Helioseismology and solar neutrinos: an update

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We review recent advances concerning helioseismology, solar models and solar neutrinos. Particularly we address the following points: i) helioseismic tests of recent SSMs; ii) predictions of the Beryllium neutrino flux based on helioseismology; iii) helioseismic tests regarding the screening of nuclear reactions in the Sun.

1. Introduction

Helioseismology has added important data on the solar structure which provide severe constraint and tests of SSM calculations. For instance, helioseismology accurately determines the depth of the convective zone \( R_b \) and the photospheric helium abundance \( Y_{ph} \). With these additional constraints there are essentially no free parameters for SSM builders.

In this paper we review recent advances concerning helioseismology, solar models and solar neutrinos. Particularly we shall address the following points: i) helioseismic tests of recent SSMs; ii) prediction of the Beryllium neutrino flux based on helioseismology; iii) helioseismic tests concerning the screening of nuclear reactions in the Sun.

2. A summary of helioseismic determinations of solar properties

While we refer to e.g. [1] for a review of the method and to [2] for the data, we recall that by measurements of thousands of solar frequencies (p-modes) with a typical accuracy of \( \Delta \nu / \nu \approx 10^{-4} \), one derives:

- properties of the present convective envelope, such as depth and helium abundance:

\[
\begin{align*}
R_b &= 0.711(1 \pm 0.4\%) R_\odot \\
Y_{ph} &= 0.249(1 \pm 4\%)
\end{align*}
\]  

(1,2)

The quoted errors, mostly resulting from systematic uncertainties in the inversion technique, have been estimated conservatively by adding linearly all known individual uncertainties, see [1]. If uncertainties are added in quadrature, the global error is about one third of that indicated in eqs. (1,2), see again [1]. This latter procedure was also used by Bahcall et al. [3] with similar results. This yields the so called “1σ” errors. We shall refer to the conservative estimate as the “3σ” determination. We remark however that this terminology is part of a slang, and it does not correspond to well defined confidence levels, as one has to combine several essentially systematic errors.

- sound speed profile. By inversion of helioseismic data one can determine the sound speed profile, \( u = P/\rho \), as derived from helioseismic observations, from [1].
speed in the solar interior. This analysis can be performed in terms of either the isothermal sound speed squared, \( u = P/\rho \), or in terms of the adiabatic sound speed squared \( c^2 = \partial P/\partial \rho \)\textsubscript{adi} = \( \gamma P/\rho \), as the coefficient \( \gamma = \partial \log P/\partial \log \rho \)\textsubscript{adiab} is extremely well determined by means of the equation of state of the stellar plasma.

In fig. 1 we show the helioseismic value of \( u \) as a function of the radial coordinate \( R/R_\odot \). The typical “3σ” errors are of order ±0.4% in the intermediate solar region, \( R/R_\odot \approx 0.4 \), and increase up to ±2% near the solar center.

3. Helioseismic tests of recent Standard Solar Models

Fig. 2 compares the results of six different observational determinations of the sound speed with the results of the best solar model of ref. [4], hereafter BP2000. This figure suggests several comments:

- i) Different measurements yield quite consistent value of the sound speed, to the level 0.1%;
- ii) The solar model BP2000 is in agreement with helioseismic data to better than 0.5% at any depth in the sun. We remark that also the properties of the convective envelope predicted by BP2000 (\( R_b/R_\odot = 0.714, Y_{ph} = 0.244 \)) are in agreement with helioseismic determinations, see eqs. (1,2).
- iii) On the other hand, the predicted sound speed differs from the helioseismic determination at the level of 0.3-0.4% just below the convective envelope.

Figure 2. Fractional difference between the sound speeds calculated for the BP2000 model and the sound speeds in six helioseismological experiments [2], from [4].

Figure 3. Comparison between BP2000 [4] and MDI helioseismic data for \( u = P/\rho \). The “1σ” and “3σ” helioseismic uncertainties [1] correspond to the dark and light areas respectively.

Concerning this last point, we remark that the difference is however at the level of the “3σ” uncertainty of the helioseismic determination, as it is shown in Fig. 3. Nevertheless it can be taken as an indication of some imperfection of the SSM. A marginally better agreement can be obtained in solar models including mixing induced by rotation, see e.g. [4] and in models including macroscopic transport term, see e.g. [5].

In summary all this means that SSM predictions are accurate to the level of one per cent or
better, although there are indications of some deficiencies at the level of per mille.

Concerning neutrino physics, the properties of the central solar region, where nuclear reactions are efficient, are relevant. Specifically, Boron and Beryllium neutrinos are produced very near to the solar center with maximal production rates respectively at \( R_B = 0.04 R_\odot \) and \( R_{Be} = 0.06 R_\odot \). Since the p-modes which are observed do not propagate (actually are exponentially damped) so deeply in the sun the question often arises if present helioseismic data can determine the sound speed in region of Beryllium and Boron production. From an extensive analysis of the inversion method and of data available at that time we already concluded in [1] that \( u(R \approx 0) \) is determined with a “1σ” accuracy of 1%. This point was further elucidated in [6] where a simplified analysis was presented in order to produce convincing evidence that helioseismology fixes the sound speed near the solar center with such an accuracy.

