Hyperon Polarization in Inclusive Hadronic Production

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Abstract. A QCD formula for the polarization in the large-\(p_T\) \(\Lambda\) hyperon production in the unpolarized nucleon-nucleon collision at large \(x_F\) is derived. We focus on the mechanism in which the chiral-odd spin-independent twist-3 quark distribution \(E_F(x, x)\) becomes the source of the transversely polarized quarks fragmenting into the polarized \(\Lambda\). A simple model estimate for that contribution shows the possibility that it gives rise to a sizable \(\Lambda\) polarization.

It is a well known experimental fact that the hyperons produced in the unpolarized nucleon-nucleon collisions are polarized transversely to the production plane \([1,2]\). In this letter we focus on the polarization of the \(\Lambda\) hyperon production with large transverse momentum in \(pp\) collision

\[ N(P) + N'(P') \rightarrow \Lambda(l, \vec{S}_\perp) + X. \]  

(1)

Ongoing experiment at RHIC is expected to provide more data on the polarization. The nonzero \(\Lambda\) polarization in this process requires the presence of particular quark-gluon correlation (higher twist effect) and/or the effect of transverse momentum either in the unpolarized nucleon or the fragmentation function for \(\Lambda\). According to the generalized QCD factorization theorem, the polarized cross section for this process consists of two kinds of twist-3 contributions:

- \(E_a(x_1, x_2) \otimes q_b(x') \otimes \delta D_{c \rightarrow \Lambda}(z) \otimes \hat{\sigma}_{ab \rightarrow c}, \)  
  \((2)\)
- \(q_a(x) \otimes q_b(x') \otimes D_{c \rightarrow \Lambda}^{(3)}(z_1, z_2) \otimes \hat{\sigma}_{ab \rightarrow c}. \)
  \((3)\)

Here the functions \(E_a(x_1, x_2)\) and \(D_{c \rightarrow \Lambda}^{(3)}(z_1, z_2)\) are the twist-3 quantities representing, respectively, the unpolarized distribution in the nucleon and the fragmentation function for the transversely polarized \(\Lambda\) hyperon, and \(a, b\) and \(c\) stand for the parton’s species. Other functions are twist-2; \(q_b(x)\) the

unpolarized distribution (quark or gluon) and $\delta D_{c\rightarrow A}(z)$ the transversity fragmentation function for $A$. The symbol $\otimes$ denotes convolution. $\delta_{ab\rightarrow c}$ etc represents the partonic cross section for the process $a + b \rightarrow c +$ anything which yields large transverse momentum of the parton $c$. Note that (A) contains two chiral-odd functions $E_a$ and $\delta D_{c\rightarrow A}$, while (B) contains only chiral-even functions.

In this report, we derive a QCD formula for the polarized cross section (1) from the (A) term in the kinematic region $|x_F| \rightarrow 1$, using the valence quark-soft gluon approximation proposed by Qiu and Sterman [3]. Employing this approximation, they reproduced the E704 data for the single-transverse spin asymmetries in the pion production at $x_F \rightarrow 1$ reasonably well. The fact that the perturbative QCD description for the pion production is valid as low as $l_T \sim 1$ GeV encouraged us to apply the method to the polarized $\Lambda$ hyperon production (1) for which the data exist only in the relatively small $l_T$ region. At large $x_F > 0$, which mainly probes large $x$ and small $x'$ region, the cross section is dominated by the particular terms in (A) which contain the derivatives of the valence twist-3 distribution $E_{Fa}(x,x)$. The reason for this observation is the relation $|\frac{\partial}{\partial x} E_{Fa}(x,x)| \gg E_{Fa}(x,x)$ owing to the behavior of $E_{Fa}(x,x) \sim (1 - x)^$ (β > 0) at $x \rightarrow 1$. We thus keep only the terms with the derivative of $E_{Fa}$ for the valence quark (valence quark-soft gluon approximation).

