RECENT DEVELOPMENTS IN SUPERSYMMETRIC DARK MATTER

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A brief review is given of some of the recent developments in the theoretical analyses of supersymmetric dark matter. These include the effects of uncertainties in the wimp velocity and wimp density and of the effects of uncertainties in the quark densities of the proton. Also analyzed are the effects of non-universalities in the gaugino sector and their effects on determining the nature of cold dark matter, i.e., if the neutralino is bino like, higgsino like, or wino like. The maximum and the minimum elastic neutralino proton cross sections are discussed and a comparison of the direct and the indirect detection arising from the capture and annihilation of neutralinos in the core of the earth and the sun is given. Some of the other recent developments are summarized.

1 Introduction

Because of the recent significant activity in dark matter searches on the experimental side\textsuperscript{1,2,3} there is renewed interest in the theoretical analyses of dark matter which are significantly more refined than in the previous years. Among the refinements is the inclusions of the effects of uncertainties in the input parameters in the theoretical predictions of event rates and of the neutralino proton cross sections as well inclusion of the effects of non-universalities, CP violating effects and the effects of coannihilation. The content of the paper is as follows: In Sec.2 we discuss the effects of uncertainties in the analyses of dark matter. These include the effects of uncertainties in the wimp velocity

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and in the wimp relic density, and the effects of uncertainties in the quark densities in the proton in the analyses of dark matter. In Sec.3 we give a discussion of the maximum and the minimum elastic neutralino-proton cross-sections. In Sec.4 we discuss the effects of non-universalities and specifically the non-universalities in the gaugino sector on dark matter analyses. A brief discussion of the effects of $\mu$ on the composition of the neutralino and its role in determining the nature of cold dark matter, i.e., if it is dominantly a bino, a wino or a higgsino is given in Sec.5. A comparison of the direct and the indirect detection of dark matter is given in Sec.6. In Sec.7 we give a discussion of the annual modulation effect in the direct detection of dark matter. In Sec.8 we give a brief discussion of the effects of CP phases on dark matter. Conclusions are given in Sec.9.

## 2 Uncertainties in Theoretical Analyses of Dark Matter

The direct detection of dark matter has been investigated by many authors (see Ref.\textsuperscript{4} for some of the recent works). Several types of uncertainties enter in these analyses such as the effects of variations in the local wimp density, the effects of variations in the wimp velocity range, and the effects of uncertainties in the quark densities in the nucleons. Other effects not considered here are the uncertainties in the nuclear form factors, and effects of halo models\textsuperscript{5} on the event rates in dark matter detection. We begin with a discussion of the analyses of uncertainties in wimp density\textsuperscript{6} and velocity\textsuperscript{7,8,9}. The current range of local wimp density lies in the range\textsuperscript{10} $(0.2 - 0.7) GeVcm^{-3}$. Defining $\xi = \rho_{\chi^0_1}/\rho_0$ one can parameterize the local wimp density in term of $\xi$ which for $\rho_0 = 0.3 GeVcm^{-3}$ gives $0.7 \leq \xi \leq 2.3$. For the wimp velocity one typically
assumes a Maxwellian velocity distribution for the wimps. Estimates of the rms wimp velocity range give $v = 270 \pm 50$ km/s. The analysis is carried out within the framework of the minimal supergravity model (SUGRA). The soft SUSY breaking sector in the minimal version mSUGRA of this theory is given by $m_0, m_{1/2}, A_0$, and $\tan \beta$ where, $m_0$ is the universal scalar mass, $m_{1/2}$ is the universal gaugino mass, $A_0$ is the universal trilinear coupling, and $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$ where $H_2$ gives mass to the up quarks and $H_1$ gives mass to the down quarks and leptons. The Higgs mixing parameter $\mu$ is determined by the constraint of radiative breaking of the electro-weak symmetry. It is also useful to define a fine tuning parameter $\Phi$ so that $\Phi = (\mu^2/M_Z^2 + 1/4)^{1/2}$. The fine tuning parameter defines how heavy the SUSY spectrum gets. The effects of the uncertainly in the event rate as a function of the fine tuning parameter due to variations in the wimp relic density and in the wimp velocity are shown in Fig. 1. One finds that the effects of this type of uncertainty can lead to a variation in the rates by a factor of 2-3.