4. Helioseismology and the beryllium neutrino flux

As well known, the production of neutrinos from \( ^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu_e \) is an important item in the context of the so called “solar neutrino puzzle” for several reasons:

i) The result of Gallium experiments would be (partially) consistent with the hypothesis of standard neutrinos if Beryllium neutrino (\( \nu_{Be} \)) production rate, \( L(\nu_{Be}) \), is suppressed by an order of magnitude with respect to the prediction of the Standard Solar Model (SSM), see e.g. [7,8].

ii) If one accepts neutrino oscillations as the solution of the solar neutrino puzzle, the determination of the neutrino mass matrix depends however on the predicted value of \( L(\nu_{Be}) \).

iii) Direct experiments aiming to the determination of the \( \nu_{Be} \) signal are in preparation and of course the interpretation of their result will rely on \( L(\nu_{Be}) \) [9,10].

The SSM prediction for Beryllium neutrinos [4],

\[
L(\nu_{Be})^{SSM} = 1.3 \times 10^{37} \text{ (1 \pm 9\%)} \text{ s}^{-1} \text{ (1\sigma)} \quad (3)
\]
is very robust, much more than that of Boron neutrinos. However any additional information which does not rely on SSM are clearly welcome.

In ref. [11] a lower limit on the Beryllium neutrino flux on earth was found, \( \Phi(Be)_{min} = 1 \times 10^3 \text{ cm}^{-2} \text{ s}^{-1} \), in the absence of oscillations, by using helioseismic data, the B-neutrino flux measured by Superkamiokande and the hydrogen abundance at the solar center \( X_c \) predicted by Standard Solar Model (SSM) calculations. We remark that this abundance is the only result of SSMs needed for getting \( \Phi(Be)_{min} \). Lower bounds for the Gallium signal, \( G_{min} = (91 \pm 3) \text{ SNU} \), and for the Chlorine signal, \( C_{min} = (3.24 \pm 0.14) \text{ SNU} \), in the absence of oscillations have also been derived. They are about 3σ above the corresponding experimental values, \( G_{exp} = (75 \pm 5) \text{ SNU} \) [12,13] and \( C_{exp} = (2.56 \pm 0.22) \text{ SNU} \) [14]. We remark that predictions for \( X_c \) are very stable among different (standard and non standard) solar models, see [11]. In fact \( X_c \) is essentially an indicator of how much hydrogen has been burnt so far. The stability of \( X_c \) corresponds to the fact that any solar model has to account for the same present and time integrated solar luminosity.

In ref. [15] a step forward was made; a determination of \( L(\nu_{Be}) \) was directly obtained by means of helioseismology without using additional assumptions. The basic idea is the following. In the SSM the pp-II termination (which is the Beryllium producing branch) accounts for an appreciable fraction of the \(^4\text{He} \) produced near the solar center. If Beryllium production is suppressed – now and in the past – less \(^4\text{He} \) would have been produced near the center. As a consequence, the molecular weight should decrease and the sound speed should increase in this region. In other words, we know that SSM calculations are in good agreement with helioseismology and we can expect that this agreement is spoiled if \( \nu_{Be} \) production is artificially changed.

In order to test this idea, solar models with artificially changed \( \nu_{Be} \) production were constructed. An efficient way for producing arbitrary variations of \( L(\nu_{Be}) \) is to vary the zero energy astrophysical S-factor (\( S_{34} \)) for the reaction \( ^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma \), since, as well known (see e.g. [16,8,17]), the production rate \( L(\nu_{Be}) \) is directly
Figure 4. The photospheric helium abundance $Y_{ph}$ and the depth of the convective envelope $R_b$ in solar models with the indicated values of $s = S_{34}/S_{34}^{SSM}$. The error bars correspond to the “1σ” helioseismic uncertainties, from ref. [1].

proportional to $S_{34}$:

$$L(\nu_{Be})/L(\nu_{Be})^{SSM} \simeq S_{34}/S_{34}^{SSM}.$$ \hspace{1cm} (4)

We remind that $S_{34}$ is measured with an accuracy of about ten per cent, ($S_{34}^{SSM} = 0.54 \pm 0.05$ KeVb [18]). It has been varied, however, well beyond its experimental uncertainty in order to simulate several effects which have been claimed to suppress $\nu_{Be}$ production, e.g. hypothetical plasma effects which could alter nuclear reaction rates. The resulting values for the quantities which can be tested by means of helioseismology are shown in Fig. 4 and Fig. 5, for several assumed values of the parameter $s = S_{34}/S_{34}^{SSM}$.