The polarized cross section for (1) is a function of three independent variables, $S = (P + P')^2 \approx 2P \cdot P'$, $x_F = 2l_T/\sqrt{S} (= (T - U)/S)$, and $x_T = 2l_T/\sqrt{S}$. $T = (P - l)^2 \approx -2P \cdot l$ and $U = (P' - l)^2 \approx -2P' \cdot l$ are given in terms of these three variables by $T = -S \left[ \sqrt{x_F^2 + x_T^2} - x_F \right]/2$ and $U = -S \left[ \sqrt{x_F^2 + x_T^2} + x_F \right]/2$. In this convention, production of $\Lambda$ in the forward hemisphere in the direction of the incident nucleon $(N(P))$ corresponds to $x_F > 0$. Since $-1 < x_F < 1$, $0 < x_T < 1$ and $\sqrt{x_F^2 + x_T^2} < 1$, $x_F \rightarrow 1$ corresponds to the region with $-U \sim S$ and $T \sim 0$.

In the valence quark-soft gluon approximation, the cross section for the (A) term reads,

$$E_l \frac{d^3 \Delta \sigma^A(S\perp)}{d l^3} = \frac{\pi M^2}{S} \sum_{a,c} \int_{z_{\mathrm{min}}}^{1} \frac{dz}{z^3} \delta D_{c\rightarrow A}(z) \int_{x_{\mathrm{min}}}^{1} \frac{dx}{x} \frac{1}{xS + U/z}$$

$$\times \int_{0}^{1} \frac{dx'}{x'} \delta \left( x' + \frac{xT/z}{xS + U/z} \right) x_{S\perp} \left[ \frac{1}{-\hat{u}} \right] \left[ -x \frac{\partial}{\partial x} E_{Fa}(x,x) \right]$$

$$\times \left[ G(x') \delta \tilde{a}_{g-\perp} + \sum_{b} q_b(x') \delta \tilde{a}_{ab-\perp} \right], \quad (4)$$

where $p$ and $n$ are the two light-like vectors defined from the momentum of the unpolarized nucleon as $P = p + M^2 n/2$, $p \cdot n = 1$ and $x_{S\perp} = x_{\mu\nu\lambda\delta} T^\mu S^\nu T^\lambda S^\delta \sim$
sin$\phi$ with $\phi$ the azimuthal angle between the spin vector of the $\Lambda$ hyperon and the production plane. The invariants in the parton level are defined as $s = (p_a + p_b)^2 \simeq (xP + x'P')^2 \simeq xx'S$, $t = (p_a - p_c)^2 \simeq (xP - l/z)^2 \simeq xT/z$, $u = (p_b - p_c)^2 \simeq (x'P' - l/z)^2 \simeq x'U/z$. The lower limits for the integration variables are $z_{\text{min}} = \frac{-\l(l+u)}{s} = \sqrt{x_T^2 + x_T'^2}$ and $x_{\text{min}} = \frac{-u/z}{s+y/z}$. $q_b(x')$ is the unpolarized quark distribution, and $G(x')$ is the unpolarized gluon distribution. $\delta\sigma_{ag\rightarrow c}$ and $\delta\sigma_{ab\rightarrow c}$ are partonic cross sections for the quark-gluon and quark-quark processes, respectively. $E_F(x, x)$ is the soft gluon component of the unpolarized twist-3 distribution defined as

$$E_{Fa}(x, x) = \frac{-i}{2M} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|\bar{\psi}^a(0) \gamma_\perp \gamma_\sigma \{ \int \frac{d\mu}{2\pi} gF^{\sigma\beta}(\mu n) \psi^\beta(\lambda n)|P\rangle. \quad (5)$$