Next we discuss the uncertainties in neutralino-proton cross-section arising from errors in the quark densities in the proton. The basic interaction governing the $\chi - p$ scattering with CP conservation is the four Fermi interaction given by $L_{eff} = \bar{\chi}\gamma^\mu\gamma^5\chi (AP_L + BP_R)q + C\tilde{\chi}\chi m_q\bar{q}q + D\tilde{\chi}\gamma^5\chi m_q\bar{q}q$. For heavy target materials the neutralino-nucleus scattering is dominated by the scalar interactions which is controlled by the scalar $\chi - p$ cross-section where

$$\sigma_{\chi p}(\text{scalar}) = \frac{4\mu_r^2}{\pi} \left( \sum_{i=u,d,s} f^P_i C_i + \frac{2}{27} (1 - \sum_{i=u,d,c} f^P_i) \sum_{a=c,b,t} C_a \right)^2$$

Here $\mu_r$ is the reduced mass and $f^P_i$ ($i=u,d,s$ quarks) are the (u,d,s) quark densities defined by $m_p f^P_i = \langle p| m_q \bar{q}_i q_i |p \rangle$. In the above C is the strength of the scalar interaction and consists of s channel contributions from the higgs $h^0$, $H^0$ exchange and t channel contributions from the sfermion exchange so that $C = C_{h^0} + C_{H^0} + C_f$. The uncertainly in the theoretical predictions of $\sigma_{\chi p}(\text{scalar})$ is dominated by the uncertainly in the quark densities $f^P_i$. To study the uncertainly in $f^P_i$ it is best first to solve the quark densities analytically in terms of some judiciously chosen parameters. One finds

$$f^P_{(u,d)} = \frac{m_{(u,d)}}{m_u + m_d} \left( 1 \pm \xi \right) \frac{\sigma_{\pi N}}{m_p}, \quad f^P_{s} = \frac{m_s}{m_u + m_d} \left( 1 - x \right) \frac{\sigma_{\pi N}}{m_p}$$

where the parameters $\xi, x$ and $\sigma_{\pi N}$ are defined by
\[ \xi = \frac{<p|\bar{u}u - \bar{d}d|p>}{<p|\bar{u}u + \bar{d}d|p>}, \quad x = \frac{<p|\bar{u}u + \bar{d}d - 2\bar{s}s|p>}{<p|\bar{u}u + \bar{d}d|p>}, \ \sigma_{\pi N} = <p|2^{-1}(m_u + m_d)(\bar{u}u + \bar{d}d)|p> \] (3)

Similarly one can analytically solve for the quark densities in the neutron and the analytic relations provide an interesting connection between the quark densities in the proton and in the neutron. One finds that independent of the details of any input one has\[ f_p^p f_d^d = f_u^n f_d^n. \] One can use the analysis on the baryon mass splittings\[ to determine the ratio \( \xi/x \). One finds\[ \xi/x = 0.196. \]

Using the determination of \( x \) from lattice gauge analyses\[ one finds \( \xi = 0.132 \pm 0.035. \] Additional uncertainties can arise from the quark mass ratios. Here results from chiral perturbation theory\[ give \( \frac{m_u}{m_d} = 0.553 \pm 0.043, \ \frac{m_u}{m_d} = 18.9 \pm 0.8. \] Using the inputs above one finds \( f_u^p = 0.021 \pm 0.004, f_d^p = 0.029 \pm 0.006, f_u^n = 0.21 \pm 0.12 \) and \( f_d^n = 0.016 \pm 0.003, f_d^n = 0.037 \pm 0.007, \ f_u^n = 0.21 \pm 0.12. \]

