EPISODIC SELF-SIMILARITY IN CRITICAL GRAVITATIONAL COLLAPSE

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We report on a new behavior found in numerical simulations of spherically symmetric gravitational collapse in self-gravitating SU(2) \( \sigma \) models at intermediate gravitational coupling constants: The critical solution (between black hole formation and dispersion) closely approximates the continuously self-similar (CSS) solution for a finite time interval, then departs from this, and then returns to CSS again. This cycle repeats several times, each with a different CSS accumulation point. The critical solution is also approximately discretely self-similar (DSS) throughout this whole process.

1 Introduction

As summarized in companion papers in these proceedings (Lechner et al.\(^1\), Thornburg et al.\(^2\)), and described in detail elsewhere (Husa et al.\(^3\)), we are studying critical phenomena in the SU(2) nonlinear \( \sigma \) model in spherical symmetry. This model is parameterized by a dimensionless coupling constant \( \eta \). We denote the matter field by \( \phi = \phi(u, r) \), where \( u \) is an outgoing null coordinate (normalized to proper time at the origin) and \( r \) is the areal radius.

This model is known to have a CSS solution for all \( \eta < 0.5 \). This solution can be explicitly constructed\(^4\), and takes the form \( \phi = \phi_{\text{CSS}}(z; u_\ast) \), where \( z = r/(u_\ast - u) \) and the (only) free parameter \( u_\ast \) gives the location of the accumulation point.

We consider a 1-parameter family of initial data, and fine-tune this parameter so the initial data’s evolution is very close to the threshold of black hole formation. At large (small) \( \eta \) the evolution of such “critical” initial data is DSS (CSS) for a time, before finally either dispersing or collapsing.

However, at intermediate \( \eta \) (\( \approx 0.16 \)) a new behavior appears, which we call “episodic self-similarity”: The field configuration closely approximates a CSS solution, \( \phi \approx \phi_{\text{CSS}}(z; u^{(1)}_\ast) \) in the inner part of the slice for some finite time interval, then departs from CSS, and then returns to closely approximate a CSS solution, \( \phi \approx \phi_{\text{CSS}}(z; u^{(2)}_\ast) \) in the inner part of the slice for another finite time interval, then departs, this cycle repeating several times. The \( u^{(k)}_\ast \) values increase from one CSS episode to the next.

In addition, a large region of the evolution (spanning several CSS episodes) is approximately DSS, but to a much lower degree of approximation than the approx-
imate CSS.

Figure 1 shows an example of this behavior. The field configuration never exactly matches a CSS solution, but on the $u = 15.893$ and $u = 16.414$ slices (where the fit is good and hence $u_* = u_*^{(k)}$ is well-defined), $|\phi - \phi_{CSS}| \lesssim 10^{-2}$ everywhere inside the self-similarity horizon (the backwards light cone of the accumulation point $u_*^{(k)}$). This region of the evolution is DSS to within $\sim 0.05$ in $\phi$.

We do not yet have a full understanding of episodic self-similarity in terms of the standard phase-space model of self-similarity$^5$, but we think this behavior is caused by a competition between nearby CSS and DSS attractors.

References


Figure 1. This figure shows selected $u = \text{constant}$ slices in a numerical evolution of $\eta = 0.16$ critical initial data (●), with the best-fitting CSS solutions (computed independently for each slice) superimposed (—). For each slice where the fit is good and hence $u_* = u_*^{(k)}$ is well-defined, the vertical dashed line shows the CSS solution’s self-similarity horizon.