The photospheric helium abundance $Y_{ph}$ is weekly sensitive to the value of $S_{34}$ whereas the depth of the convective envelope $R_b$ is altered by more than 1σ if $S_{34}$ is reduced below one half of the SSM value, i.e.:

$$L(\nu_{Be}) = 1.3 \cdot 10^{37} (1 \pm 50\%) \text{ s}^{-1} \text{ at } 1\sigma.$$ \hspace{1cm} (5)

As previously mentioned, one expects that the sound speed is altered, particularly near the solar center. In fact, stringent constraints arise from the sound speed profile, particularly near $R \simeq 0.2R_\odot$. The requirement that the sound speed is not changed by more than 1σ yields:

$$L(\nu_{Be}) = 1.3 \cdot 10^{37} (1 \pm 0.25) \text{ s}^{-1} \text{ at } 1\sigma.$$ \hspace{1cm} (6)

In conclusion, helioseismology directly confirms the production rate of Beryllium neutrinos as predicted by SSMs to within $\pm 25\%$ (1σ error). This constraint is somewhat weaker than that estimated from uncertainties of the SSM, see Eq.3, however it relies on direct observational data.

5. Helioseismology and screening of nuclear reactions in the sun

The study of screened nuclear reaction rates was started with the pioneering work of Salpeter [19], who discussed both the extreme cases of ”weak” and ”strong” screening, providing suitable expressions for the screening factors

$$f_{ij} = \langle \sigma v \rangle_{ij,\text{plasma}} / \langle \sigma v \rangle_{ij,\text{bare}}.$$ \hspace{1cm} (7)

The solar core is not far from the weak screening case, however it does not satisfy the usual conditions under which the weak screening approximation holds. This is the reason why the possibilities of large deviations from weak screening have been investigated by several authors, especially as an attempt to avoid or mitigate the ”solar neutrino problem”, see e.g. [20] and references therein.

We remind that solar models are built by using stellar evolutionary codes which include specific expressions for the nuclear reaction rates. If
one uses different formulas for the screening factors $f_{ij}$ one obtains different solar models. On the other hand, helioseismology provides precise information on the sound speed profile and on the properties of the convective envelope which have to be reproduced by the correct solar model. Therefore helioseismology can potentially provide a test of screening models.

In ref. [21], solar models using different screening assumptions were constructed and compared with helioseismic data. Specifically, four different model were considered:

i) The weak screening approximation (WES). The screening factors $f_{ij}$ are given by:

$$\ln f_{WES}^{ij} = Z_i Z_j e^2 / (a_D kT)$$  \hspace{1cm} (8)

where $Z_i, Z_j$ are the charges of the interacting nuclei, $T$ is the temperature and $a_D$ is the Debye radius. As clear from equation above, the screening factors are always larger than unity, i.e. the plasma provides enhancement of the thermonuclear reaction rates.

ii) The Mitler result [22] (MIT), obtained with an analytical method which goes beyond the linearized approach and which correctly reproduces both the limits of weak and strong screening.

iii) Neglect completely any screening effect (NOS), i.e. nuclear reactions occur with rates $\langle \sigma v \rangle_{\text{bare}}$. This case is considered in connection with the suggestions that screening can be much smaller than Salpeter’s estimate, see e.g. [23].

iv) The Tsytovich model (TSY) [24,25], which provides a decrease of all the thermonuclear reaction rates with respect to the case of bare nuclei.

In Table 1 the screening factors at the solar center for the various models are shown. One sees that the weak screening approximation always yields the largest enhancement factors, as physically clear due to the fact that electrons and ions are assumed to be free and capable of following the reacting nuclei. By definition there is no enhancement in the NOS model, whereas in TSY model there is a decrease of the reaction rate, as already remarked.

Recent Standard Solar Models calculated by using the weak screening prescription are in agreement with helioseismic constraints on the properties of the convective envelope and on the sound speed profile, see fig. 3. This shows that the weak screening model is in agreement with data and deviations from WES cannot be too large. From the comparison of different models, see Table 2 and fig. 6, one obtains the following results:

i) The difference between the Tsytovitch model (TSY) and the weak screening model (WES) exceeds the “conservative” uncertainty on $u$ in a significant portion of the solar profile. We remark that also the depth of the convective envelope is significantly altered. In other words the anti-screening predictions of ref. [24,25] can be excluded by means of helioseismology.

ii) Also the difference between the no-screening model (NOS) and WES is significant for both $u$ and $R_b$ in comparison with helioseismic uncertainty. In other words the existence of a screening effect can be proved by means of helioseismology.

iii) The Mitler model of screening (MIT) cannot be distinguished from the weak screening model within the present accuracy of helioseismology.

### Table 1

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<th>Reaction</th>
<th>WES</th>
<th>MIT</th>
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<th>TSY</th>
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<td>1.171</td>
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### 6. Concluding remarks

We summarize here the main points of our discussion:

i) The comparison of recent SSMs with helioseismic measurements shows that SSMs prediction are accurate to the level of one per cent or better, although there are indications of some deficiencies at the level of per mille.

ii) Helioseismology directly confirms the production rate of Beryllium neutrinos as predicted by...
Figure 6. Comparison of different screening models with the WES model for $u = P/\rho$. The "1σ" and "3σ" helioseismic uncertainties [1] correspond to the dark and light areas.

SSMs to within ±25% (1σ error).

iii) Models for screening of nuclear reactions in the Sun can be tested by means of helioseismology. In particular, the anti-screening predictions of [24,25] can be excluded. In addition, the existence of a screening effect can be proved by helioseismology.

REFERENCES