3 Maximum and Minimum Neutralino-proton Cross Sections

We discuss now the numerical results of the neutralino proton cross-sections with the quark densities as discussed in Sec. 2. In Fig.2 \( \sigma_{\chi - p} \) is plotted exhibiting the effects of the variations of the quark density as a function of \( m_{\chi} \). The above analysis shows that the \( \chi - p \) cross section cannot be computed to an accuracy of better than a factor of 3-5 with the current level of uncertainties in the input data. Fig.2 also gives a plot of the maximum and the minimum of \( \sigma_{\chi - p} \) as a function of the neutralino mass. In the analysis of Fig.2 we have
allowed $m_0$ and $m_{1/2}$ to vary in the range up to 1 TeV and the constraint from the flavor changing neutral current process $b \rightarrow s\gamma$ is imposed\textsuperscript{24}. The analysis of this figure does not include the effects of the spin dependent contribution which could change the scatter plot. Specifically, the minimum cross sections are sensitive to the inclusion of the spin dependent part. Similar analyses can be found in other recent references\textsuperscript{25,26}.

4 Effects of Non-universalities on Dark Matter

mSUGRA is based on the assumption of a flat Kahler potential. However, the nature of physics at the Planck scale is not fully understood. Thus in general one should allow for the possibility of a curved Kahler potential. This would lead to non-universalities in the scalar sector. However, there are stringent constraints on the types of non-universalities allowed in the scalar sector due to the FCNC constraint. For example, the FCNC constraint in very strong on the amount of non-universality allowed in the first vs the second generation sector. However, this constraint is not so strong for the Higgs sector and for the third generation sector. Effects of these constraints have been studied in detail in Refs.\textsuperscript{27,28}. One finds that in general the presence of non-universalities can increase the cross-sections by a factor of 10 or more.

In addition to modifying the Kahler potential Planck scale physics can also modify the gauge kinetic energy function. In general the gauge kinetic energy function transforms as the symmetric product of two adjoint representations. For the case of SU(5) one finds that the gauge kinetic energy function $f_{\alpha\beta}$ transforms as the symmetric product of $24 \times 24$ in SU(5) which contains the following irreducible representations of SU(5):

\begin{equation}
(24 \times 24)_{\text{symm}} = 1 + 24 + 75 + 200
\end{equation}

The SU(5) singlet in the product on the right hand side of Eq.(4) leads to a universal gaugino mass while, the additional terms generate non-universalities in the gaugino masses at the GUT scale. Thus the $SU(3)_C \times SU(2)_L \times U(1)$ gaugino masses at the GUT scale are in general admixtures of all the allowed representations. This admixture leads to the following relation for the gaugino masses at the GUT scale

\begin{equation}
\tilde{m}_i(0) = m_\perp(1 + \sum_c c_r n_i^r)
\end{equation}

where $n_i^r$ depend on one of the representations on the right hand side in Eq.(4) and on the subgroup $i$. In addition to the appearance of gaugino mass
non-universalities of the above type in supergravity models, one also finds quite naturally non-universalities in a broad class of string models: heterotic, Horava-Witten and as well in brane models based on TypeI/Type IIB string compactifications. Now the value of $\mu$ is in general very sensitively dependent on non-universalities. For the case of non-universalities in the scalar sector one can exhibit in an analytic fashion the dependence of $\mu$ on non-universalities in the Higgs sector and in the third generation sector. The analysis shows that $\mu$ is sensitively dependent on the non-universalities and their effects can significantly decrease the value of $\mu$ and thus affect gaugino vs higgsino composition of the neutralino. An analysis of the effects of non-universalities in the scalar sector is given in Ref.27. A similar phenomenon occurs for the case of gaugino sector non-universalities. Here also one can exhibit the dependence of $\mu$ on non-universalities. One finds

$$\tilde{\mu}^2 = \mu_0^2 + \sum_r \frac{\partial \tilde{\mu}^2}{\partial c_r} c_r + O(c_r^2)$$

