing C/O white dwarf. When the white dwarf mass is sufficiently close to the Chandrasekhar mass, gravitational contraction sets in, and the central density becomes high enough to ignite carbon fusion. The environment of a degenerate electron gas provides a pressure that depends only on the density; therefore, the initial heat generation does not lead to the pressure increase and expansion that would lead to a controlled and stable burning. Instead, a thermonuclear runaway occurs. The burning front propagates through the whole star, causing complete disruption without a remnant.

The high Fermi energy of the degenerate electron gas in the white dwarf leads to efficient electron capture on nuclei in the high density burning regions and reduces \( Y_e = \langle Z/A \rangle \), the electron fraction, or equivalently, the average proton-to-nucleon ratio, during explosive burning in the center. This important factor controls the isotopic composition ejected from such explosions. Thus one test of theoretical models is whether they reproduce the observed isotopic compositions. If the central density exceeds a critical value, electron capture can cause a dramatic reduction in the pressure of degenerate electrons and can therefore induce collapse of the white dwarf. Thus, electron capture on intermediate-mass and Fe-group nuclei plays a crucial role for the burning front propagation in Type Ia supernovae. Beta-decay is also relevant when the \( Y_e \) values correspond to \( Z/A \) ratios of nuclei that are more neutron-rich than stability.

Electron capture on nuclei in the energy regime relevant in Type Ia environments occurs primarily through Gamow-Teller transitions. These transitions connect an initial nuclear state to final states through a spin-isospin operator \( \vec{\sigma} \tau_z \). The Gamow-Teller strength distributions enter directly into the rate calculations for electron-capture by folding with the electron energy distribution in the stellar environment of interest.

The centroid of the Gamow-Teller strength distribution in nuclei resides at several MeV of excitation energy in the daughter systems. From the mid-1980s, this distribution was modeled as a single matrix element at a given excitation energy, carrying the total strength calculated from the non-interacting shell model \([4]\). These electron capture and \( \beta \)-decay rates, known as Fuller, Fowler, and Newman (FFN) rates, are commonly used in astrophysical calculations. About six years ago, research using the shell-model Monte Carlo (SMMC) technique indicated that two ingredients were necessary to obtain Gamow-Teller results that were compatible with experimental data: the complete \( 0 \hbar \omega \) model space needed to be used, and calculations had to be performed using a reliable interaction for the model space of interest. If these two conditions are met, then one can reliably reproduce experimental total Gamow-Teller strengths to within a constant factor \([5]\) and the strength distributions \([6]\). Systematic deviations from the FFN parameterization of the Gamow-Teller strength were observed in the SMMC calculations. In particular, for even-even nuclei the strength was found to be lower than estimated by FFN, and in odd-A and odd-odd nuclei the centroid of the strength was significantly higher than estimated.
Recently, stellar electron capture and $\beta$-decay rates were calculated by shell-model diagonalization using a slightly improved version of the $pf$-shell two-body interaction for all $pf$-shell nuclei [2] and confirm the trend already observed in SMMC studies. Systematic deviations from the Gamow-Teller parameterization assumed by the FFN compilation lead to significantly smaller electron capture rates on odd-A nuclei and odd-odd nuclei, and slightly smaller capture rates on even-even nuclei than FFN.

Using Gamow-Teller strengths generated from either SMMC calculations or from the shell-model diagonalization, we implemented new electron capture and beta-decay rates into Type Ia evolution codes [8]. We compared the FFN and shell-model rates for several typical nuclei and, based on this comparison, multiplied the FFN electron capture rates by 0.2 (for even-even nuclei), 0.1 (odd-A), and 0.04 (odd-odd), while the FFN $\beta$-decay rates were scaled by 0.05 (even-even), 0.025 (odd-A), and 1.7 (odd-odd).

We then used these modified rates in Type Ia explosion calculations. The effects of the new rates on nucleosynthesis produced by the explosions are shown in Fig. 1. The figure shows the ratio of abundances, relative to solar, as predicted for the WS15 model. Electron capture and $\beta$-decay rates were modeled with FFN rates (left panel) and with the modified FFN rates (SMFA, right panel). We immediately see that the over-production of $^{54}$Cr and $^{54}$Fe are cured by an inclusion of the interacting shell-model rates.

3. Conclusion

While several astrophysical parameters are also involved in calculations of Type Ia explosion mechanisms, the nuclear structure component of these reactions is now much more robust than it has been in the past. This eliminates one set of parameterization and reduces the uncertainty of the Type Ia nucleosynthesis and explosion models.

Figure 1. The ratio of abundances, relative to solar, as predicted by the WS15 model. The lines connect isotopes of one element, and the ordinate is normalized to $^{56}$Fe. The left panel shows results for the nucleosynthesis calculation performed with FFN weak rates, while the right panel shows results using scaled shell-model rates.
In the case of Type II explosions, similar and important research is required. While some of this research is being performed presently, it will be necessary in the near future to update all nuclear reaction rates (electron-capture, beta-decay, neutrino scattering, nuclear matter opacity to neutrinos) that enter the various explosion models and pre-collapse progenitors. Work in this direction is under way [9]. Since very neutron-rich nuclei become important during Type II collapse, it will become very important to go beyond the $pf$-shell.

While increasing computational power has moved us forward, it is important to realize that the shell-model problem scales somewhat like $\exp(N)$ where $N$ is the number of valence particles, while single processor speed on high-performance machines scales as $1.8^Y$ and memory per processor scales as $1.3^Y$, where $Y$ is years. In 1971, Whitehead and Watt [10] performed the first shell-model calculations for $^{24}\text{Mg}$ in the $sd$-shell. Today we can reach nuclei near $^{52}\text{Fe}$ in a full $pf$ shell-model diagonalization using the $M$-scheme technique. A simple analysis of the required memory and time necessary to perform the calculation indicates that the size of problem in a $0h\omega$ space that we can completely tackle with diagonalization is keeping pace with single-processor performance and memory. One near-term need in our field is the development of shell-model codes that scale in distributed-memory environments. When this transformation occurs, we will be able to tackle much more ambitious problems in very neutron-rich systems.

REFERENCES