The summation for the flavor indices of $E_{Fa}(x, x)$ is to be over $u$- and $d$-valence quarks, while that for the twist-2 distributions is over $u, d, \bar{u}, \bar{d}, s, \bar{s}$. $\delta\sigma_{ab\rightarrow c}$ and $\delta\sigma_{ag\rightarrow c}$ can be obtained from the $2 \rightarrow 2$ cut diagrams. The result reads

$$\delta\sigma_{qq'\rightarrow q} = \left( \frac{\hat{s}u}{\hat{t}^2} \right) \left[ \frac{2}{9} + \frac{1}{9} \left( 1 + \frac{\hat{u}}{\hat{t}} \right) \right], \quad \delta\sigma_{qq'\rightarrow q} = \left( \frac{7}{9} + \frac{1}{9} \left( 1 + \frac{\hat{u}}{\hat{t}} \right) \right), \quad \delta\sigma_{qq\rightarrow q} = \left( \frac{10}{27} + \frac{1}{27} \left( 1 + \frac{\hat{u}}{\hat{t}} \right) \right), \quad (6)$$

for $\delta\sigma_{ab\rightarrow c}$, and

$$\delta\sigma_{ag\rightarrow c} = \frac{9}{8} \left( \frac{\hat{s}u}{\hat{t}^2} \right) + \frac{9}{8} \left( \frac{\hat{u}}{\hat{t}} \right) + \frac{1}{8} \left[ \frac{1}{4} \left( \frac{\hat{s}u}{\hat{t}^2} \right) + \frac{1}{4} \left( 1 + \frac{\hat{u}}{\hat{t}} \right) \right] \left( 1 + \frac{\hat{u}}{\hat{t}} \right). \quad (7)$$

We now present a simple estimate of the $\Lambda$ polarization. To this end we use a model for $E_F(x, x)$ introduced in Ref. [7]. It is based on the comparison of the explicit form (5) with the transversity distribution

$$\delta q_a(x) = \frac{i}{2} \varepsilon_{S\perp \sigma \mu} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}^a(0) \gamma_\perp \gamma_\mu \psi^\mu(\lambda n)|PS\rangle, \quad (8)$$

where $\varepsilon_{S\perp \sigma \mu} \equiv \varepsilon_{\mu\nu\lambda\sigma} S^\perp_{\mu \nu} n^\lambda$. We make an ansatz

$$E_{Fa}(x, x) = K_a \delta q_a(x), \quad (9)$$

with a flavor-dependent parameter $K_a$ which simulates the effect of the gluon field with zero momentum in $E_F(x, x)$. We note that even though $E_F(x, x)$ is an unpolarized distribution, the quarks in $E_F(x, x)$ is “transversely polarized” which eventually fragments into the transversely polarized $\Lambda$. The relation (9) is in parallel with the ansatz originally introduced in [3]

$$G_F(x, x) = K_a q_a(x). \quad (10)$$
FIGURE 1. (a): The R608 data for the polarization $P_\Lambda$ of the produced $\Lambda$ hyperon in the unpolarized proton-proton collision at $\sqrt{S} = 62$ GeV [2]. The transverse momentum of the $\Lambda$ is $l_T = 1.1$ GeV. The curves show the calculated polarization with three scenarios for $\delta D_{c \rightarrow \Lambda}$.

(b): The $\Lambda$ polarization in the unpolarized proton-proton collision at $\sqrt{S} = 40, 80, 200$ GeV with $l_T = 1.1$ GeV. Scenario 2 is used for $\delta D_{c \rightarrow \Lambda}$.