(6)

where $\partial \mu_{24}^2 / \partial c_{24} > 0, \partial \mu_{75}^2 / \partial c_{75} > 0, \partial \mu_{200}^2 / \partial c_{200} < 0$. Again with an appropriate choice of the sign of $c_r$ one finds that $\mu$ becomes smaller relative to its universal value. An analysis of the effects of non-universalities in the gaugino sector on $\sigma_{\chi-p}$ is given in Ref.16. Fig.3 gives a typical illustration of the effects of gaugino non-universality showing that a significant enhancement of $\sigma_{\chi-p}$ can occur in the presence of non-universalities.

Figure 3: Exhibition of the effects of non-universalities on $\sigma_{\chi-p}$ as a function of the neutralino mass for different values of $c_{200}$. 
5 Bino, Higgsino, and Wino Dark Matter

The neutralino is in general a mixture of gauginos and higgsinos,

\[ \chi_1 = X_{11}\tilde{B} + X_{12}\tilde{W} + X_{13}\tilde{H}_1 + X_{14}\tilde{H}_2 \]  

(7)

where \( \tilde{B} \) is the bino, \( \tilde{W} \) the neutral wino, and \( \tilde{H}_1, \tilde{H}_2 \) are higgsinos. Now an analysis of the neutralino mass matrix shows that for large values of the Higgs mixing parameter \( \mu \) one finds that the neutralino is essentially a bino and this situation is realized over a large part of the SUGRA parameter space\(^29\). Thus SUGRA models in general predict a bino like cold dark matter at least over a major portion of the parameter space. However, in certain limited regions of the SUGRA parameter space \( \mu \) can become small. In this case the higgsino and the wino components can become large and one may have cold dark matter which is higgsino like or wino like. This phenomenon can be easily understood by examining the \( |\mu| >> M_Z \) limit of the components \( X_{1n} \). Here one finds that the bino component in the neutralino is given by \( X_{11} \approx 1 - (M_Z^2/2\mu^2)\sin^2\theta_W \), the wino component is of size \( X_{12} \approx (M_Z^2/2m_{\chi}^2, \mu)\sin 2\theta_W \sin \beta \), and the higgsino components have the sizes \( X_{13} \approx -(M_Z/\mu)\sin 2\theta_W \sin \beta \), and \( X_{14} \approx (M_Z/\mu)\sin 2\theta_W \sin \beta \). The above illustrates the sensitive dependence of the bino, the wino and the higgsino components on \( \mu \). One consequence of the effect of large higgsino components is that the scalar cross section which depends on the product of the gaugino and the higgsino components increases as the higgsino components increase\(^27\).

An analysis of CDM with large higgsino components in the context of MSSM is given in Ref.\(^30\) and with large higgsino and wino components in the context of anomaly mediated breaking of supersymmetry in Ref.\(^31\)

6 Comparison of Direct and Indirect Detection

Indirect detection is complementary to the direct detection in the search for dark matter. The most interesting indirect signal for dark matter arises from the capture and the subsequent annihilation of the neutralinos in the center of the Sun and the Earth. Some of the remnants in the annihilation of the neutralinos are the neutrinos which propagate and undergo charged current interactions in the rock surrounding the detector and produce upward moving muons. The outgoing muon flux can be written in the form \( \Phi_\mu = \Gamma_A f \) where \( \Gamma_A \) is the \( \chi_1 - \chi_1 \) annihilation rate in the center of the Earth or the Sun and \( f \) is the product of remaining factors. It is \( \Gamma_A \) which is the quantity sensitive...
to SUSY. One can parameterize $\Gamma_A$ by

$$\Gamma_A = \frac{C}{2} \tanh^2(t/\tau)$$

(8)

Here $C$ is the neutralino capture rate, $t$ is the lifetime of the Earth or the Sun, and $\tau = (CC_A)^{-1/2}$ where $C_A$ is determined by the wimp annihilation cross section. An equilibrium between capture and annihilation is reached for the case $t >> \tau$ and in this case one has $\Gamma_A \sim \frac{C}{2}$. The equilibrium condition is strongly dependent on the susy parameter space. For the case of the sun the equilibrium condition is satisfied essentially over all of the parameter space while for the Earth it is satisfied only over part of the parameter space. This disparity shows up very strongly in the profile of the muon flux from the Earth and the Sun. In Fig.4 we exhibit the muon flux for the Earth and the Sun plotted against the direct detection rate for the germanium detector. The analysis of Fig.4 shows that for relatively large direct detection rates, it is the indirect detection from the Earth which is competitive with the direct detection while for relatively low direct detection rates it is the indirect detection from the Sun which is competitive with the direct detection. Thus indirect detections from the Earth and the Sun are complementary. For other recent analyses of indirect detection see Refs.33,34,35.

7 Annual Modulation Effect in Direct Detection

The annual modulation effect in the direct detection of dark matter is a potentially important signal for the observation of WIMP like dark matter. The effect arises due to the periodicity of the velocity of the Earth $v_E$ relative to the galaxy, i.e., $v_E = v_S + v_0 \cos \gamma \cos \omega (t - t_0)$ where $v_0 (= 30 \text{ km/s})$ is Earth's
orbital velocity around the Sun, $v_S (= 232 \text{ km/s})$ is the Sun’s velocity relative to the galaxy, $\gamma (\approx 60^0)$ is the inclination of Earth’s orbit relative to the galactic plane, and $t_0$ is June 2. The effects of this motion can produce a modulation effect of about 7% in the scattering event rates for wimps. The DAMA experiment claims to see an effect\(^1\). The annual modulation signal has been analyzed theoretically in several papers\(^{36,6,37,38}\). In the future one expects further data on this exciting possibility. Further, one needs to reduce the current ambiguities in the theoretical predictions of dark matter such as those discussed in Sec.2. This requires more accurate chiral perturbation and lattice gauge analyses.

8 Effects of Large CP Phases on Dark Matter

It is well known that supersymmetry brings in new sources of CP violation since the soft SUSY parameters are in general complex. These CP phases induce additional corrections to the electric dipole moments of the electron and of the neutron. We consider here the case where the CP phases are large and the edm constraints are satisfied. (For an abbreviated set of references on large phases see Ref.\(^{39}\)). Large phases affect the analysis of dark matter and the detailed analyses of event rates for direct detection show that the effects of the phases can change the event rates by an order of magnitude or more\(^{40,41}\). However, with the inclusion of the edm constraints one finds that the effects are significantly reduced although they are still substantial\(^{41}\). Large phases also induce a mixing between the CP even and the CP odd higgs states\(^{42,43}\) which can affect dark matter\(^{43}\).

9 Conclusion

In this review we have presented the very recent developments in the theoretical analyses of supersymmetric dark matter. These include the effects of uncertainties on the wimp density and on the wimp velocity for the Milky Way wimps, effects of uncertainties in quark densities in the theoretical predictions of $\sigma_{\chi-p}$, and the effects of non-universalities in the gaugino sector. A analysis of the maximum and of the minimum cross sections for $\chi - p$ scattering was given. We have also given a comparison of the direct and of the indirect detection of dark matter and discussed briefly other recent developments for the detection of dark matter. An important topic not discussed is the subject of coannihilation\(^{44}\) which can significantly extend the domain of the allowed neutralino masses. In the future one expects that more sensitive detectors\(^{15,46}\).
will be able to probe more deeply the parameter space of SUGRA and other
SUSY models.

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