Here we assume the same parameter for $K_u$ in (9) and (10) [7]. Since we are only interested in the estimate of order of magnitude, we do not pay much attention to the scale dependence of each distribution and fragmentation function and use those functions at the scale $1 \sim 2$ GeV which is a typical size of the transverse momentum of the produced $\Lambda$ and pion. For the unpolarized distribution $q_a(x)$ and $G(x)$, we use the GRV LO distribution at the scale $\mu = 1.1$ GeV ($= l_T$ of $\Lambda$ in R608 data below) [4]. For the transversity distribution $\delta q_a(x)$, we use the GRSV helicity distribution $\Delta q_a(x)$ (LO, standard scenario) [5] assuming $\delta q_a(x) = \Delta q_a(x)$ at the scale $\mu = 1.1$ GeV. For the fragmentation function of the unpolarized and transversely polarized $\Lambda$, we use, respectively, the fragmentation function of the unpolarized and longitudinally polarized $\Lambda$, $D_{c \rightarrow \Lambda}(z)$ and $\Delta D_{c \rightarrow \Lambda}(z)$ (three scenarios for $\Delta D$), given by de Florian et al. [6] with the assumption that $\delta D_{c \rightarrow \Lambda}(z) = \Delta D_{c \rightarrow \Lambda}(z)$ at the scale $\mu = 1.1$ GeV. Following our recent paper [7], we determine $K_{u,d}$ to fit the FNAL E704 data of the single-transverse spin asymmetry in the pion production [8] using $G_{Fa}(x,x)$ with (10) at the scale $\mu = 1.5$ GeV and the fragmentation function of the pion given at $\mu = 2$ GeV in [9]. The result is $K_u = -K_d = 0.06$. The obtained $\Lambda$ polarization is shown in Fig. 1(a) for the three scenarios of $\delta D_{c \rightarrow \Lambda}(z)$ in [6] with the CERN R608 data [2]. The scenario 1 corresponds to the expectation from the naive non-relativistic quark model, where only strange quarks can fragment into a polarized $\Lambda$. In our approximation, $E_F(x,x) = 0$ for the $s$-quark and thus the polarization is zero in this scenario. The scenario 2 is based on the assumption that the
flavor-dependence of $\Delta D_{c\to\Lambda}(z)$ is the same as that of the polarized quark distributions in $\Lambda$ obtained by the SU(3) symmetry as proposed in Ref. [10]; $\delta D_{u\to\Lambda} = \delta D_{d\to\Lambda} = -0.2\delta D_{s\to\Lambda}$. In the scenario 3, three flavors of quarks equally fragment into the polarized $\Lambda$; $\delta D_{u\to\Lambda} = \delta D_{d\to\Lambda} = \delta D_{s\to\Lambda}$. From this figure, one sees that both scenario 2 and 3 give rise to increasing polarization at large $x_F$ as expected, the former being slightly more favored because of the sign of the polarization. For a complete understanding on the hyperon polarization, combined analysis of both (A) and (B) contributions together with more sophisticated parametrization of the participating distributions and fragmentation functions is certainly necessary.

In Fig. 1(b), we plotted the polarization from the term (A) for various values of $\sqrt{S}$ at $b_T = 1.1$ GeV with scenario 2 for $\delta D_{c\to\Lambda}$. One sees that the result is almost independent of the value of $\sqrt{S}$ in this kinematic region. This tendency is the same as the experimental data.

A different approach to the $\Lambda$ polarization introduces the so-called T-odd distribution or fragmentation functions with the intrinsic transverse momentum instead of twist-3 distributions introduced here. Similarly to (A) and (B), this approach starts from the factorization assumption for the two types of contributions to the polarization; (i) $h_{\perp 1}(x, p_{\perp}) \otimes q(x') \otimes \delta D(z) \otimes \hat{\sigma}$; (ii) $q(x) \otimes q(x') \otimes D_{\perp T 1}(z, k_{\perp}) \otimes \hat{\sigma}'$, where $h_{\perp 1}$ represents distribution of a transversely polarized quark with nonzero transverse momentum inside the unpolarized nucleon, and $D_{\perp T 1}$ represents a fragmentation function for an unpolarized quark fragmenting into a transversely polarized $\Lambda$ with the transverse momentum (“polarizing fragmentation function”). Anselmino et al. fitted the experimental data for the $\Lambda$ polarization assuming the above (ii) is the sole origin of the polarization [11]. We expect from the present study, however, that the large portion of the $\Lambda$ polarization should be ascribed to the twist-3 distribution in the unpolarized nucleon and $\delta D(z)$ which should be related to the above contribution (i). It is interesting to explore the connection between the present approach and that in [11].

To summarize, we have derived a cross section formula for the polarized $\Lambda$ production in the unpolarized nucleon-nucleon collision at large $x_F$. A simple model estimate for this contribution suggests a possibility that the contribution from the soft gluon pole gives sizable $\Lambda$ polarization.

REFERENCES